Negation
Datalog with Negation

Okay. Let us add "not" to the Datalog language (Datalog¬).

E.g.,

\[
\text{cousin} \ (X, \ Y) \leftarrow \text{grandparent} \ (?, \ X), \\
\text{grandparent} \ (?, \ Y), \\
X \neq Y, \\
\text{not} \ \text{sibling} \ (X, \ Y).
\]

We only allow use of "not" on the right-hand side of the "\leftarrow".

The intuitive meaning of "not" is quite clear.

How to handle it formally is far from clear.

- What are the models of a Datalog¬ database?
- What should the proof procedure be for Datalog¬?

This "not" is not logical negation (\lnot).!

Safeness
Extended for Datalog¬

We require that Datalog¬ programs be safe.

We need to extend the definition of safeness for Datalog¬:

Any variable that appears either in the head atom of the rule (on the left-hand side) or in a negated atom must also appear in a non-negated atom in the body (on the right-hand side). Thus,

\[
h \ (X_1, \ldots, X_k) \leftarrow b_1 \ (Y_1, \ldots, Y_{j_1}), \ldots, b_m \ (Y_{j_m-1}, \ldots, Y_{j_m}), \\
\text{not} \ a'_{1} \ (Z_1, \ldots, Z_{j_1}), \ldots, \text{not} \ a'_{n} \ (Z_{j_n-1}, \ldots, Z_{j_n}),
\]

is safe if

\[
\{X_1, \ldots, X_k\} \cup \{Z_1, \ldots, Z_{j_n}\} \subseteq \{Y_1, \ldots, Y_{\overline{j_m}}\}
\]
Non-Monotonicity

Non-classical Logic!

Adding a new fact may or may not require that we retract other things that we used to know.

\[ \mathcal{P} : \ a \leftarrow b, \ \text{not} \ c. \]

From \( \mathcal{P} \), a follows.

\[ \mathcal{P}' : \ a \leftarrow b, \ \text{not} \ c. \]

\[ \mathcal{P}' : \ b. \]

However, from \( \mathcal{P}' \), a does not follow. In fact, we want to say that \( \neg a \) follows.

Classical logic is monotonic. Thus this is a change from classical logic.

This also means that what we have in mind for “not” really is different from classical negation (\( \neg \)).

Stratification

No cycles through “not”

The Grandmother database is \textit{statically stratified}, even with the predicate \textit{cousin}.

A program is \textit{statically stratified} iff the predicates can be ordered such that no predicate employs another predicate negated that appears before it in the ordered list.

\[ \text{integer} (0). \]

\[ \text{integer} (I \leftarrow \text{integer} (J), \ I \ is \ J+1). \]

\[ \text{even} (0). \]

\[ \text{even} (I \leftarrow \text{integer} (J), \ I > 0, \ J \ is \ I-1, \ \text{not even} (J). \]

\[ \text{odd} (I \leftarrow \text{integer} (J), \ \text{not even} (J). \]

This odd-even program is clearly not statically stratified. However, it is \textit{locally stratified}.

A program is \textit{locally stratified} iff for any ground atom \( A \) (e.g., \( \text{even} (7) \)), it is not possible for the negation of atom \( A \) (e.g., \( \text{not even} (7) \)) to appear in a resolution path from \( A \).

In other words, no “proof” of \( A \) relies on \( \neg A \).
The Perfect Model
For Stratified Datalog→ Programs

Just as there is one minimum model for a Datalog program, there exists one special model named the perfect model for each Datalog→ program.

Let $P$ denote the perfect model of program $\mathcal{P}$. The interpretation in which $A$ is assigned true when $A \in P$ and is assigned false when $A \notin P$ is a model of $\mathcal{P}$ (in which the not's are treated as logical $\neg$), and is, in a sense, minimal.

Negation-as-finite-failure (NAF) remains a sound proof strategy for stratified datalog→ programs.

Non-mon Negation in Datalog→
Extends Expressiveness

Modeling

- Can ask queries with negative components.
- Can express many views (e.g., cousins) that we cannot in Datalog.
- Can model databases more succinctly

Towards capturing SQL

- Of course, we now can do except.
- Can express aggregation using not.

NULLs and full-bladed arithmetic in SQL are still a problem.
Negation

Example: Game of Peggy

The game of Peggy is played by two players with a pile of \( k \) coins.

- The players alternate turns.
- On a player’s turn, the player removes one, two, or three coins.
- If only one coin remains, the player whose turn it is must take it.
- The player to take the last coin loses. (And thus the other player is the winner.)

Generic.

\[
\text{win} (X) \leftarrow \text{move} (X, Y), \text{not win} (Y).
\]

Peggy Rules.

\[
\text{move} (X, Y) \leftarrow X \geq 1, Y \equiv X - 1.
\]

\[
\text{move} (X, Y) \leftarrow X \geq 2, Y \equiv X - 2.
\]

\[
\text{move} (X, Y) \leftarrow X \geq 3, Y \equiv X - 3.
\]

\[
\text{win} (Y).
\]

Non-monotonic Negation in Datalog—

Computationally Expensive!

Why? To prove a \texttt{not}, one must show that \textit{every} possible proof path fails.

\[
\begin{align*}
\text{state(11) wins because} & \text{state(9) loses,} \\
\text{state(9) loses because} & \text{state(8) wins,} \\
& \text{state(8) wins because state(5) loses,} \\
& \text{state(5) loses because} \\
& \text{state(4) wins,} \\
& \text{state(4) wins because state(1) loses,} \\
& \text{state(1) loses because} \\
& \text{that is all,} \\
& \text{AND} \\
& \text{state(3) wins,} \\
& \text{state(3) wins because state(1) loses,} \\
& \text{state(1) has been shown to lose,} \\
& \text{AND} \\
& \text{state(2) wins,} \\
& \text{state(2) wins because state(1) loses,} \\
& \text{state(1) has been shown to lose,} \\
& \text{AND} \\
& \text{that is all,} \\
& \text{AND} \\
& \text{state(7) wins,} \\
& \text{state(7) wins because state(5) loses,} \\
& \text{state(5) has been shown to lose,} \\
& \text{AND} \\
& \text{state(6) wins,} \\
& \text{state(6) wins because state(5) loses,} \\
& \text{state(5) has been shown to lose,} \\
& \text{AND} \\
& \text{that is all.}
\end{align*}
\]
Non-Stratified

Of course there are Datalog\(-\) programs (databases) that are not even locally stratified.

\[ a \leftarrow \text{not} \ b. \]

\[ b \leftarrow \text{not} \ a. \]

Do we ever need a non-stratified Datalog\(-\) program?

Unfortunately, there are natural cases.

Also, the decision problem to determine whether an arbitrary Datalog\(-\) program is locally stratified is undecidable.

For non-stratified Datalog\(-\) programs:

- What is the semantics?
  - Well, we have choices...
- What is the proof procedure?
  - NAFT no longer works.