## EECS 4422/5323

## Unit 7: Spatiotemporal analysis

## Outline

- Introduction
- Orientation in visual space-time
- A representation for spatiotemporal patterns
- Spatiotemporal boundaries
- A framework for spatiotemporal analysis
- Applications
- Summary


## Introduction

- We have considered the analysis of spatial structure.
- Oriented, bandpass representations.
- We have considered analysis of the temporal dimension
- Motion
- Now we consider the integrated analysis and interpretation of the spatial and temporal dimensions.
- Spatiotemporal analysis


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## Orientation in visual space-time: Basics

- The local orientation (or lack thereof) of a pattern is one of its most salient characteristics.
- Geometrically, orientation captures the local firstorder structure of a pattern.
- For vision, local spatiotemporal orientation can have additional interpretations.
- Image velocity is manifest as spatiotemporal orientation.
- And more...


## Orientation in visual space-time: Graphic



## Orientation in visual space-time: Representation

- Goal is to analyze spatiotemporal data according to its local orientation structure.
- Consider orientation in x-t and y-t planes, with local weighted averaging in orthogonal spatial dimension.
- Filter for multiple bands each tuned for certain orientations in a spatiotemporal plane.
- For example, select 4 orientations/plane: horizontal, vertical, 2 diagonals
- Consider single spatiotemporal scale (for now).


## Orientation in visual space-time: Filtering

- Apply filters tuned to 4 different
orientations in both $x-t$ and $y$-t domains.
- In general, might consider additional directions.
- Filter specifics:
- Oriented bandpass filters in spatiotemporal slice.
- Lowpass filter in orthogonal spatial dimension.
- Pointwise squared to yield local "oriented energy".



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## Orientation in visual space-time: Normalization

- For any given orientation, the filter response is a joint function of
- orientation
- contrast
- Normalization yields purer measure of orientation

$$
R(x, y, t)=\frac{r(x, y, t)}{r(x, y, t)+l(x, y, t)+s_{x}(x, y, t)+f_{x}(x, y, t)+\varepsilon}
$$

with 3 a small bias added for stability.

- Similarly for $l, s_{x}, f_{x}$ and their y -t counterparts.


## Orientation in visual space-time: Velocity

- Consider the response to a horizontally moving pattern of $r, l$ and $s$
 filters.


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## Orientation in visual space-time: Velocity

- Consider the response to a horizontally moving pattern of $r, l$ and $s$ filters.
- By taking the difference $r-l$ we get a single response that is properly signed WRT velocity.
- Dividing to get $(r-l) / s$ yields a response that is (approximately) linear
 with velocity.


## Orientation in visual space-time: Velocity

Comparison with optical flow constraint equation (OFCE)

- Recall the OFCE as derived from constant brightness assumption

$$
E_{x} u+E_{y} v+E_{t}=0
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## Orientation in visual space-time: Velocity

Comparison with optical flow constraint equation (OFCE)

- Recall the OFCE as derived from constant brightness assumption

$$
E_{x} u+E_{y} v+E_{t}=0
$$

- Let us restrict consideration to one spatial dimension + time

$$
E_{x} u+E_{t}=0
$$

- Now, we can directly solve for (1D) velocity

$$
u=-E_{t} / E_{x}
$$

## Orientation in visual space-time: Velocity

Comparison with optical flow constraint equation (OFCE)

- In contrast, let us consider simple differential filters for leftward and rightward movement


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## Orientation in visual space-time: Velocity

Comparison with optical flow constraint equation (OFCE)

- In contrast, let us consider simple differential filters for leftward and rightward movement

$$
l=\left(E_{t}+E_{x}\right) / 2 \quad r=\left(E_{t}-E_{x}\right) / 2
$$

- Indeed, as we are concerned not with sign for a given direction, but rather magnitude, it suffices to consider the squared filter responses

$$
l=\left(E_{t}^{2}+2 E_{x} E_{t}+E_{x}^{2}\right) / 4 \quad r=\left(E_{t}^{2}-2 E_{x} E_{t}+E_{x}^{2}\right) / 4
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- Finally, to avoid being biased by locally large values of image contrast, we divide through by the square of a first-order measure of local contrast $S=E_{x}^{2}$

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(r-l) / s=-E_{t} / E_{x}
$$

- We now recognize that this computation of velocity is equivalent to that based on the OFCE

$$
E_{x} u+E_{t}=0 \Rightarrow u=-E_{t} / E_{x}
$$

## Orientation in visual space-time: Velocity

Conclusion

- Oriented filters in visual space-time support the recovery of image velocity, along a particular direction.
- A "bank" of such filters can be used to span direction and provide an approach to optical flow estimation.


Filter


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## A representation for spatiotemporal patterns: Motivation

- When confronted with spatiotemporal data, an intelligent system can be overwhelmed by sheer quantity.
- An initial organization would be a key enabler for dealing effectively with data of this nature.
- The organization should afford distinctions that can guide subsequent processing.
- Distinctions that go beyond that of simple velocity.


## A representation for spatiotemporal patterns: General approach

- Parse stream of spatiotemporal data into primitive, yet semantically meaningful categories at the earliest stages of processing.
- Make distinctions along the following lines
- What is moving and what is stationary?
- Are the moving objects behaving coherently?
- How much of the variance in the data is due to temporal brightness change?
- Which portions of the data are simply too unstructured to support further analysis?


## A representation for spatiotemporal patterns: General approach

- Integrate information across both the spatial and temporal dimensions.
- Build on analysis of local orientation.
- The simplest non-trivial characterization of local geometric structure.


## A representation for spatiotemporal patterns: Primitive patterns

- Consider a spatiotemporal slice
- As an observer views a uniform pattern



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## A representation for spatiotemporal patterns: Primitives

|  | Unstruc- <br> tured | Static | Flicker | Coherent <br> Motion | Incoherent <br> Motion | Scintil- <br> lation |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\square$ |  |  |  |  |  |  |
| $\square$ |  |  |  |  |  |  |
| $\square$ |  |  |  |  |  |  |

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with 3 a small bias added for stability.

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## A representation for spatiotemporal patterns: Opponency and summation

- The R and L (U and D) components are ambiguous WRT coherent and incoherent motion.



## A representation for spatiotemporal patterns: Opponency and summation

- The R and L (U and D) components are ambiguous WRT coherent and incoherent motion.
- Solution: Combine via
- opponency R-L (U-D)
- summation $\mathrm{R}+\mathrm{L}(\mathrm{U}+\mathrm{D})$
- Geometrically a rotation of coordinate axes.



## A representation for spatiotemporal patterns: Spatiotemporal representation

- Proposal: A four band representation for both the $x-t$ and $y$-t dimensions.



## A representation for spatiotemporal patterns: Primitives projected on representation

|  | Unstruc- <br> tured | Static | Flicker | Coherent <br> Motion | Incoherent <br> Motion | Scintil- <br> lation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\square$ |  |  |  |  |  |  |
| $\square$ | $\square$ | - |  | + |  |  |
| $\square$ |  |  |  |  |  |  |
| $\|R-L\|$ | 0 | 0 | 0 | ++ | 0 | 0 |
| $R+L$ | 0 | ++ | ++ | ++ | ++++ | ++ |
| $S_{x}$ | 0 | ++ | 0 | + | + | + |
| $F_{x}$ | 0 | 0 | ++ | + | + | + |

## A representation for spatiotemporal patterns: Examples I

- A natural image sequence of each proposed class was acquired, $(x, y, t)=(64,64,40)$.
- Unstructured: featureless sky
- static: motionless tree
- flicker: smooth surface illuminated by lightning flashes
- coherent motion: field of flowers under camera motion
- incoherent motion: overlapped legs in complex motion
- scintillation: rain striking a puddle
- Each sequence brought under proposed representation.


## Unstructured



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Static


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Flicker


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## Coherent motion



## Incoherent motion



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## Scintillation



## A representation for spatiotemporal patterns：Results x－t

|  | Unstruc－ tured | Static | Flicker | Coherent Motion | Incoherent Motion | Scintil－ lation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 5 F |  |  | 1.3 |  |
|  |  | 11 | 三 | 致教 |  |  |
| $\|R-L\|$ | $0.00$ | 0.00 | $0.00$ | $0.37$ |  |  |
| $R+L$ | $0.01$ | $0.40$ | $0.36$ |  | $0.58$ |  |
| $S_{x}$ | $0.00$ | $0.55$ | $0.00$ |  | $0.17$ | 0.25 |
| $F_{x}$ | $0.00$ | $\begin{aligned} & 2.04 \end{aligned}$ | 0.63 |  |  |  |

## A representation for spatiotemporal patterns: Results y-t

|  | Unstructured | Static | Flicker | Coherent Motion | Incoherent Motion | Scintillation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 507 |  |  | 13 |  |
|  |  | 11 | 를 | ERy |  |  |
| $\|U-D\|$ | 0.00 | $0.00$ | $0.00$ | $\begin{aligned} & \text { 原解 } \\ & 0.34 \end{aligned}$ |  | $0.02$ |
| $U+D$ | $0.01$ | $0.38$ | $0.36$ | 0.52 | $0.45$ | $0.50$ |
| $S_{v}$ | $0.00$ | $0.59$ | $0.00$ | $\begin{gathered} \text { Kit } \\ 0.19 \end{gathered}$ | $0.24$ |  |
| $F_{v}$ | $0.00$ | $0.03$ | 0.64 |  |  | 0.21 |

## A representation for spatiotemporal patterns: Remarks

- Have described a representation for distinguishing primitive spatiotemporal patterns.
- The representation makes use of oriented bandpass image decomposition.
- Initial empirical results support the hypothesis that the proposed representation can afford the desired distinctions.


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## Boundaries: Motivation

- Detection and localization of spatiotemporal boundaries is an important aspect of chunking information into meaningful pieces.
- Complimentary to the area-based analysis considered so far.
- Differential operators matched to the juxtaposition of contrasting spatiotemporal structure can be assembled from the primitive filter responses, S, F, R$\mathrm{L}, \mathrm{R}+\mathrm{L}$, etc.


## Boundaries: Coherent motion

- Boundaries in coherent motion discriminate foreground/background.
- Coherent motion related to opponent bands R-L and U-D.
- Combine a spatial Laplacian with opponent filtering to yield double opponent operators.



## Boundaries: Coherent motion

- Boundaries in coherent motion discriminate foreground/background.
- Coherent motion related to opponent bands R-L and U-D.
- Combine a spatial Laplacian with opponent filtering to yield double opponent operators.



## Boundaries: Signature

- Zero-crossings in the double-opponent motion operator output indicate coherent motion boundaries.
- Slope magnitude taken

$$
D_{x}(R-L)
$$ as strength of boundary signal.

- Sum signals from x-t and $y$-t dimensions.

$$
R-L
$$



$$
D_{x x}(R-L)
$$

## Boundaries: Examples II

- Two image sequences depicting boundaries of coherent motion, $(x, y, t)=(256,256,16)$.
- random dot cinematogram: left and right sides of display in opposite horizontal motion.
- natural image sequence: aerial view of tree canopy with movement relative to undergrowth due to camera motion; homogeneous texture of vegetation obscures boundary in any one image.
- Each sequence processed by proposed method for indicating coherent motion boundaries.


## Boundaries: Results



Frame of input sequence.


Motion boundary signal intensity.

## Boundaries: Results



Frame of input sequence.


Motion boundary signal intensity.

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## Framework: Local orientation

unstructured

Appeal to orientation is not arbitrary

- The local orientation (or lack thereof) of a pattern is one of its most salient characteristics.
- Geometrically, orientation captures the local first-order correlation structure of a pattern, alternatively the local tangent.
- Provides a formal prerequisite to analysis of higher-contact constructions (e.g., curvature)
- For vision, local spatiotemporal orientation can have additional interpretations.
- Image velocity is manifest as spatiotemporal orientation.
- And more...


## Framework: Filtering

## Orientation selectivity

- Goal is to analyze spatiotemporal data, $I(x, y, t)$, according to its local orientation structure.
- Choose a representation with multiple bands each tuned for certain orientations, $\theta$, and scales in 3D $(x, y, t)$.
- Filter specifics:
- 3D Gaussian second derivatives, $G_{2 \theta}$.
- Corresponding Hilbert transforms, $H_{2 \theta}$.
- Rectified and summed in quadrature pairs to yield local "oriented energy".

$$
E_{\theta}(x, y, t)=\left[G_{2 \theta} * I(x, y, t)\right]^{2}+\left[H_{2 \theta} * I(x, y, t)\right]^{2}
$$

input spatiotemporal volume

oriented filter bank
output oriented energy volumes

## Framework: Architecture



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## Framework: Example


spatiotemporal volume

oriented energy volume

## Framework: Distributions of oriented energy

## Key ideas

- Build a distributed representation (i.e., histogram) at each point that measures the amount of energy for various kinds of spacetime oriented structures.
- Base subsequent analysis on the distribution of oriented energies across space and time.



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## Applications

## Examples provided in lecture.

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