EECS 4422/5323 Computer Vision

Unit 6: Motion

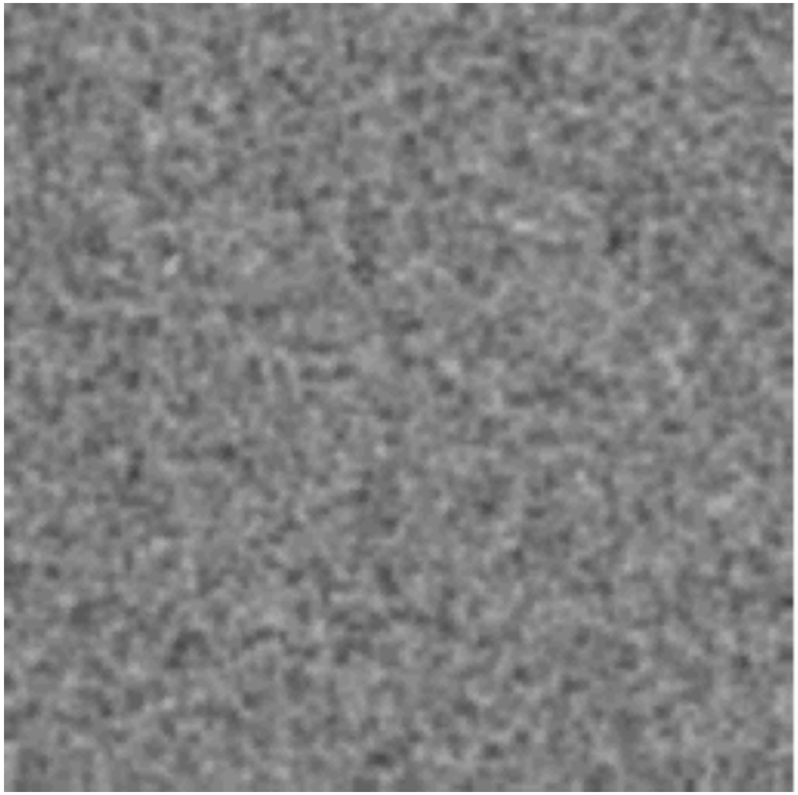
Outline

- Introduction
- Motion field vs. optical flow
- Brightness constancy
- Gradient-based optical flow estimation
- Finite displacement and feature-based methods
- 3D Structure and motion
- Summary

Introduction: Motivation

Time-varying imagery

- A great deal of useful information can be extracted from time-varying imagery (e.g., video).
  - Temporal image sequences of a dynamic world acquired from a stationary camera.
  - Temporal images sequences of a stationary world acquired from a moving camera.
  - Temporal image sequences of a dynamic world acquired from a moving camera.
- It might seem foolhardy to consider processing multiple images when extracting information from even one is so challenging.
- However, multiple images imply additional data on which to base our inferences.
  - Typically, the results are well worth the effort.



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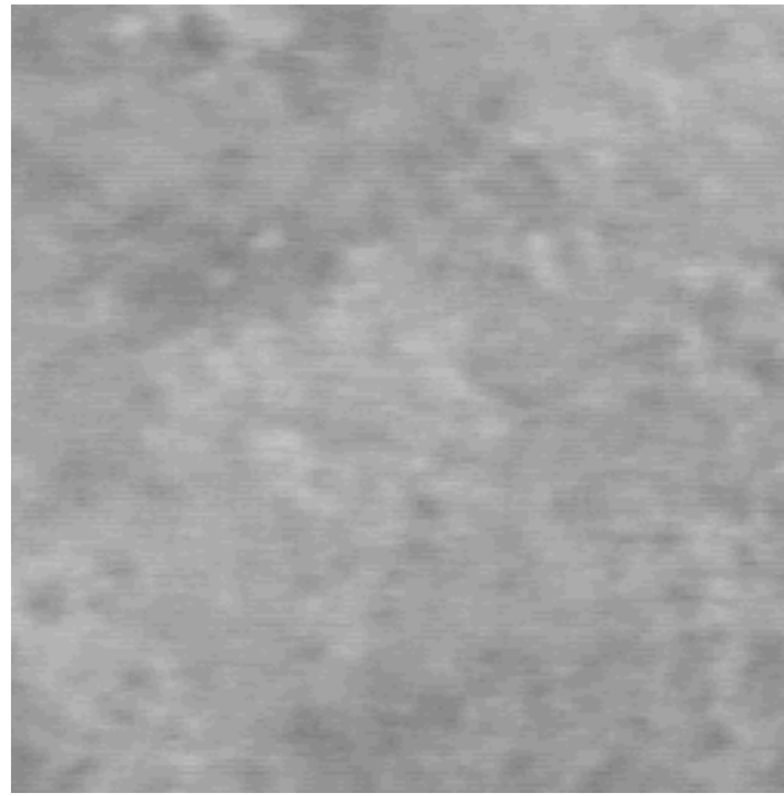

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Motion field vs. optical flow: Motion field

Basics

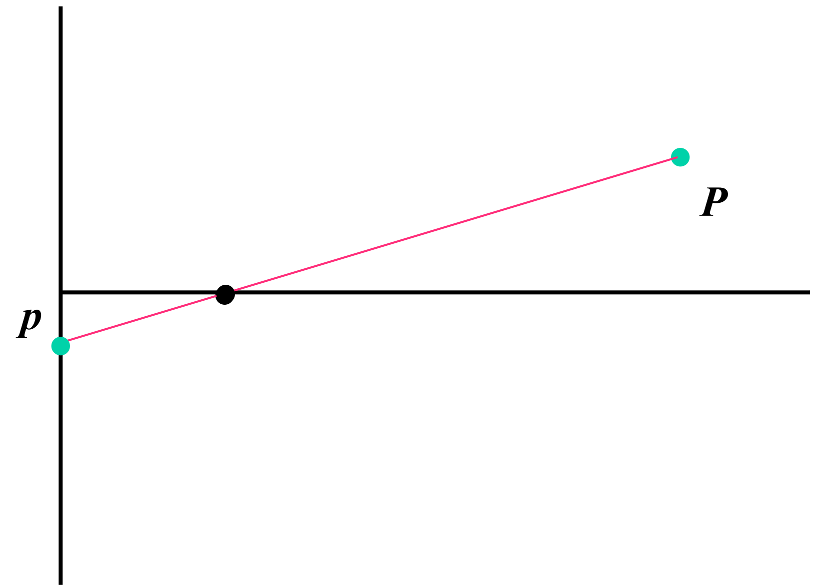
- When objects move in the environment or a camera moves through the environment there are corresponding changes in the images.
- These changes can be used to capture the relative motions as well as the shape of the objects.

Motion field vs. optical flow: Motion field

Definition

- The motion field assigns a velocity vector to each point in the image according to how the corresponding point in 3D moves.
- At a particular instance in time a point $\boldsymbol{p}$ in the image corresponds to some point $\boldsymbol{P}$ in the world according to some operative model of image projection,
  - We have

$$\boldsymbol{p} = \Pi(\boldsymbol{P})$$



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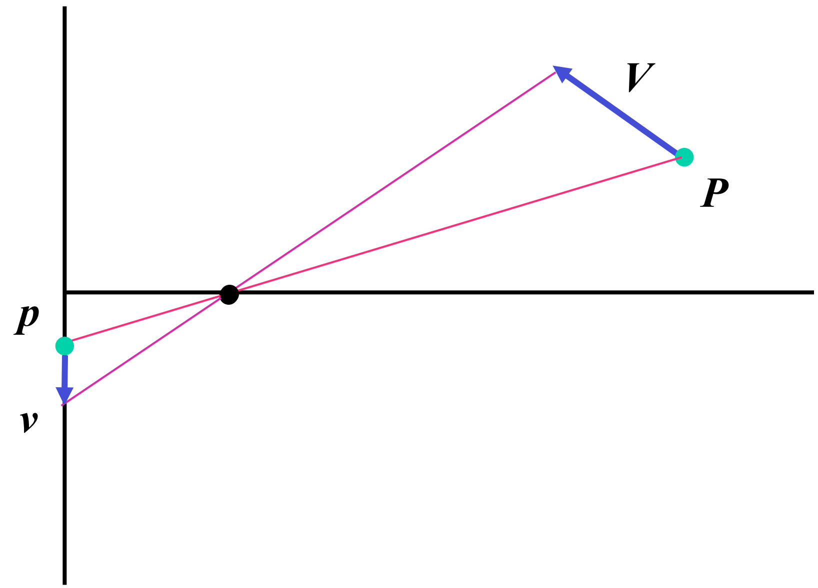
- Let the point in the world have velocity $\boldsymbol{V}$ relative to the camera, then the image point will have a corresponding velocity, $\boldsymbol{v}$.
  - We have

$$\boldsymbol{v} = \frac{d\boldsymbol{p}}{dt} \quad \text{and} \quad \boldsymbol{V} = \frac{d\boldsymbol{P}}{dt}$$

with

$$\frac{d\boldsymbol{p}}{dt} = \frac{d\Pi(\boldsymbol{P})}{dt}$$



Motion field vs. optical flow: Optical flow

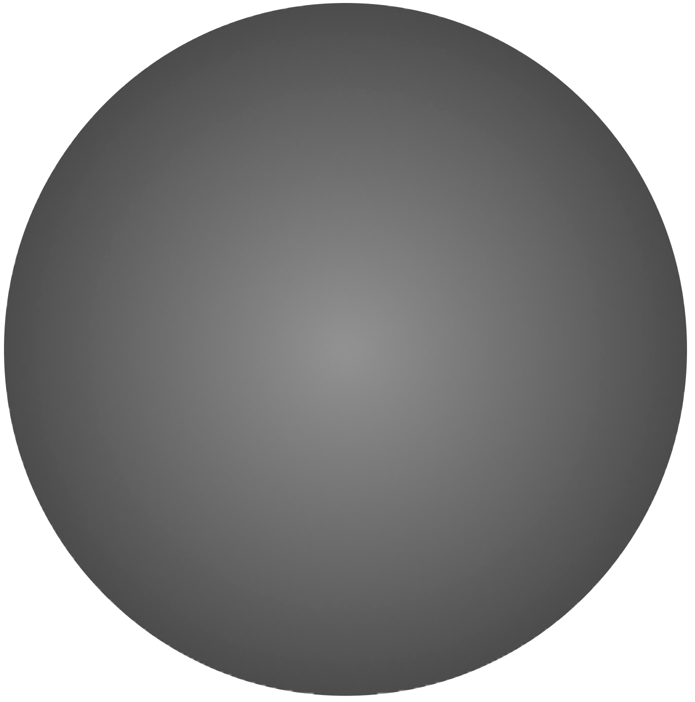
Basics

- Brightness patterns in the image move as the objects in the scene that give rise to them move.
- Optical flow is the apparent motion of the brightness pattern.
  – The motion that is present in the image.
- Ideally, the optical flow will correspond to the motion field.
  – But this is not always the case.

Motion field vs. optical flow: Optical flow

Rotating sphere

- Consider perfect sphere in front of a fixed imaging system (camera and illumination)
- There will be a smooth spatial variation in image brightness (shading) since the surface is curved.



Motion field vs. optical flow: Optical flow

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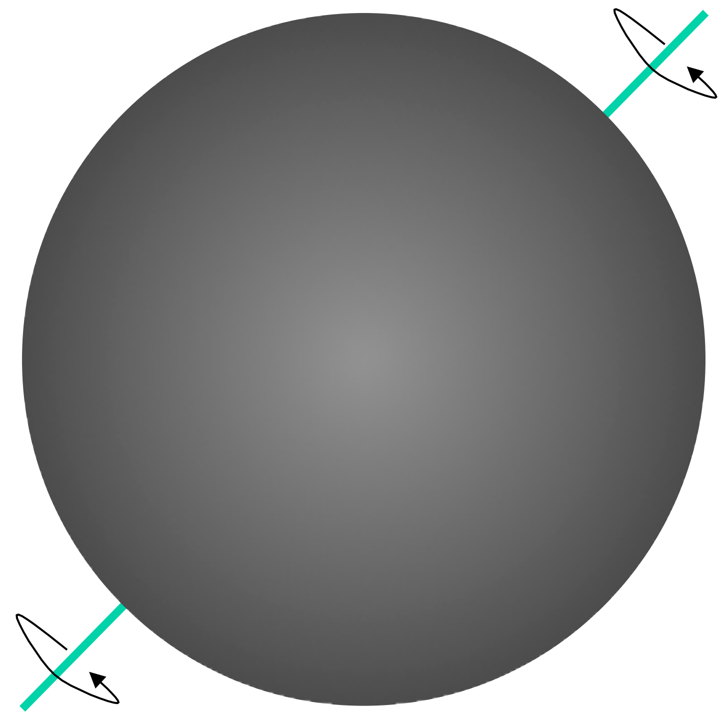

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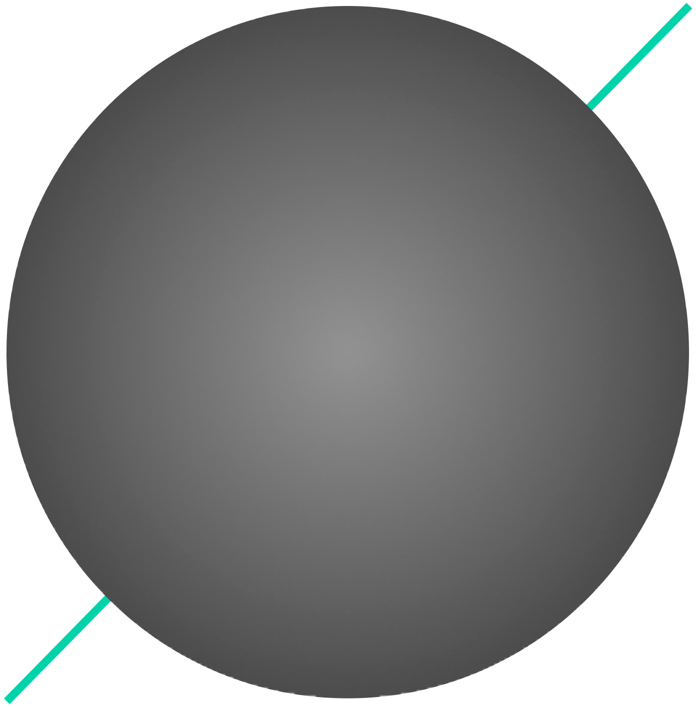

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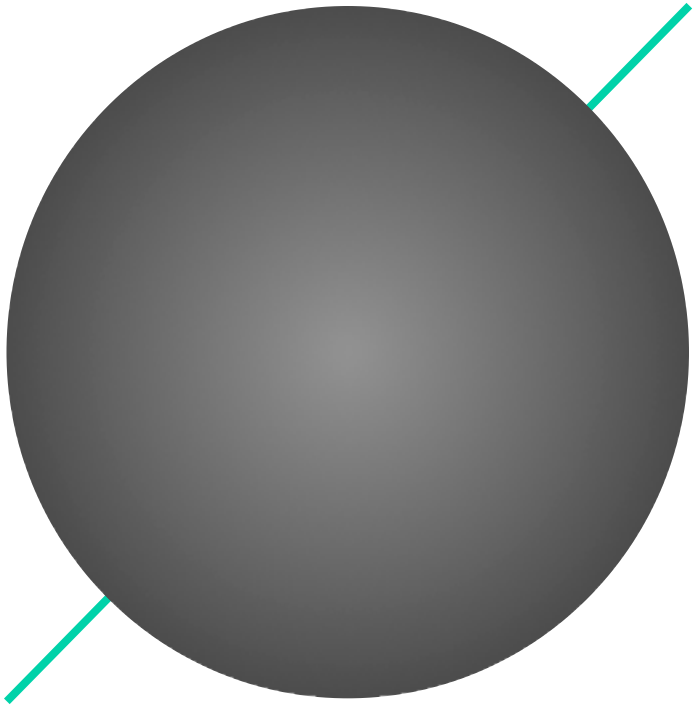
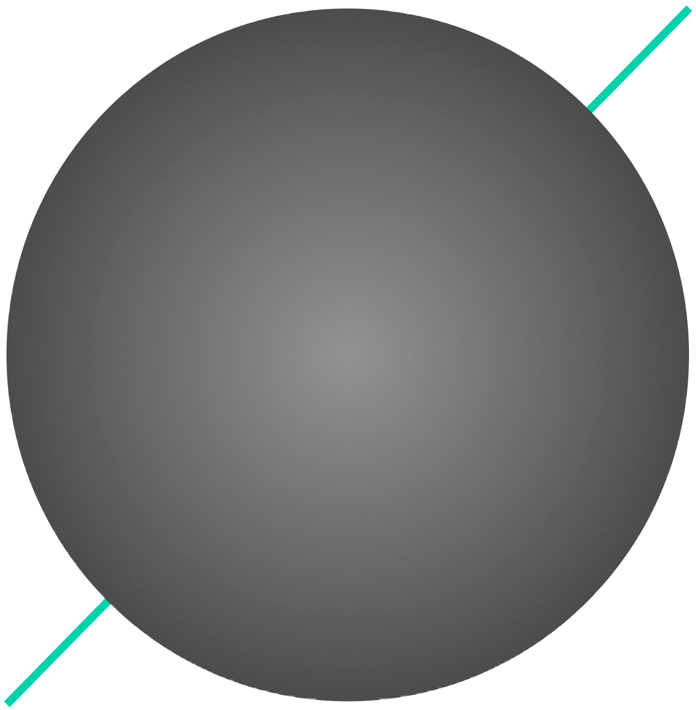


Motion field vs. optical flow: Optical flow

Rotating sphere

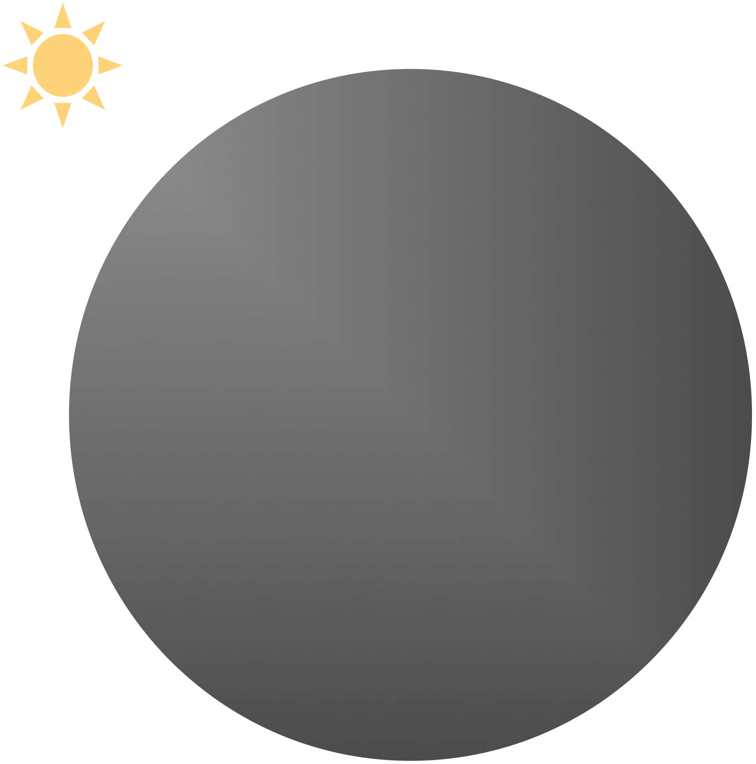
- Consider perfect sphere in front of a fixed imaging system (camera and illumination)
- There will be a smooth spatial variation in image brightness (shading) since the surface is curved.
- Let the sphere rotate.
- There is no change in the shading pattern.
  - The relationship between the local surface orientation and the imaging system does not vary.
- The optical flow is zero every where…
- …despite a nonzero motion field.



Motion field vs. optical flow: Optical flow

Moving light source

- Consider a perfect sphere in front of a stationary camera, but moving light source.



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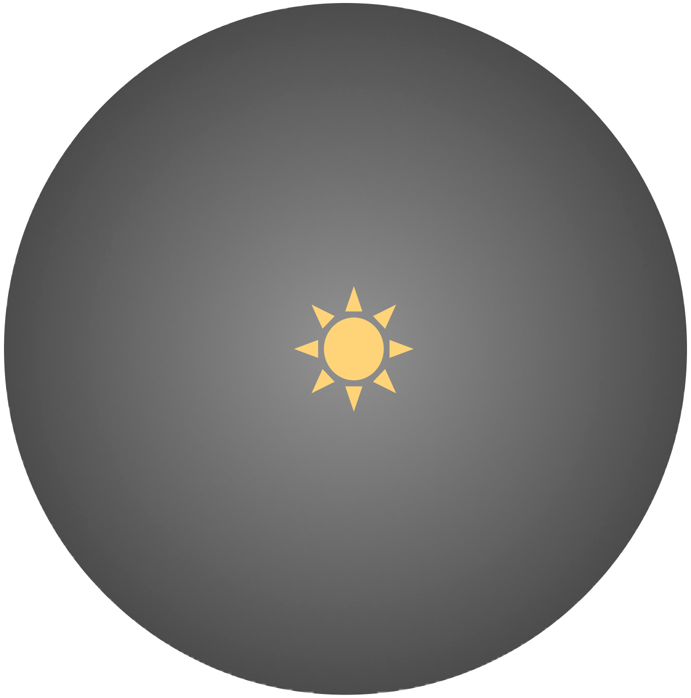

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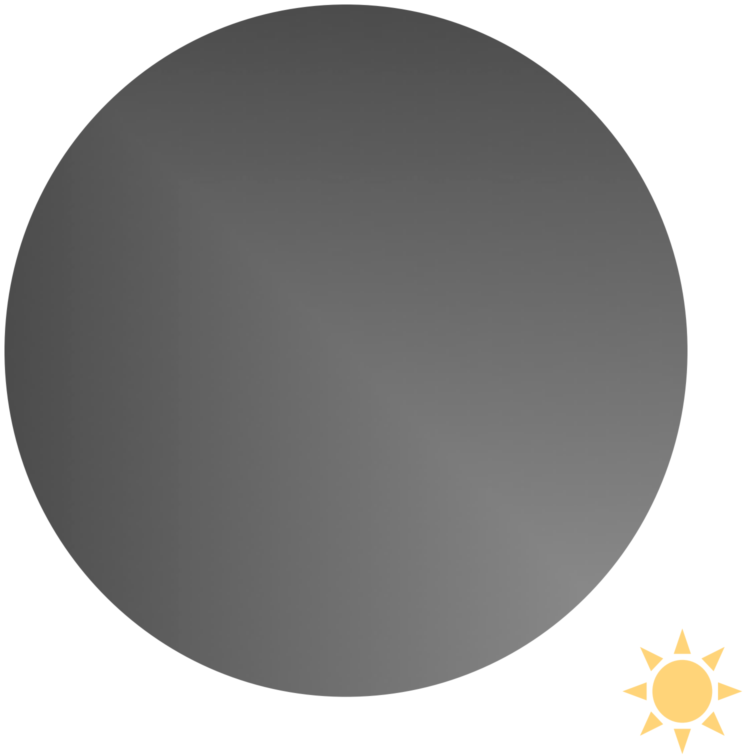
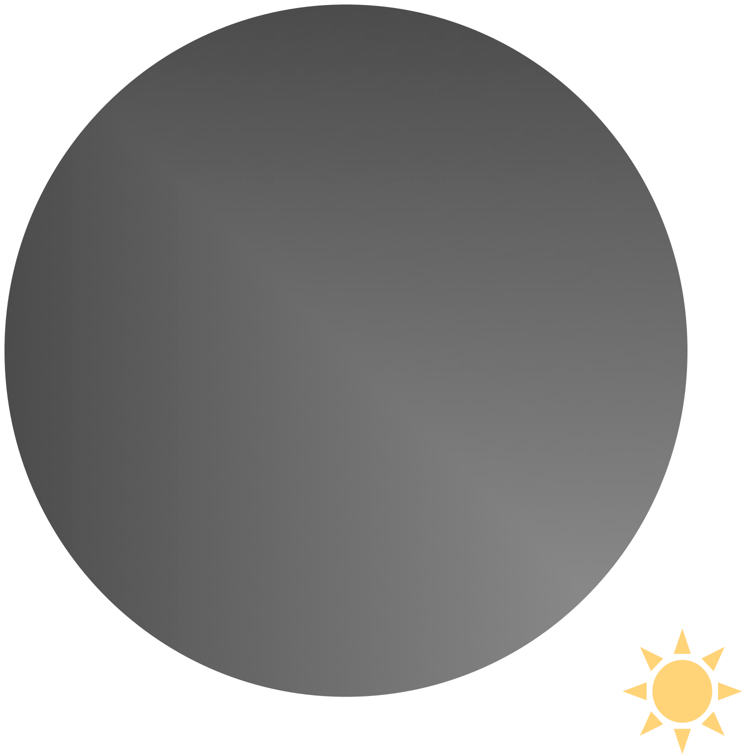


Motion field vs. optical flow: Optical flow

Moving light source

- Consider a perfect sphere in front of a stationary camera, but moving light source.
- Now the shading pattern changes with the variation in source position.
- The optical flow is nonzero everywhere…
- …although the motion field is zero.



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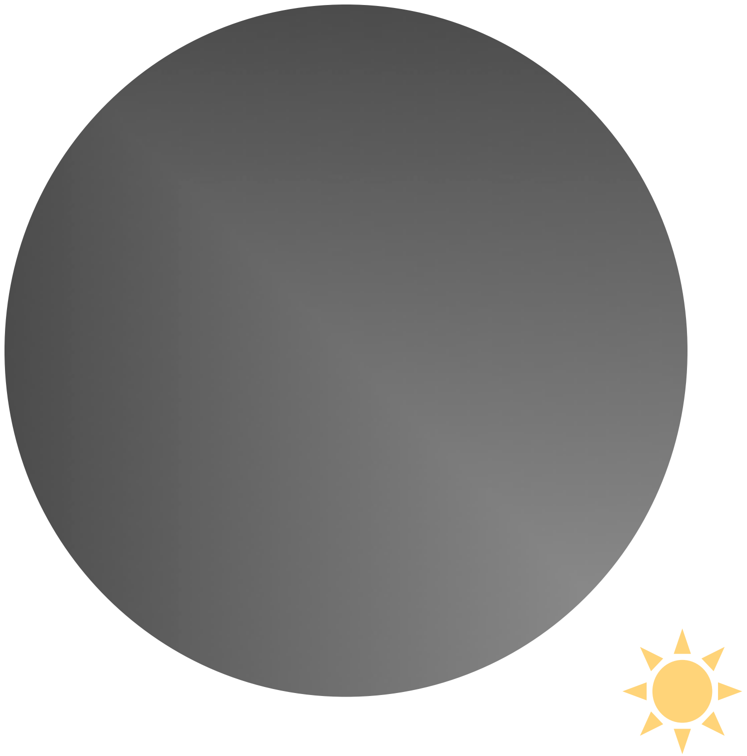

Other sources of discrepancy

- Shadows
- Specular reflection
- Virtual images
- Etc.

Motion field vs. optical flow: Conclusion

Life is tough, but not too…

- We are interested in the motion field
  - A purely geometric concept
  - That relates to the structure and dynamics of the scene
- What we have access to is the optical flow
  - A photometric concept
  - The thing that we can measure in an image.
- Typically, the motion field and optical flow are in close correspondence
  - But not always
  - As our examples have shown

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Brightness constancy: Constraint equation

Where are we headed?

- Accepting the limited correspondence between the motion field and the optical flow
- We seek to relate optical flow to measurements of image irradiance.
- This will provide constraint on the recovery of flow from the data that we can sense.

Brightness constancy: Constraint equation

Relating temporal brightness change to optical flow

- Let
  - $E(x,y,t)$ be image irradiance at time t and image location (x,y)
  - $u(x,y)$ and $v(x,y)$ be the x and y components of the optical flow, respectively

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$$u = \frac{dx}{dt}, v = \frac{dy}{dt}$$

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$$E(x + u\delta t, y + v\delta t, t + \delta t) = E(x, y, t)$$

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$$E(x + u\delta t, y + v\delta t, t + \delta t) = E(x, y, t)$$

- If brightness varies smoothly with *x, y,* and *t*, then we can expand the LHS in a Taylor series to obtain

$$E(x, y, t) + \delta x \frac{\partial E}{\partial x} + \delta y \frac{\partial E}{\partial y} + \delta t \frac{\partial E}{\partial t} + h.o.t. = E(x, y, t)$$

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Remarks:

- We are seeking constraint on optical flow in terms of things we can calculate from image data.
- We want to reduce this expression to something that can be calculated directly from the image.
- We know how to calculate derivatives; and we have some.
- Let's try for a complete expression in terms of differentials.

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Brightness constancy: Constraint equation

Optical flow constraint equation

- We can rewrite our differential constraint

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to good advantage.

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- Also, for notational convenience, let

$$E_x = \frac{\partial E}{\partial x} \qquad E_y = \frac{\partial E}{\partial y} \qquad E_t = \frac{\partial E}{\partial t}$$

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- Then we have the standard form for the optical flow constraint equation

$$E_x u + E_y v + E_t = 0$$

  which relates spatial and temporal derivatives of irradiance to optical flow.

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  which relates spatial and temporal derivatives of irradiance to optical flow.
  - Subject to the brightness constancy assumption.

Brightness constancy: Aperture problem

Equation counting

- We have derived one equation

$$E_x u + E_y v + E_t = 0$$

- But have two unknowns of interest

$$(u, v)$$

- The solution is under constrained.
- But how so?

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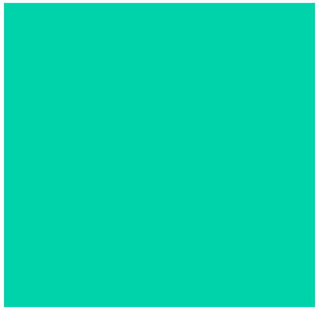

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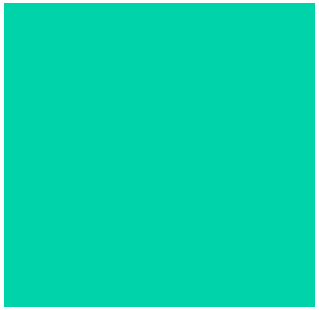

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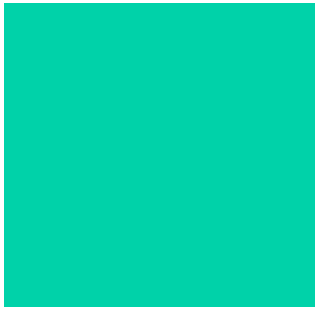

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Brightness constancy: Aperture problem

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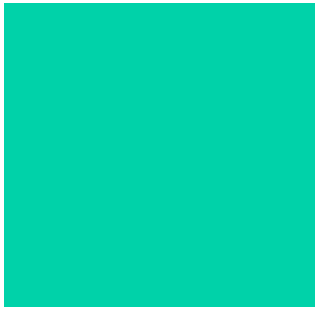

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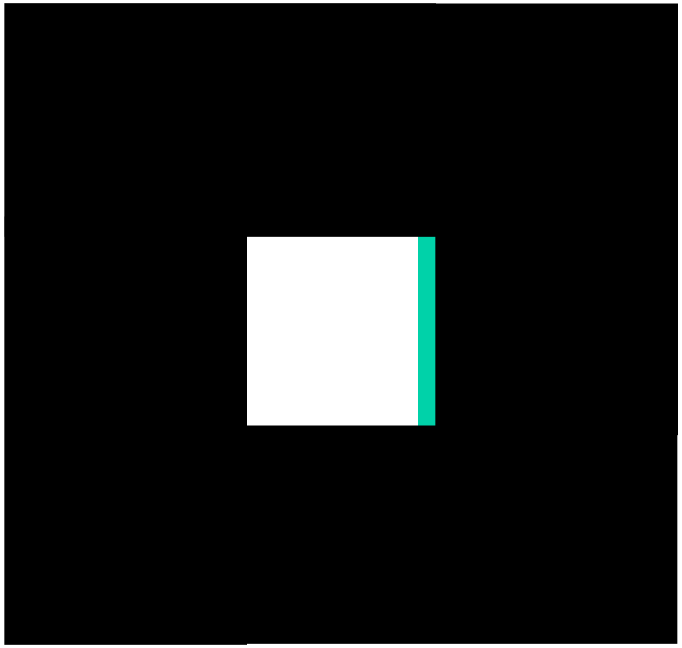

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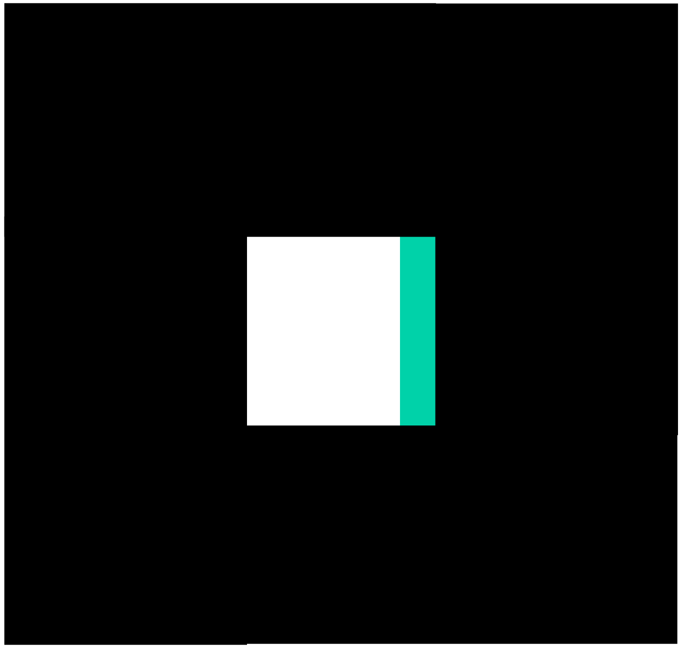

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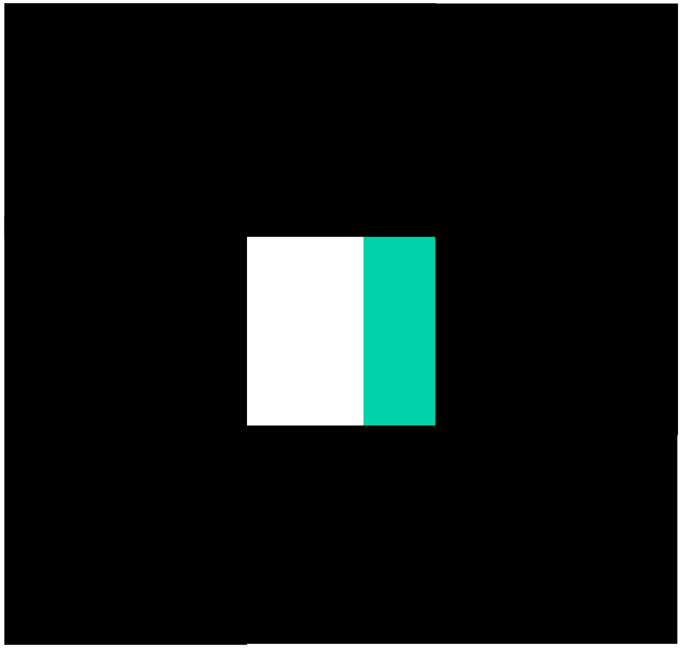

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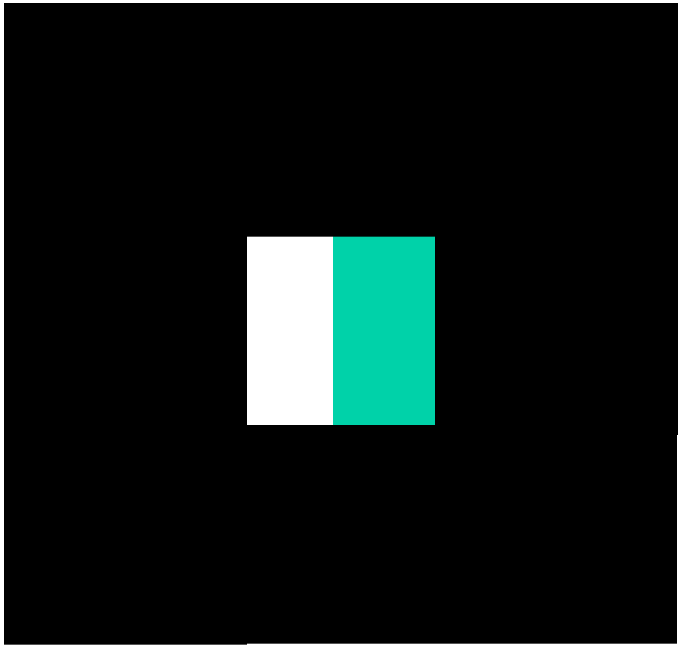

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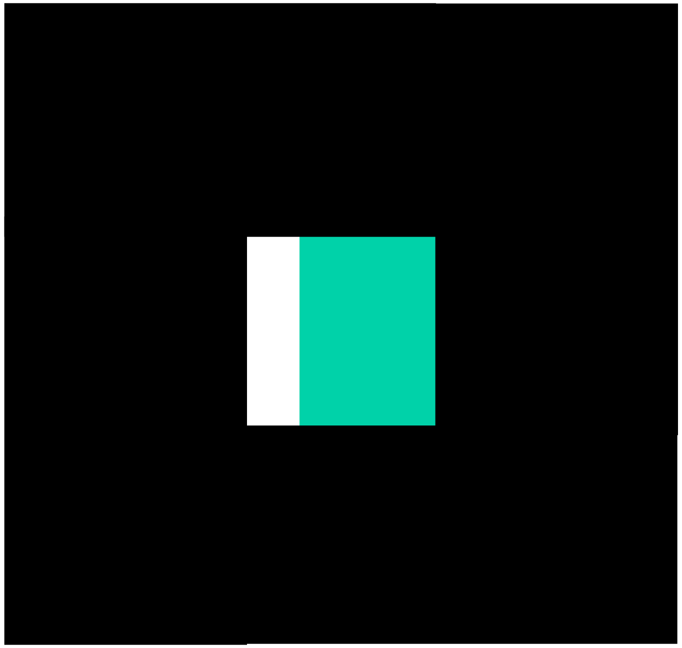

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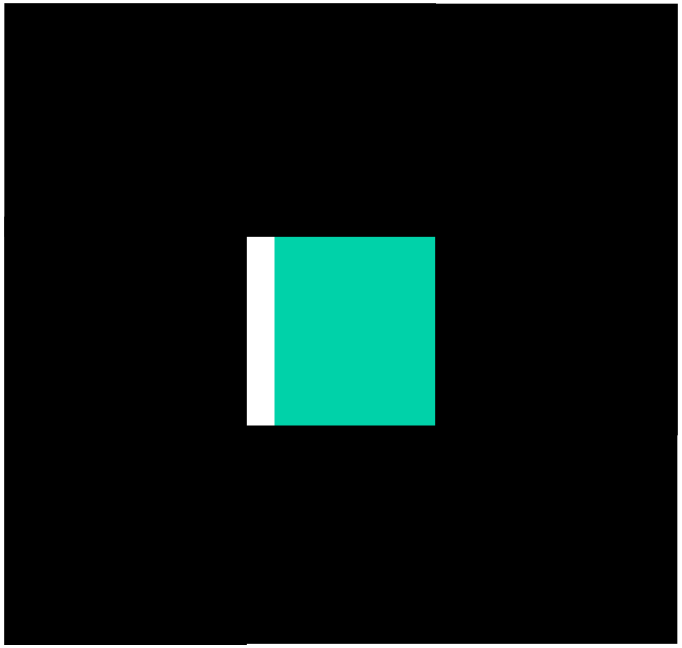

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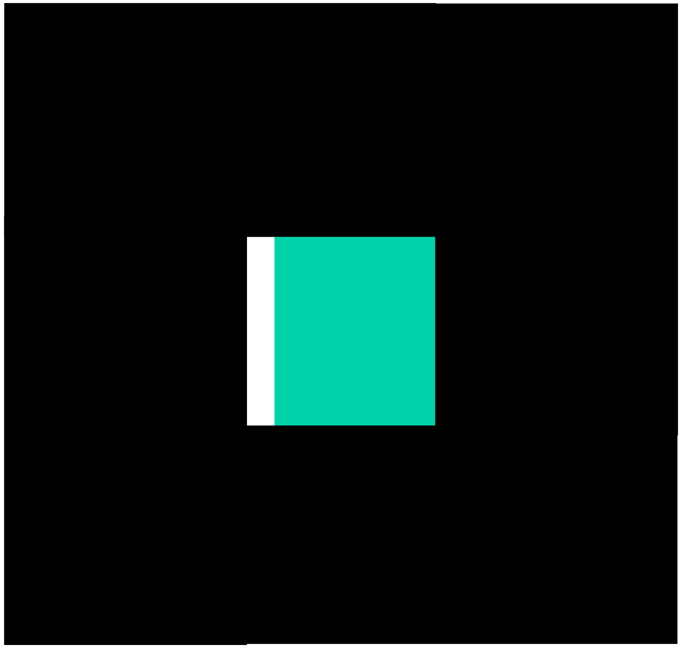

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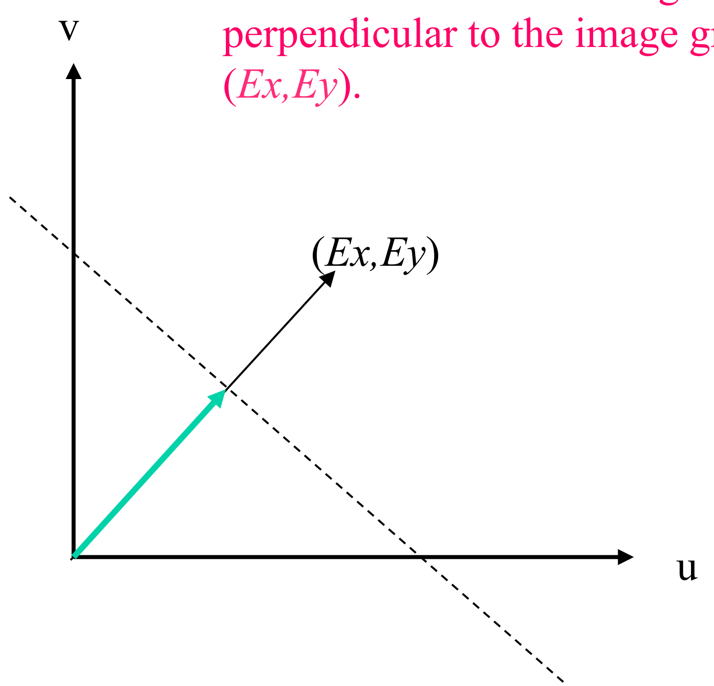

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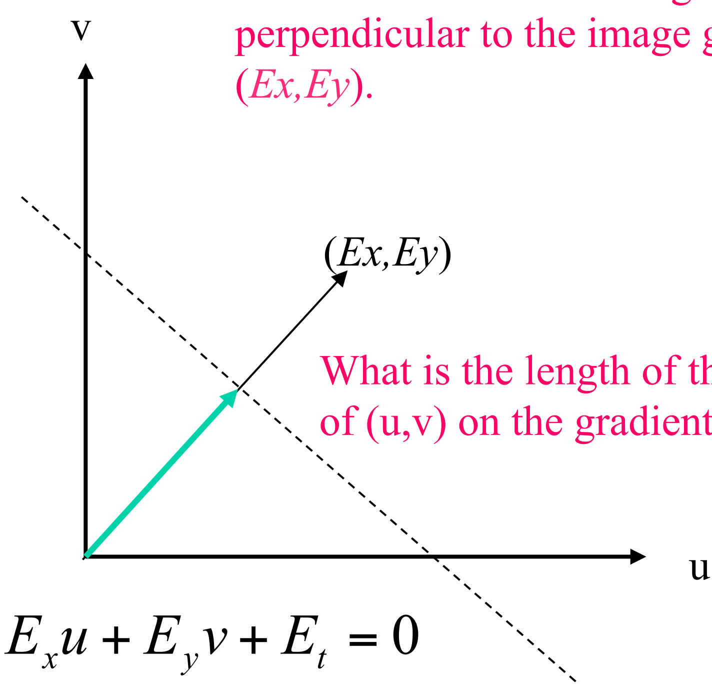

What is the length of the projection of (u,v) on the gradient direction?

$$E_x u + E_y v + E_t = 0$$

$$\Rightarrow (u, v) \bullet (E_x, E_y) = -E_t$$

$$\Rightarrow (u, v) \bullet \frac{(E_x, E_y)}{\left|(E_x, E_y)\right|} = \frac{-E_t}{\sqrt{E_x^2 + E_y^2}}$$

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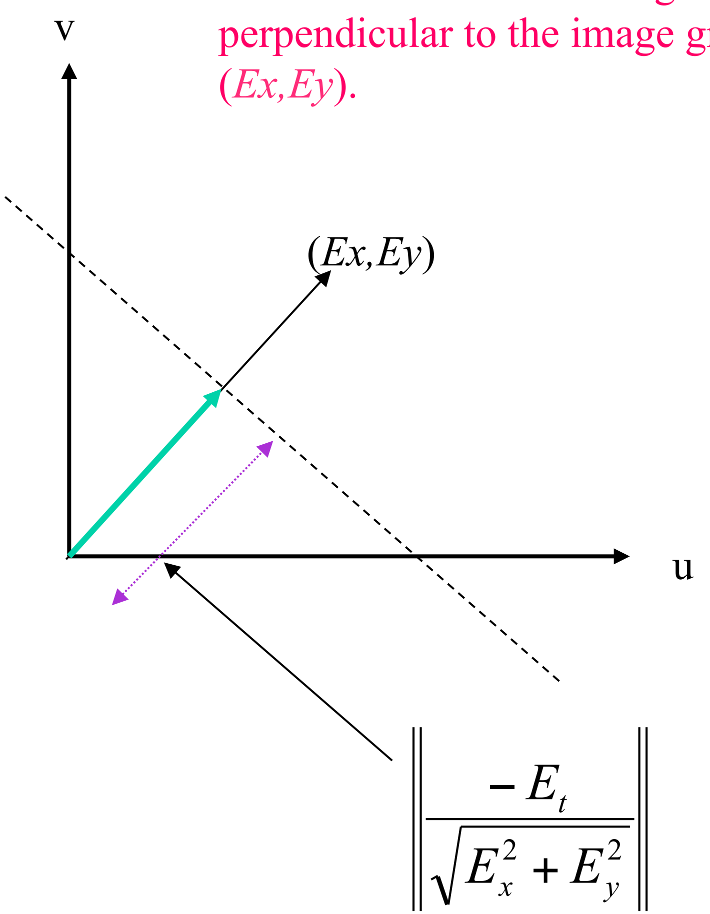

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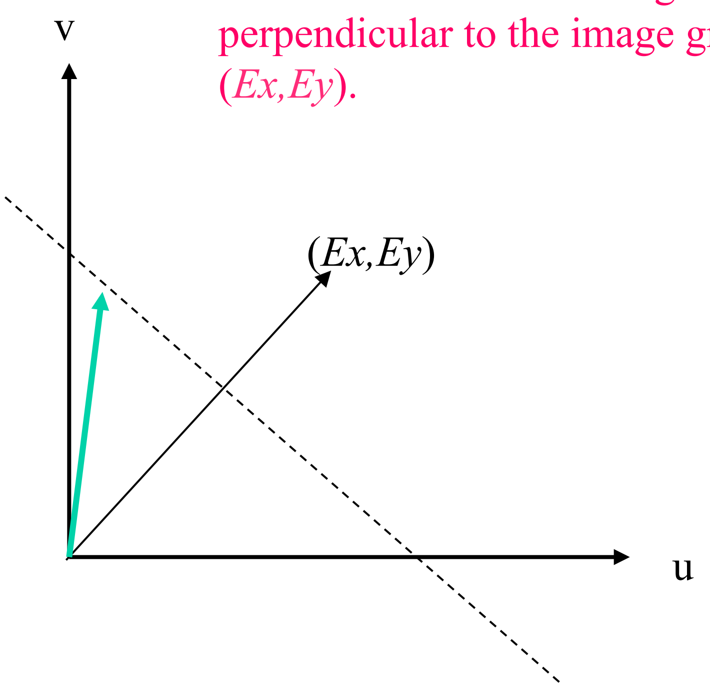

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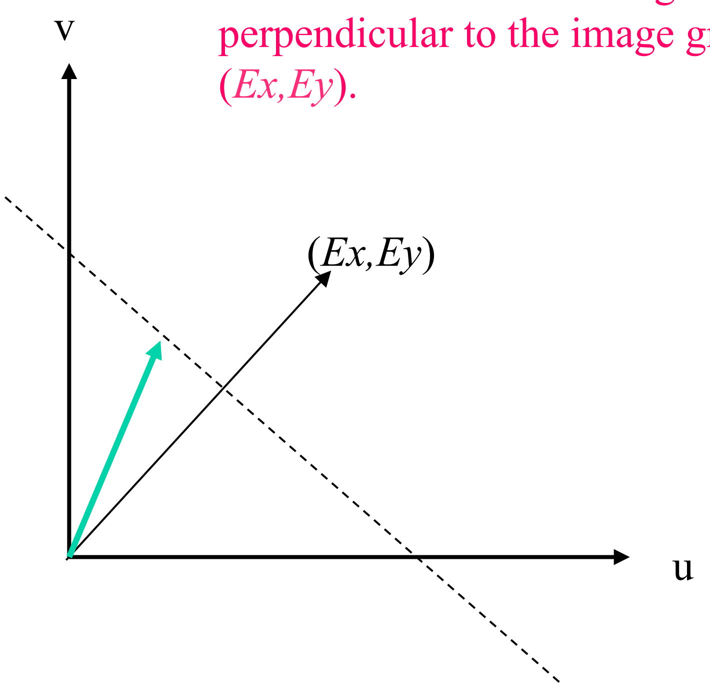

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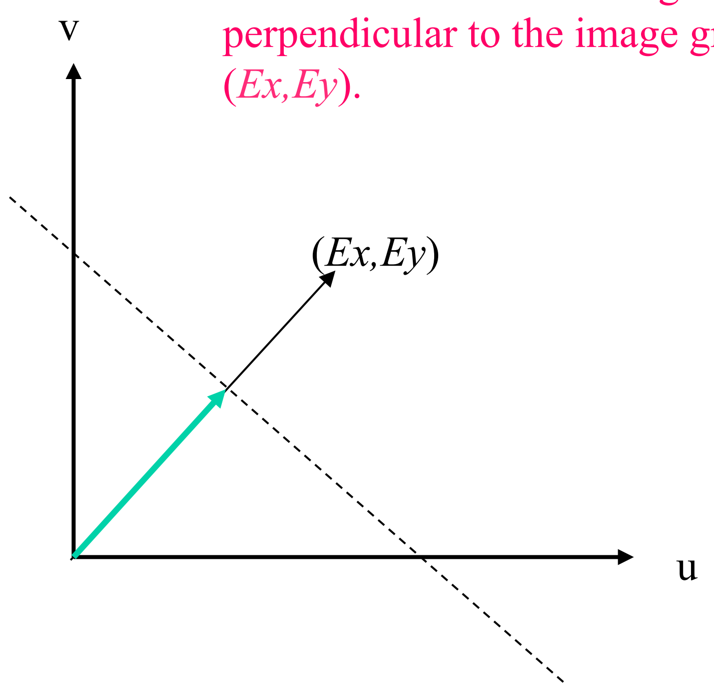

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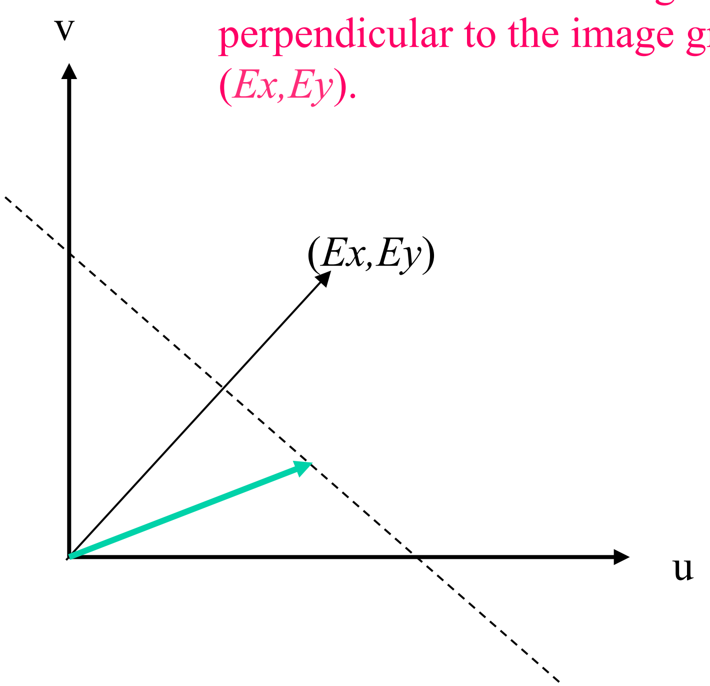

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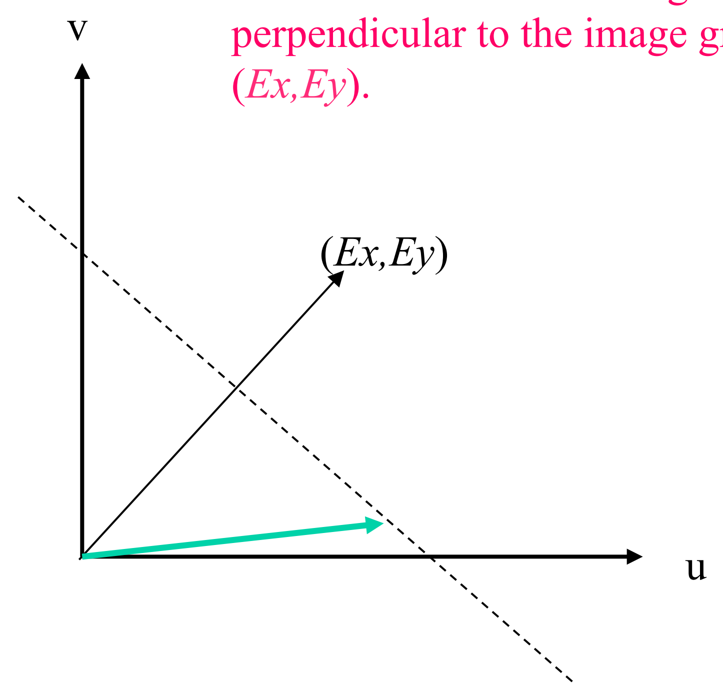

Brightness constancy: Recap

Where are we?

- We are trying to develop constraints that allow us to recover optical flow from measurements of image irradiance.
- We have introduced the assumption that a local pattern of image intensity remains constant across an instance of time (brightness constancy).
- This allowed us to derive a fundamental equation that relates derivatives of irradiance to optical flow (the optical flow constraint equation)

$$E_x u + E_y v + E_t = 0$$

- However, there is not enough constraint to unambiguously determine the flow (aperture problem).

Where to next?

- We seek additional constraint to uniquely define the optical flow.
- To allow for algorithmic recovery.

Outline

- Introduction
- Motion field vs. optical flow
- Brightness constancy
- **Gradient-based optical flow estimation**
- Finite displacement and feature-based methods
- 3D Structure and motion
- Summary

Gradient optical flow estimation: Approaches

We have

- We have derived the optical flow constraint equation

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- However, this amounts to one equation in two unknowns, (u,v).
- This is not enough to uniquely define the optical flow solution.

We need

- Additional constraint so that at (every image location) we have two equations in two unknowns to define a solution.
- Several approaches have been developed
  - Variational smoothness with boundary conditions
  - Differentiate the present constraint equation to generate additional constraint equations
  - Assume flow constancy over some finite window

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Gradient optical flow estimation: Flow constancy

Error measure

- We seek (u,v) that satisfies the constraint equation

$$E_x u + E_y v + E_t = 0$$

- We choose to do this by minimizing the squared violation of this constraint WRT the variable of interest

$$\min_{(u,v)} (E_x u + E_y v + E_t)^2$$

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- Flow constancy says that over some window, W, the values of (u,v) are constant.
- Correspondingly, we seek to minimize violation of the optical flow constraint over the window

$$\min_{(u,v)} \sum \sum_{W} (E_x u + E_y v + E_t)^2$$

Gradient optical flow estimation: Flow constancy

Generation of constraint equations

- To find (u,v) that minimize the flow constancy error

$$\min_{(u,v)} \sum \sum_{W} (E_x u + E_y v + E_t)^2$$

we follow standard procedure of differentiating WRT the variables of interest, setting to zero and solving.

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- Differentiating with respect to v and setting to zero yields

$$\sum \sum_{W} 2E_y(E_x u + E_y v + E_t) = 0$$

- We now have two equations in the two unknowns of interest (u,v).

Gradient optical flow estimation: Flow constancy

Solving for optical flow

- We have our two equations in two unknowns

$$\sum \sum (E_x u + E_y v + E_t) E_x = 0$$
$$\sum \sum (E_x u + E_y v + E_t) E_y = 0$$

- To complete our task, we need to explicitly solve for (u, v).
- Let us more cleanly isolate these variables from the other terms.

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- To complete our task, we need to explicitly solve for (u,v).
- Let us more cleanly isolate these variables from the other terms.
- Since (u,v) is assumed constant over the window of summation, we can move them outside the summation.

$$\left(\sum\sum E_x^2\right)u + \left(\sum\sum E_x E_y\right)v + \sum\sum E_x E_t = 0$$
$$\left(\sum\sum E_x E_y\right)u + \left(\sum\sum E_y^2\right)v + \sum\sum E_y E_t = 0$$

Gradient optical flow estimation: Flow constancy

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now suggest the matrix form

$$\begin{pmatrix} \sum\sum E_x^2 & \sum\sum E_x E_y \\ \sum\sum E_x E_y & \sum\sum E_y^2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -\sum\sum E_x E_t \\ -\sum\sum E_y E_t \end{pmatrix}$$

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- And our solution becomes

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \sum\sum E_x^2 & \sum\sum E_x E_y \\ \sum\sum E_x E_y & \sum\sum E_y^2 \end{pmatrix}^{-1}\begin{pmatrix} -\sum\sum E_x E_t \\ -\sum\sum E_y E_t \end{pmatrix}$$

Gradient optical flow estimation: Flow constancy

Remarks

- An optical flow algorithm follow trivially from our derivation
  - **Input:** A temporal sequence of two images
  - **Output:** A pair of optical flow images; a U image and a V image
  - For all pixels (i,j) in the first image solve the flow constancy equation

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- How do we choose the window size?
  - Smaller windows provide greater precision
  - Larger windows provide better performance in presence of low signal-to-noise
  - A data dependent trade-off (as with stereo matching)

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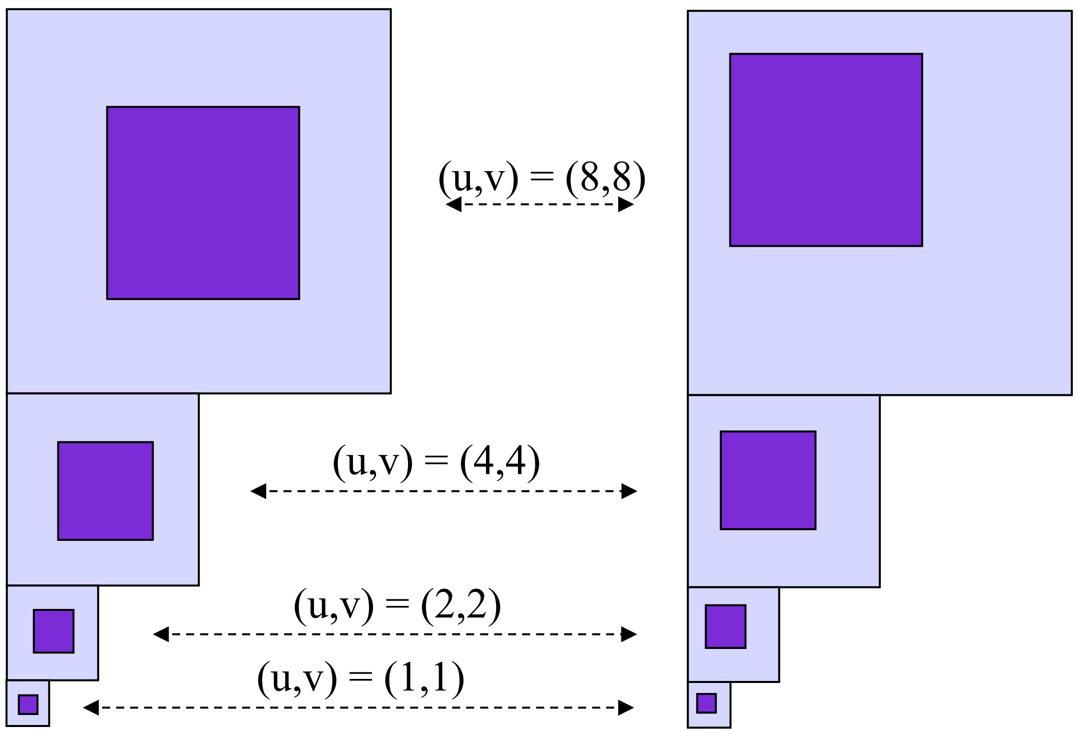
  - Larger windows provide better performance in presence of low signal-to-noise
  - A data dependent trade-off (as with stereo matching)
- This is yet another excellent place to exploit coarse-to-fine processing
  - Build a pyramid representation
  - Initially recover (a coarse estimate) of flow with the lowest resolution images
  - Use the initial estimate to seed the next highest resolution estimate
  - Etc.

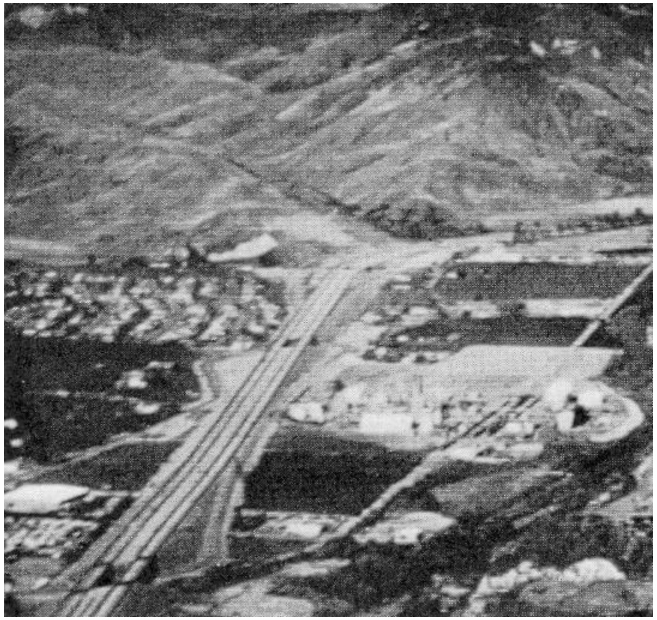
Gradient-based optical flow estimation: Flow constancy

Benefit of coarse-to-fine flow estimation



Gradient-based optical flow estimation: Example

Source image



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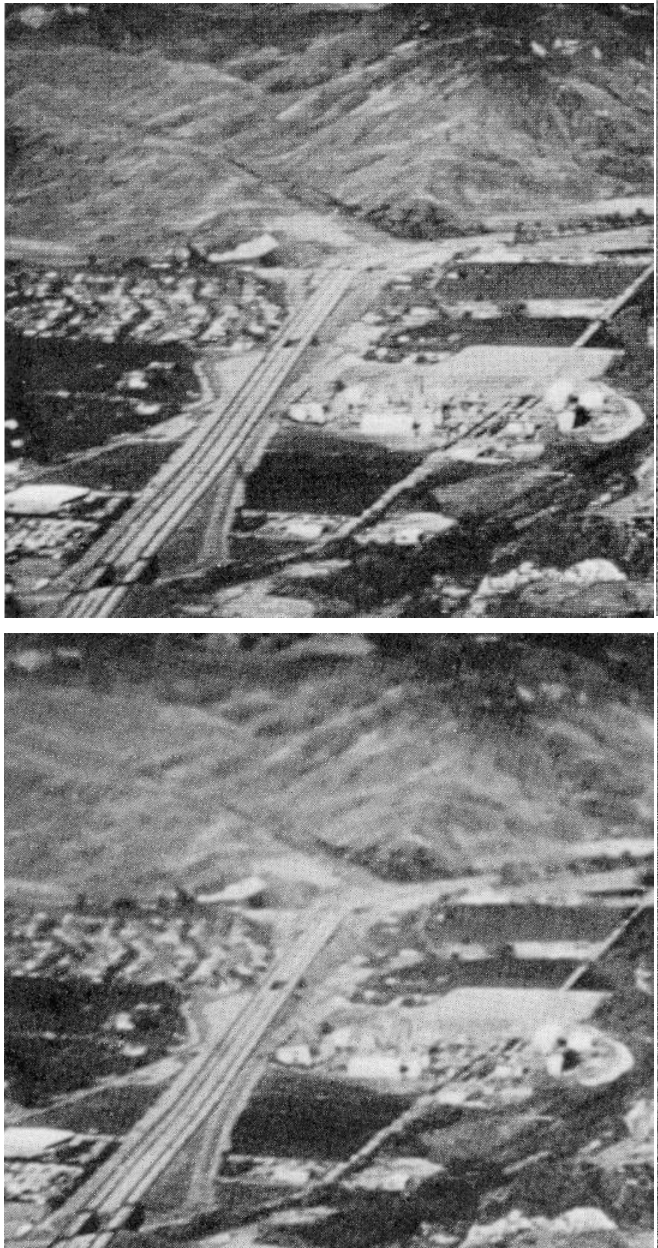
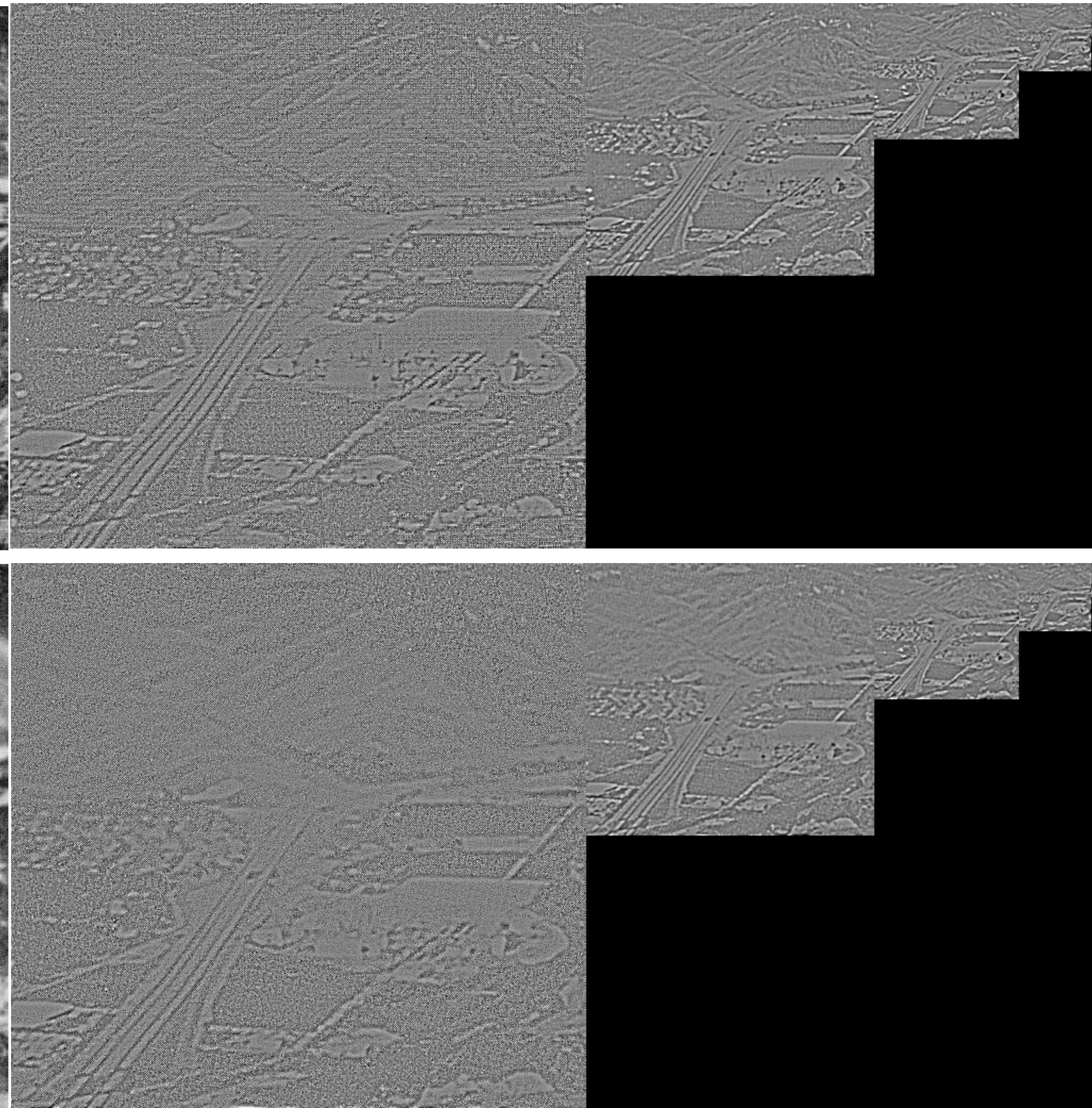


Pyramid representation



Gradient-based optical flow estimation: Example

U

V

Gradient-based optical flow estimation: Example

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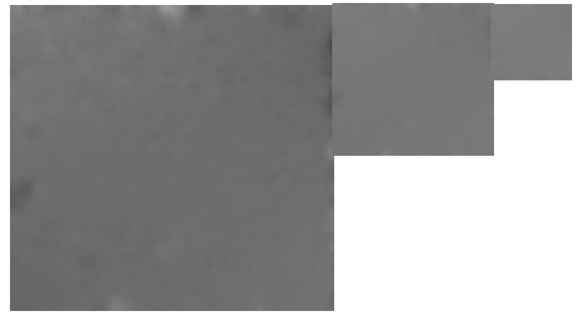

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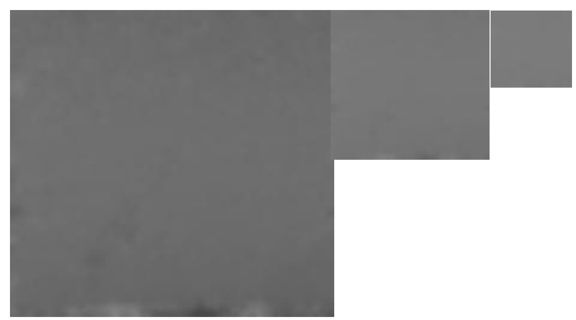

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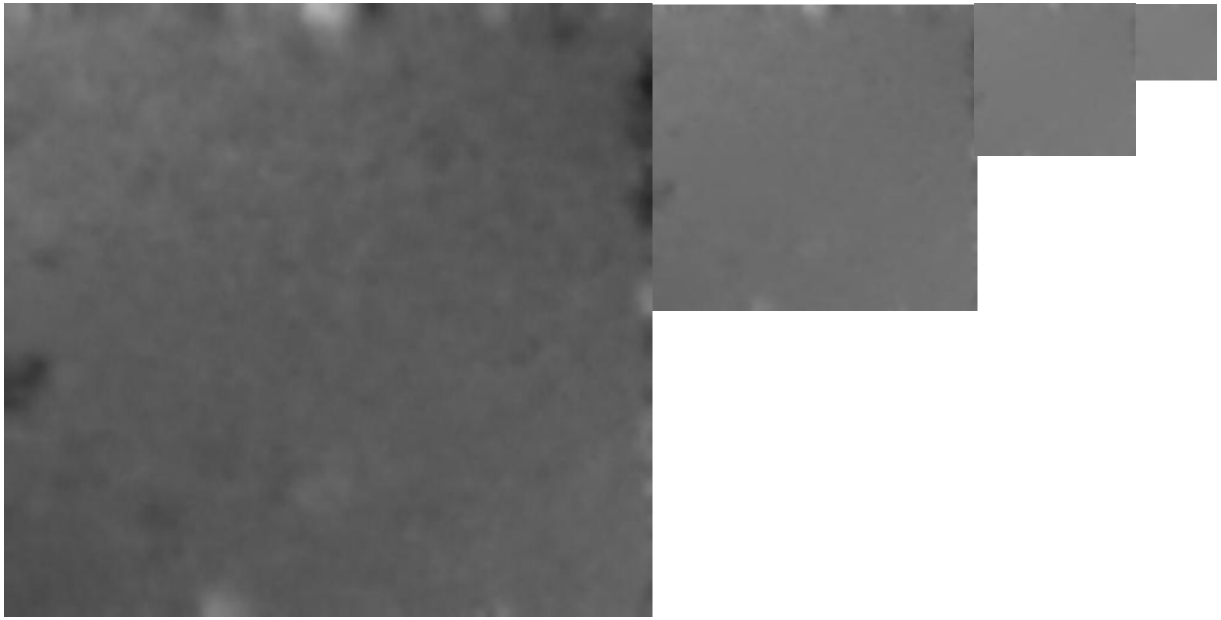

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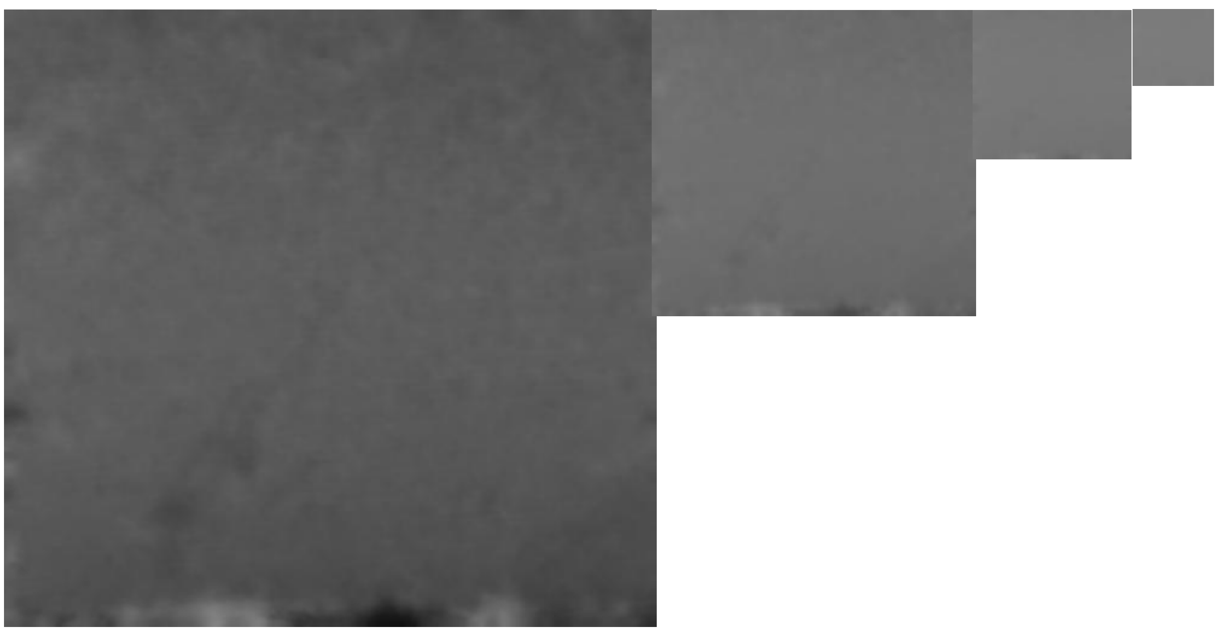

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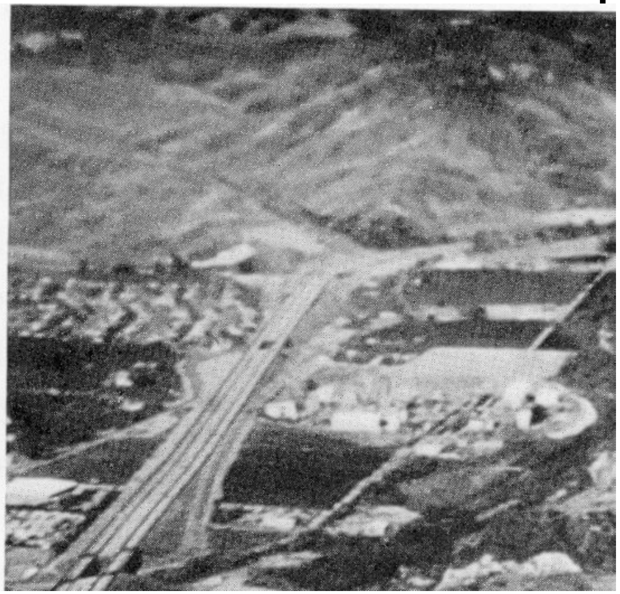

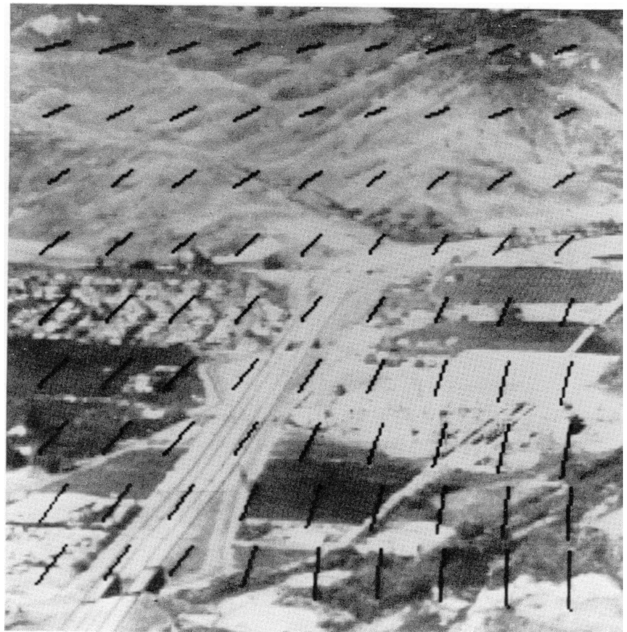

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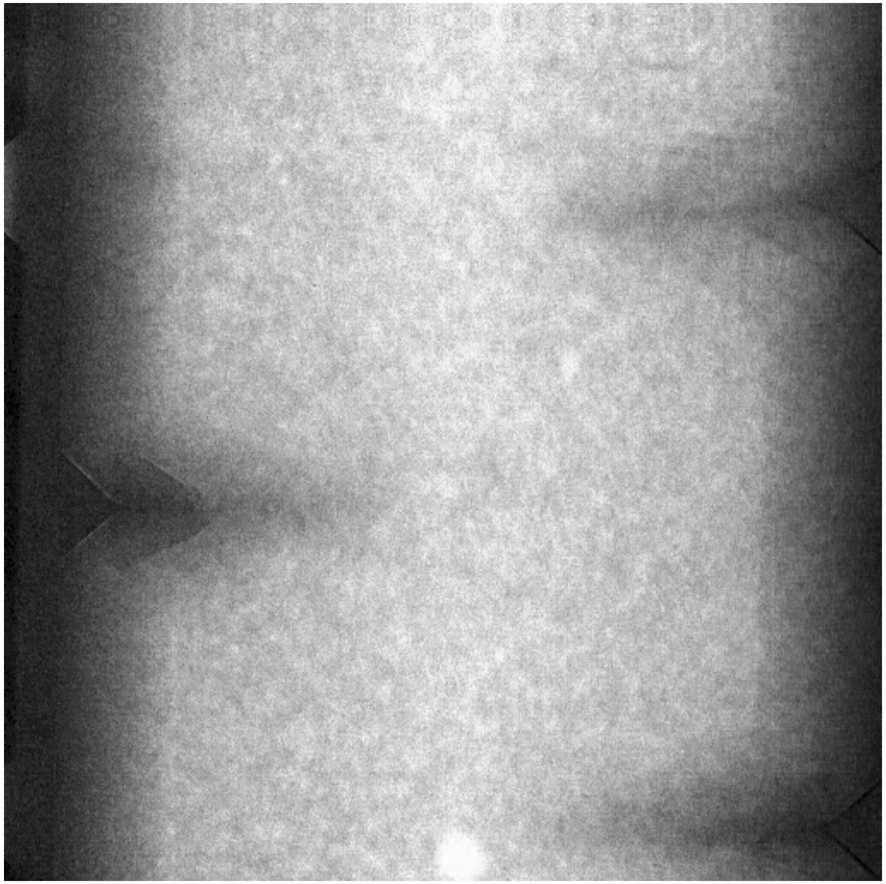

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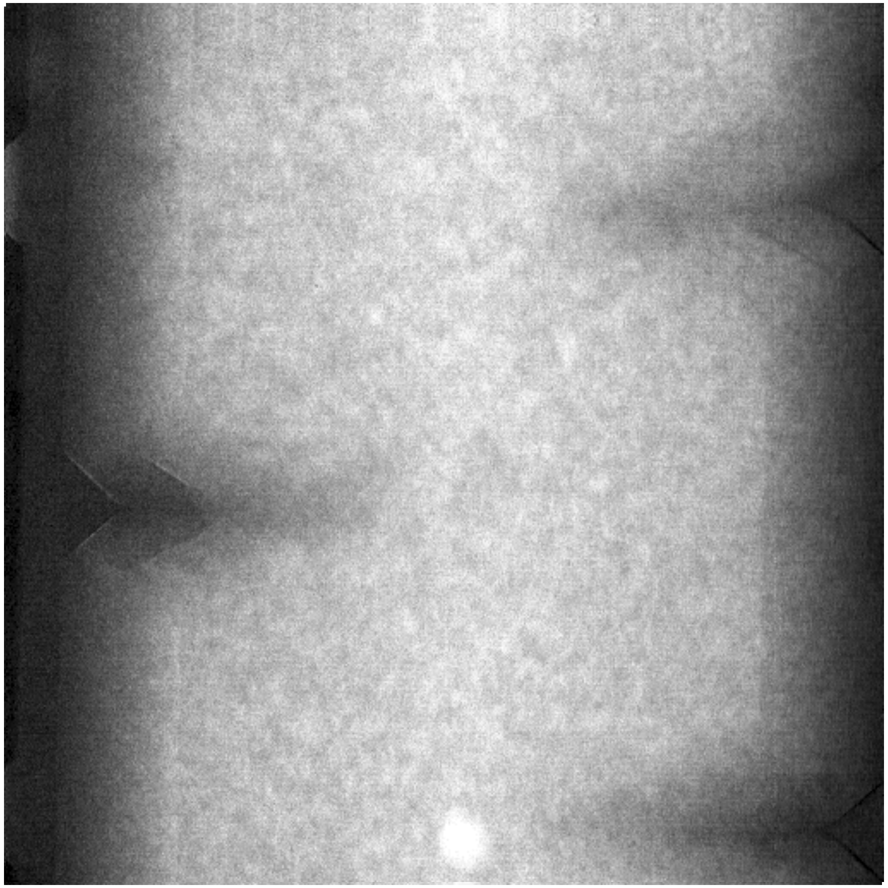

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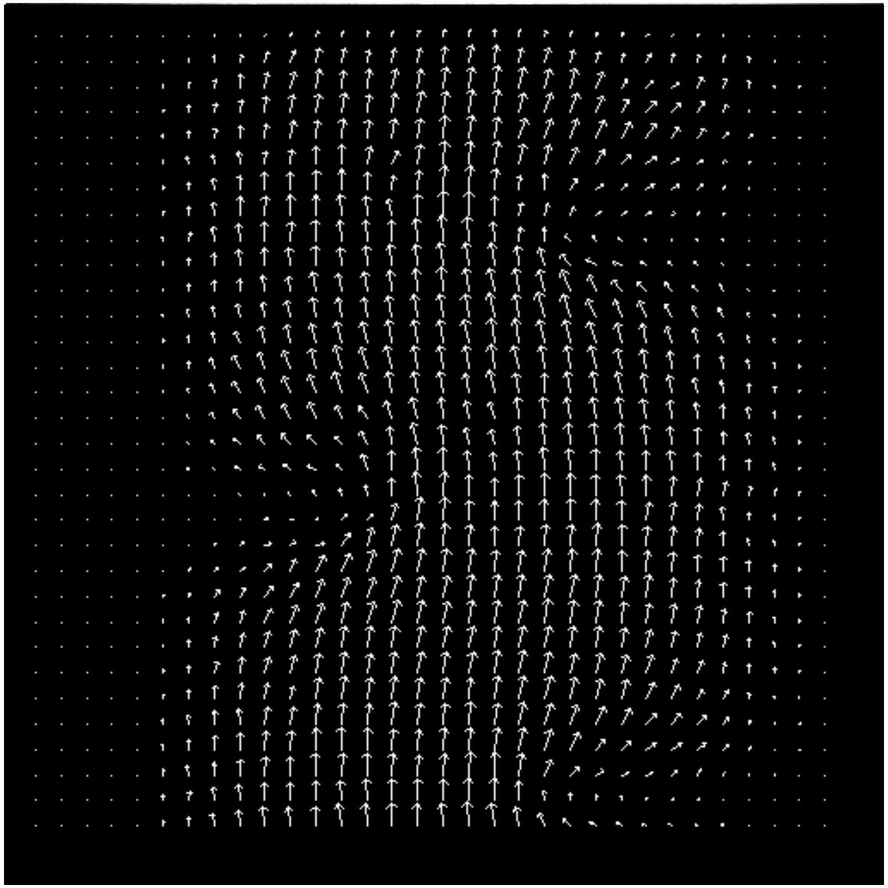

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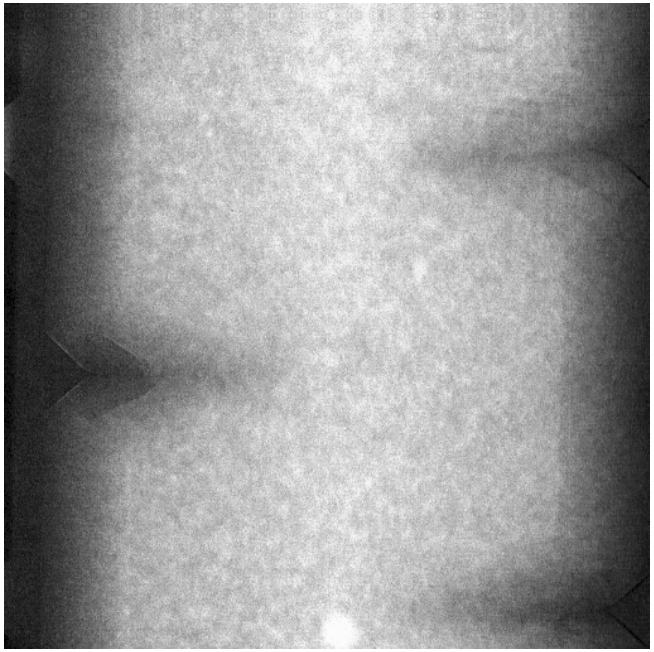

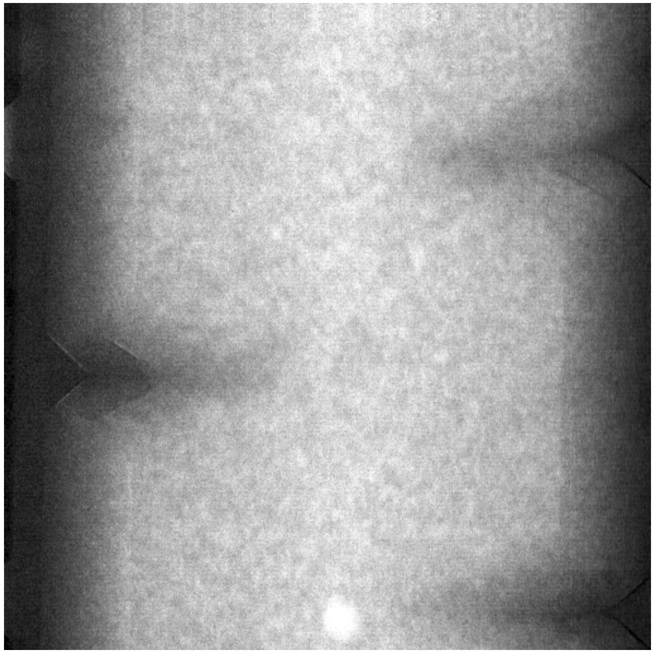

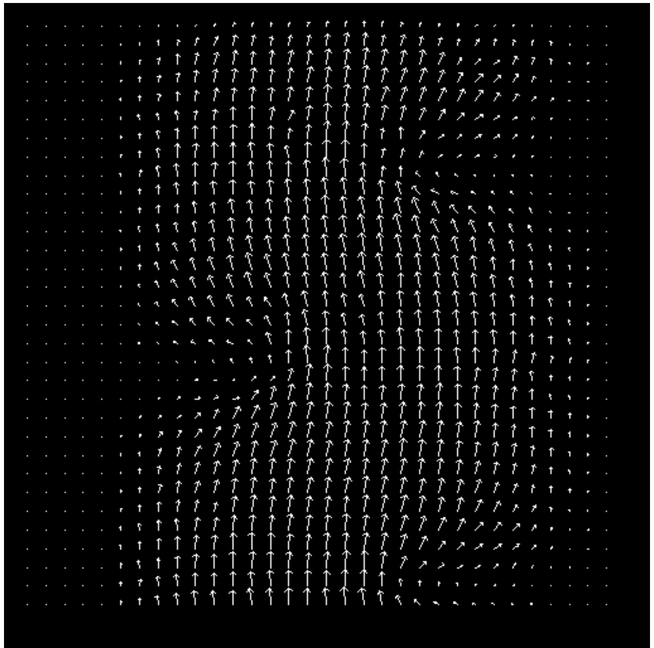

Gradient-based optical flow estimation: Additional examples presented in lecture

Outline

- Introduction
- Motion field vs. optical flow
- Brightness constancy
- Gradient-based optical flow estimation
- **Finite displacement and feature-based methods**
- 3D Structure and motion
- Summary

Finite displacement and features: Approaches

Motivation

- Gradient-based techniques work best when the displacements between the image are relatively small
  - This is implicit in the derivation of the optical flow constraint equation via differentials
  - Although course-to-fine processing can help with this limitation
- Well detected and localized features have the potential to be reasonably matched between images even in presence of much large displacements.
- Therefore, such approaches have received attention in conjunction with larger motion displacements.

Two broad classes of approach

- Methods for matching between binocular stereo pairs can be adapted to finite displacement image motion.
  - For example, the feature-based methods are particularly applicable
- Also of interest is the iteration of gradient-based optical flow
  - But restricted to interesting feature points

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Finite displacement and features: Iterated gradient

Basic idea

- We begin by extracting feature points of interest in image 1 of the input pair.
  - For example, the corner/line detector developed earlier in this class is well suited for this purpose
- We then center windows about a feature of interest and about the same location in the other image.

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$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \sum\sum E_x^2 & \sum\sum E_x E_y \\ \sum\sum E_x E_y & \sum\sum E_y^2 \end{pmatrix}^{-1} \begin{pmatrix} -\sum\sum E_x E_t \\ -\sum\sum E_y E_t \end{pmatrix}$$

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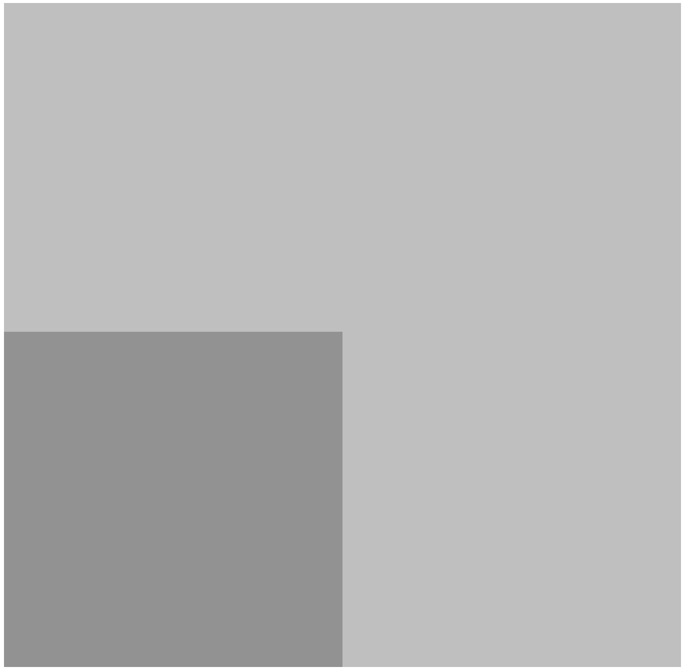
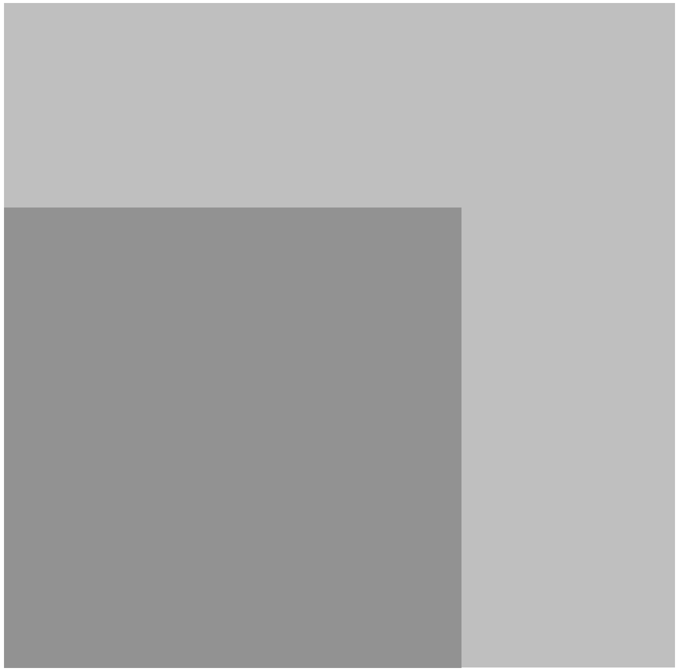
- We then calculate a similarity measure between the shifted window in image image 1 and the window in image 2.
- If the similarity is above some threshold, then we say that the match has been found and we exit.
- If the similarity measure is below some threshold, then we iterate the gradient-based calculation, but now making use of the shifted window in image 1.

Finite displacement and features: Iterated gradient





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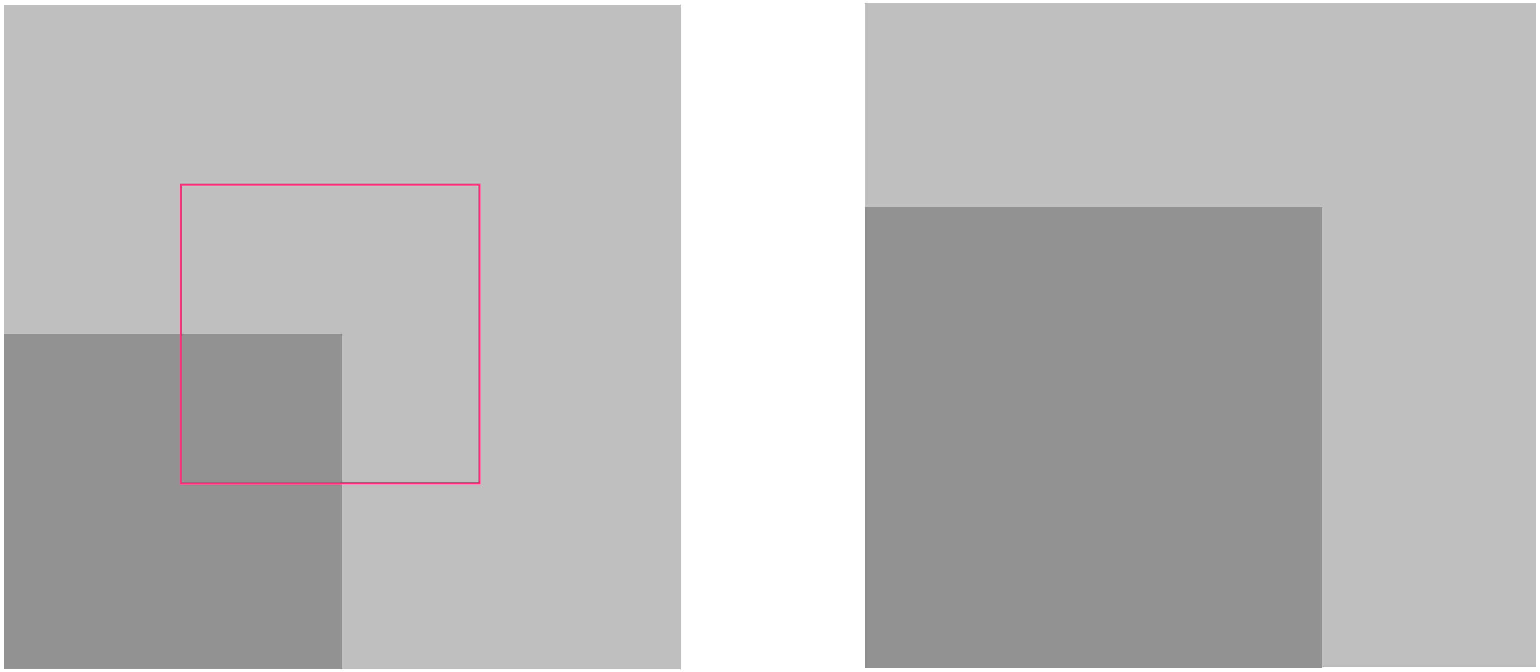

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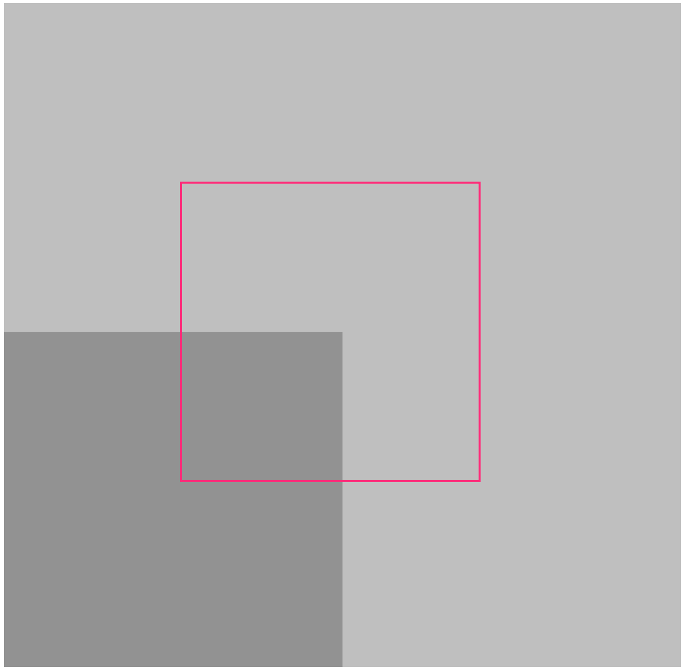

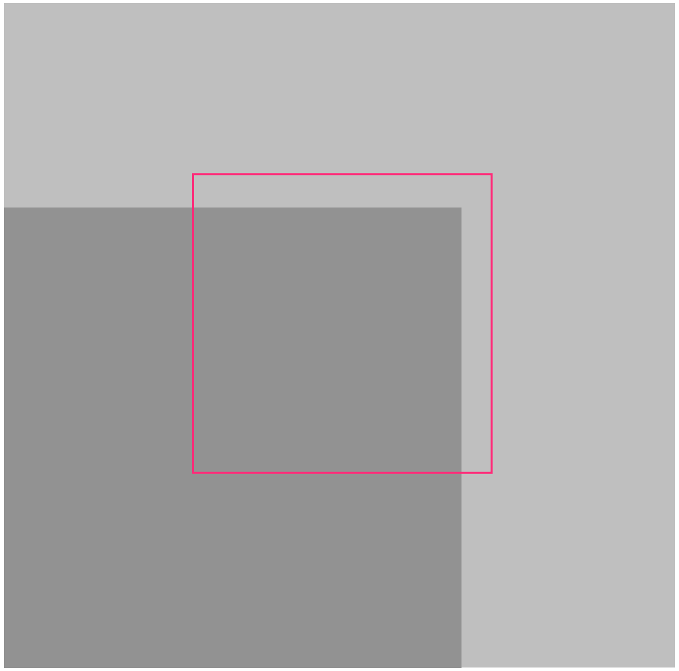

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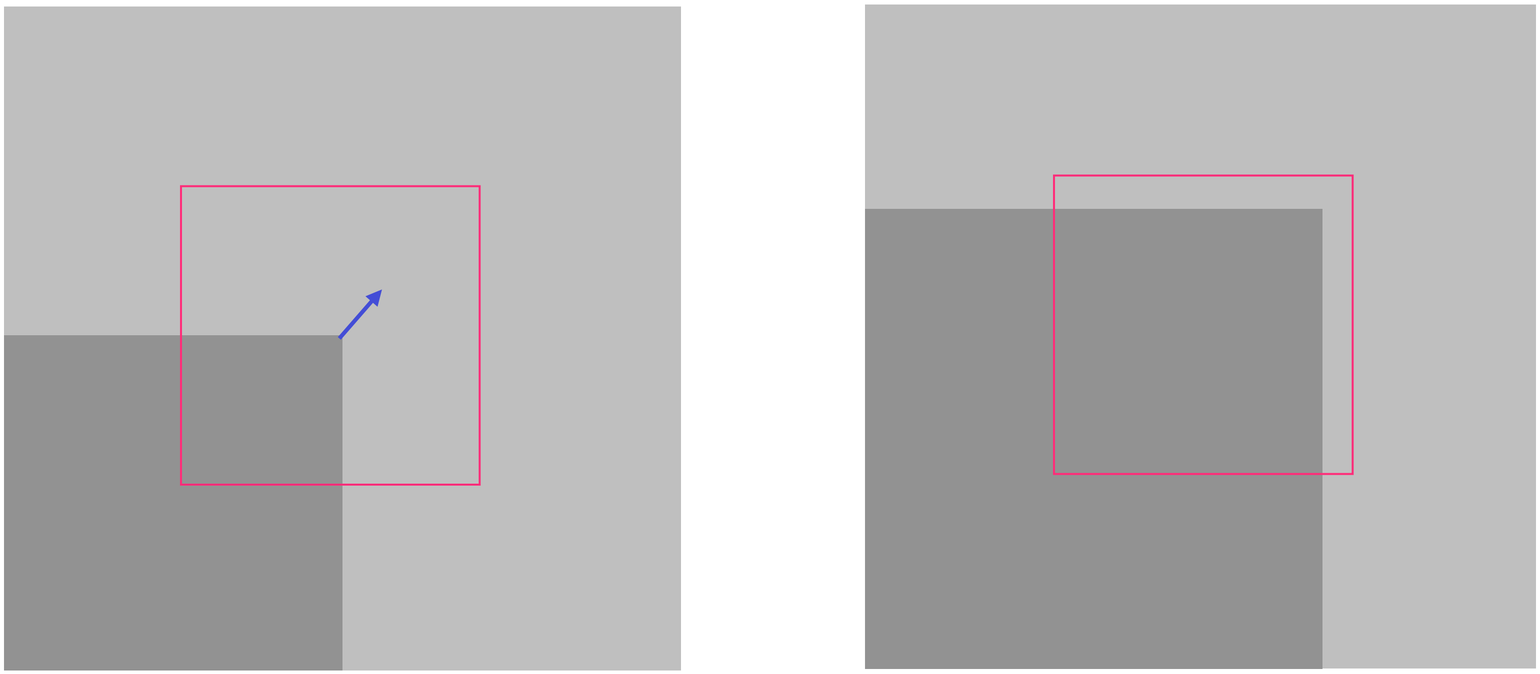

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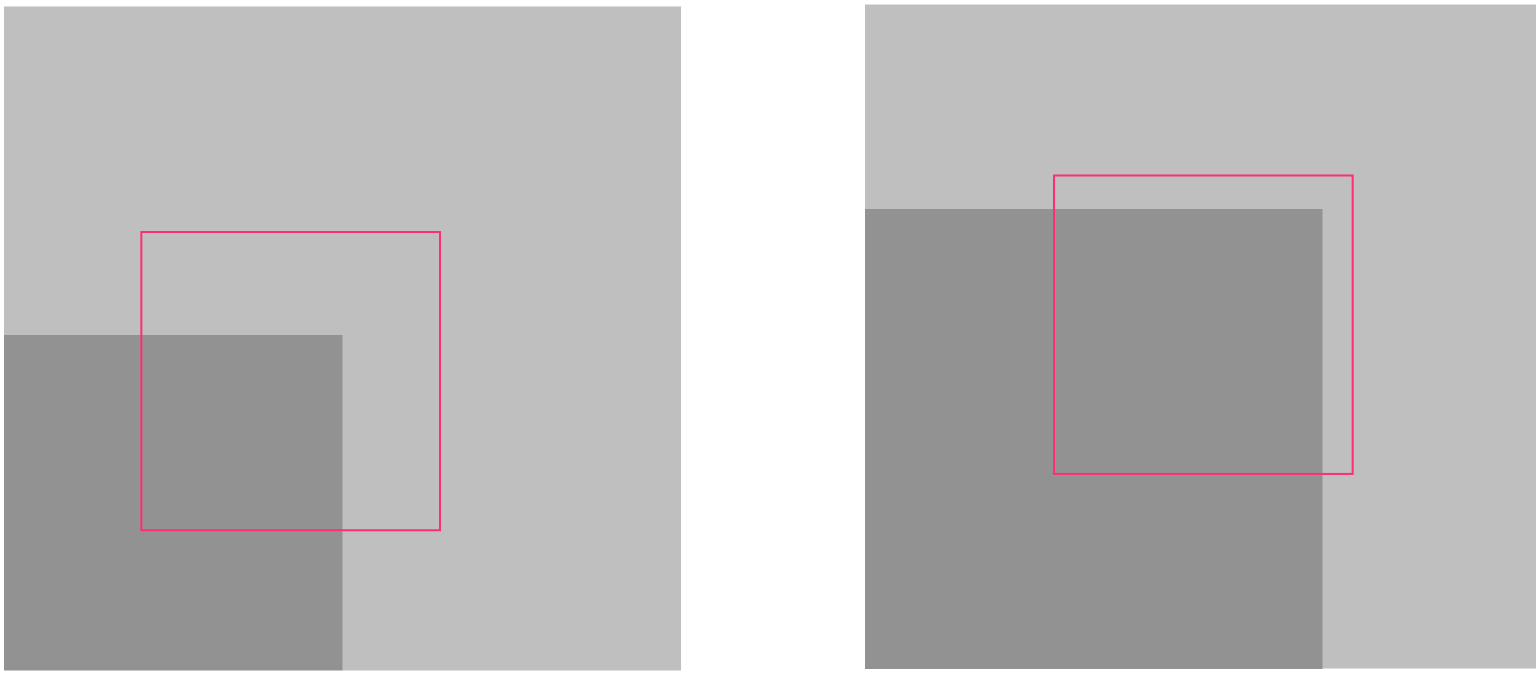

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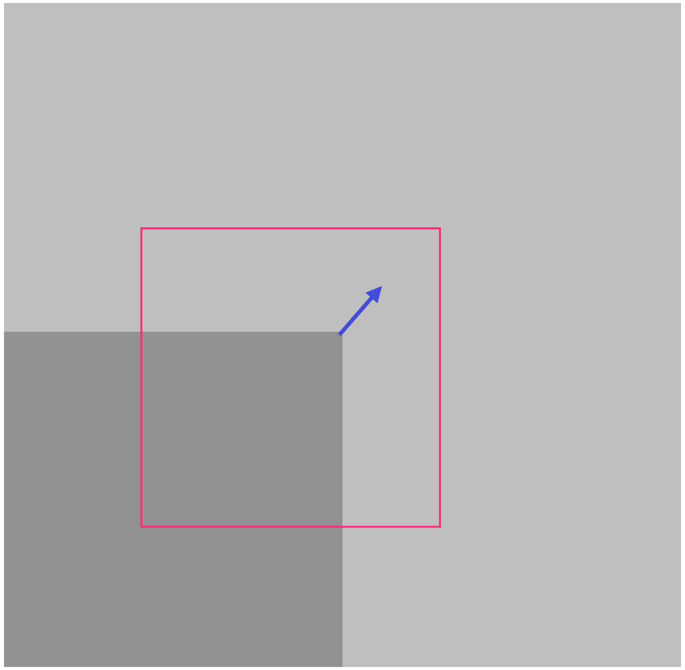

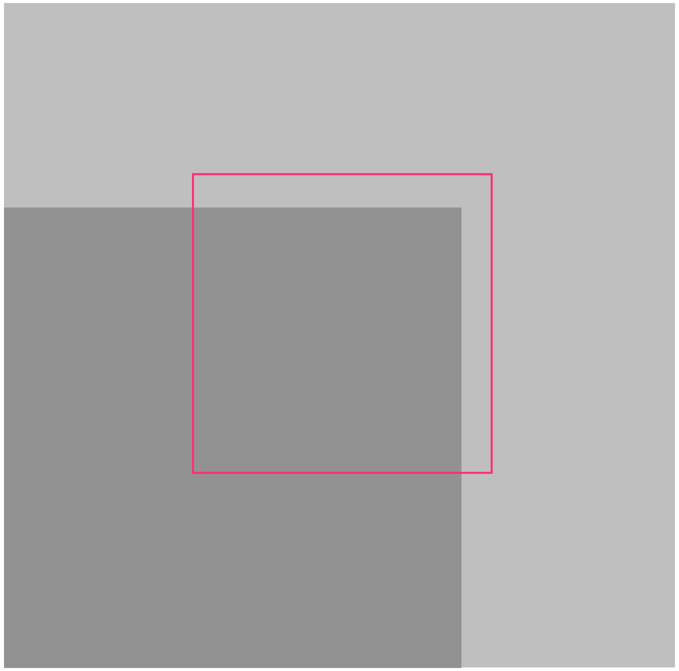

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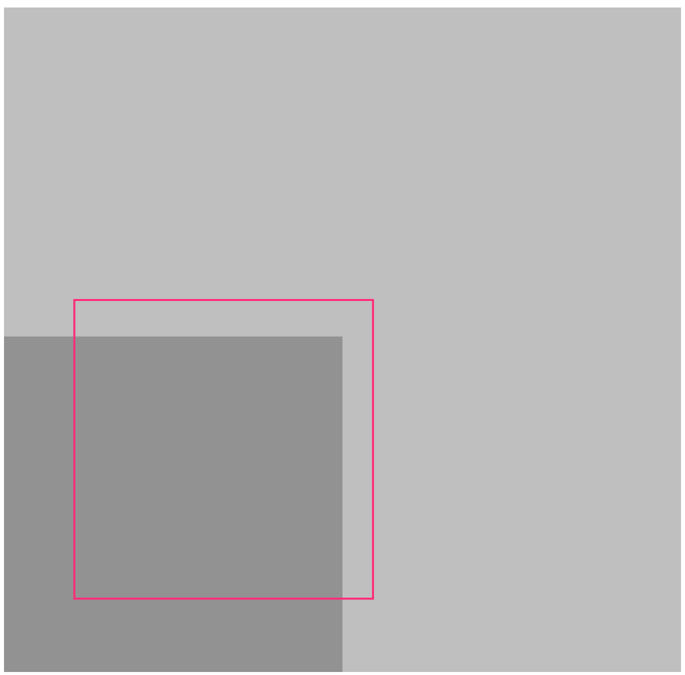

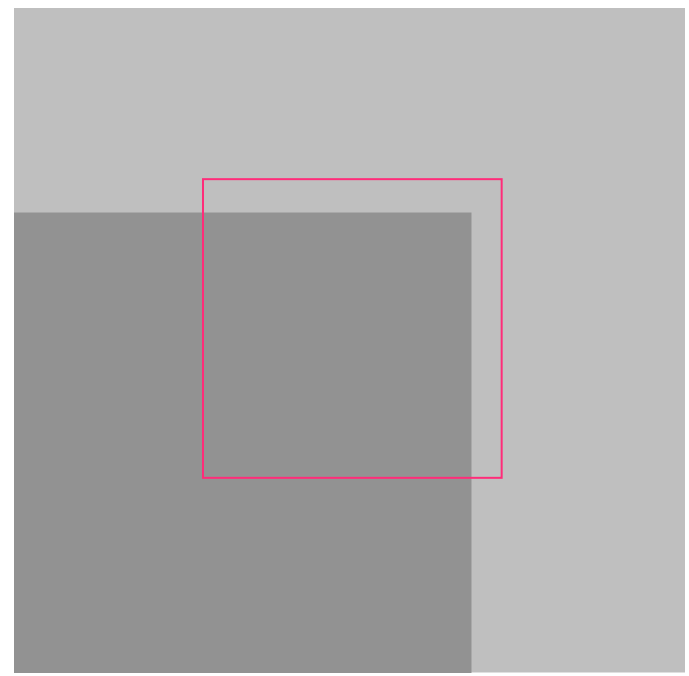

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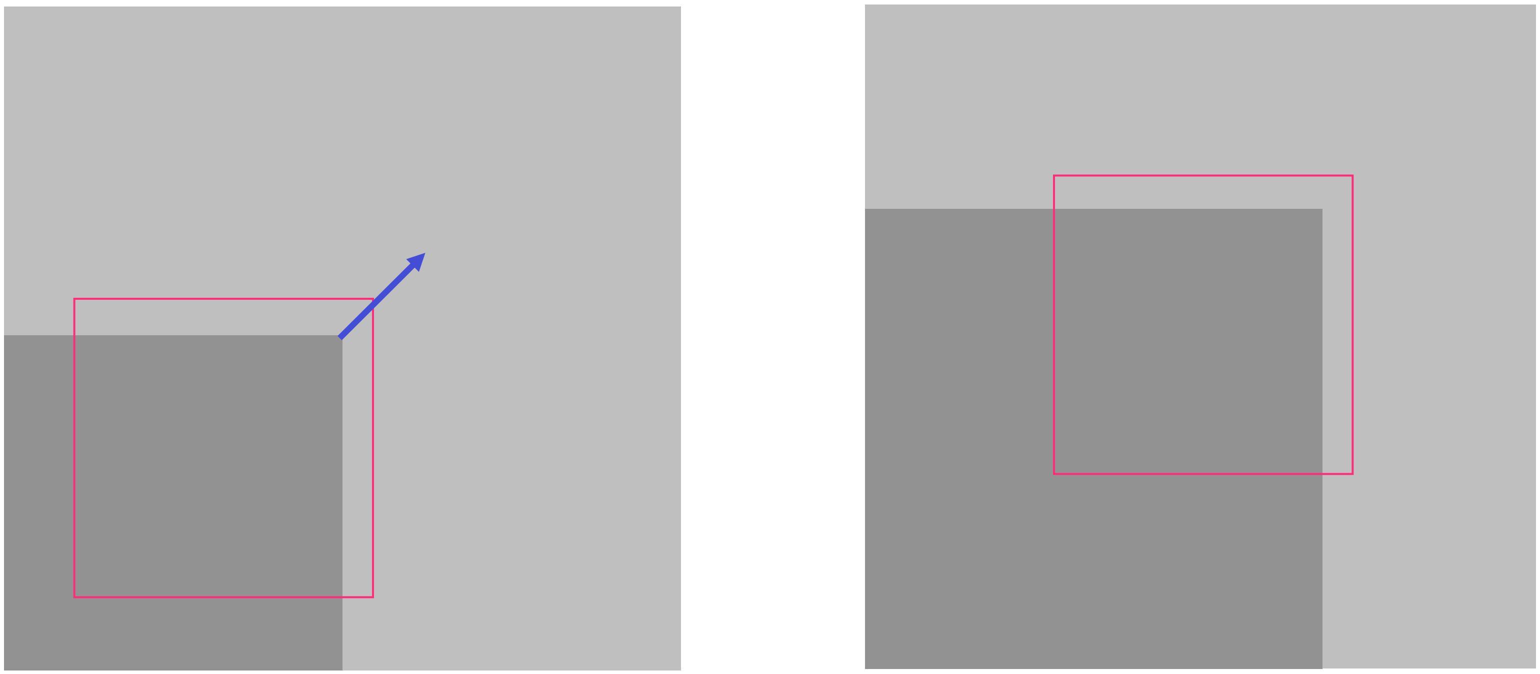

Finite displacement and features: Iterated gradient

Algorithm

- **Input:** Two images I1 and I2 and a set of features for I1
- **Output:** A set of displacements, one for each feature of I1.
- **Notation:** Let
  - *Q1, Q2 and* be two image windows
  - *t* be a threshold, a fixed positive real number
  - ***p*** be a feature point in I1
  - ***d*** be the unknown displacement for ***p***
- For each feature point ***p***
  1. Set ***d*** = 0 and centre *Q1* on ***p***
  2. Estimate the displacement ***d0*** of ***p*** centre of *Q1* according to the gradient-based algorithm
  3. Set ***d=d+d0***
  4. Let *Q2* be the image patch obtained by shifting *Q1* according to ***d0***.
     - Calculate the similarity, *S*, of *Q2* and the corresponding patch in I2
  5. If *S<t* then set *Q1=Q2* and goto 2; else exit.

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Representative similarity measure

- 1/(Sum of Squared Differences) within the windows of interest is a reasonable choice for this algorithm.

Finite displacement and feature-based estimation:

Example presented in lecture