EECS 4422/5323 Computer Vision

Unit 3: Image Features

Outline

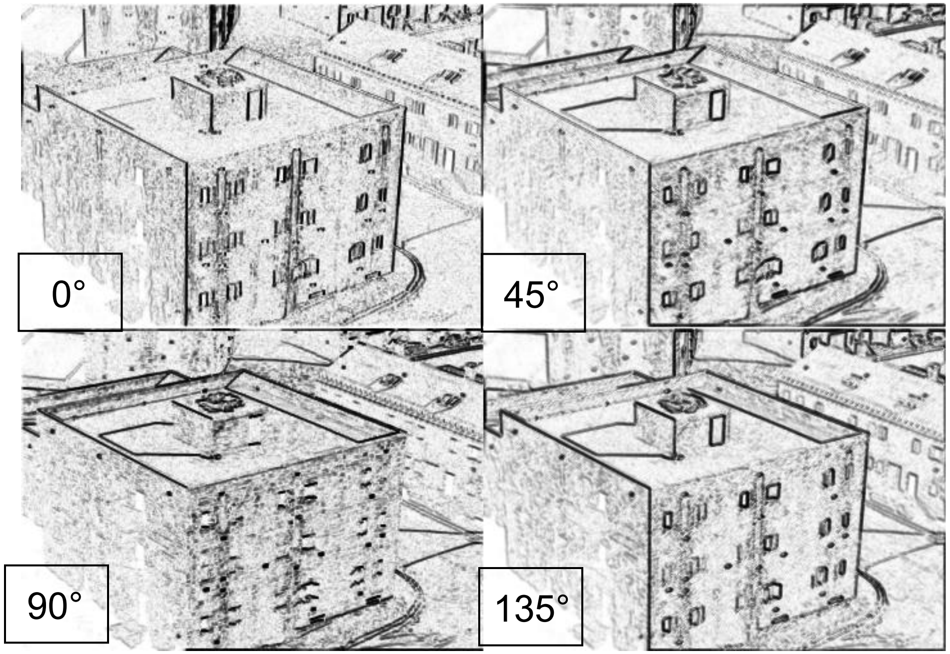
- Introduction
- Edge detection
- Corner detection
- Hough transform
- Deformable templates

Introduction: Motivation

Incremental abstraction

- We have investigated approaches to representing images so as to make local stuctural information explicit.
  - Local scale
  - Local orientation
- Information content in images can be fairly localized.
  - Abrupt changes in image irradiance: edges, corners,…
  - Configurations of intensity changes corresponding to simple patterns: extended lines, circles,…

Source image



Edge features detected at 4 orientations

Definition

- Image features are local, meaningful and detectable parts of an image.
- Local implies limited spatial support.
- Meaningful implies that they can be of use to subsequent operations.
- Detectable implies that we can develop an algorithm for extracting the position and description of these entities given image data.

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Edge detection: Basics

What is an edge

- Intuitively, an edge is a border between two regions, each of which have approximately uniform brightness.
- In an image, edges often arise as the result of occluding contours in an image
  - The two image regions correspond to two different surfaces.
- Other sources of image edges include
  - Abrupt changes in surface orientation
  - Discontinuities in surface reflectance

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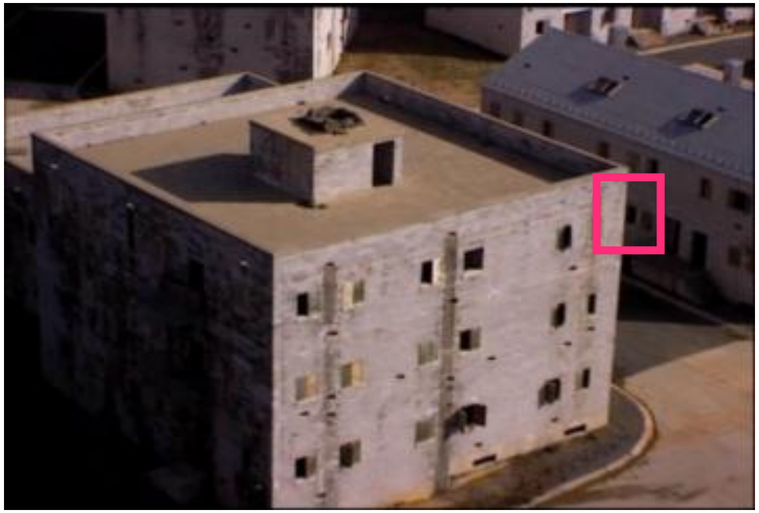

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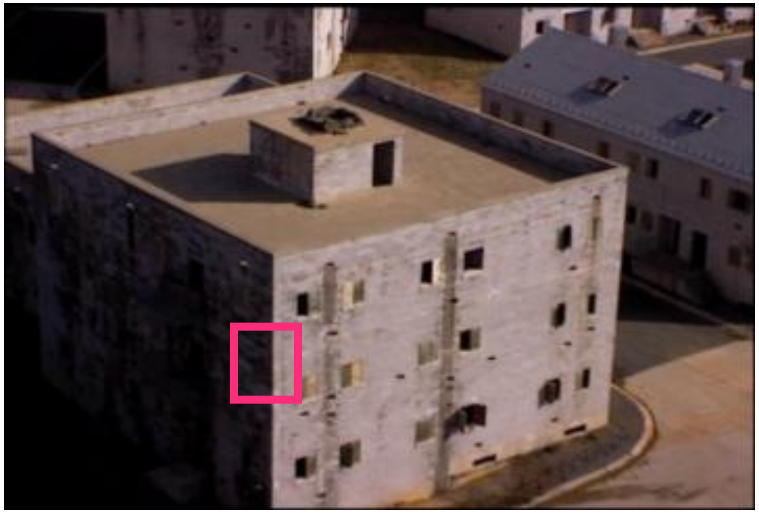

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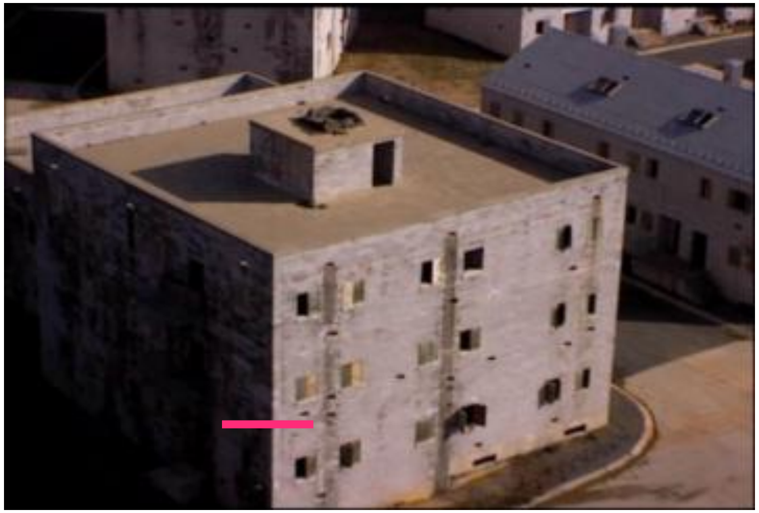

Simple model

- Consider a one-dimensional slice through an image in the vicinity of an abrupt change in image brightness.

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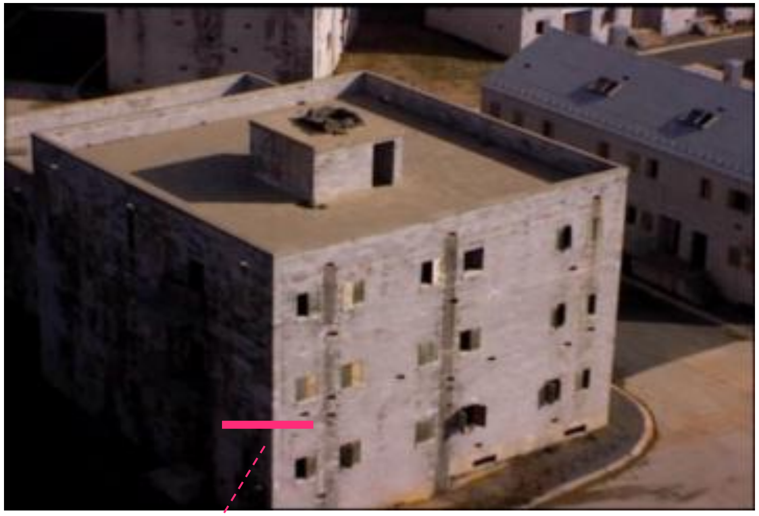
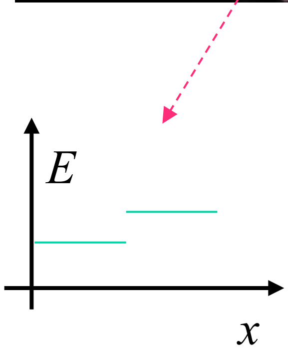
- Ideally, we might expect to find a step change in a plot of brightness vs. position.





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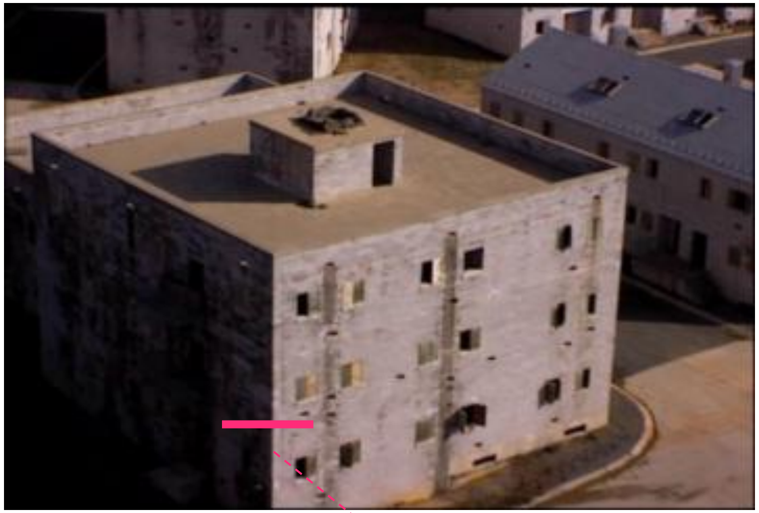
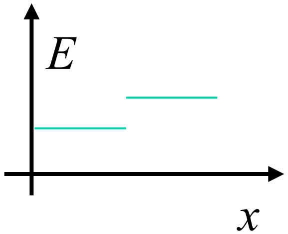
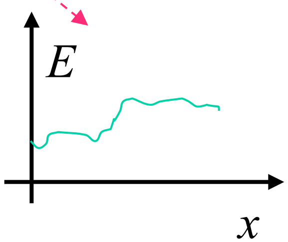


Simple model

- Consider a one-dimensional slice through an image in the vicinity of an abrupt change in image brightness.
- Ideally, we might expect to find a step change in a plot of brightness vs. position.
- In practice, we find a corrupted version of this ideal
  - Step transition smoothed
  - Noise transitions superimposed.



Ideal step edge



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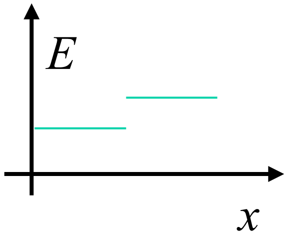
General approach

1. Suppress noise.
2. Enhance edges.
3. Locate edges.



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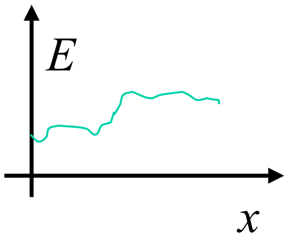

Real step edge

Edge detection: Differential operators for enhancement

Formalizing the edge model

- Keeping in mind the limitations of the ideal step edge model…
- We formalize the model as follows.

Edge detection: Differential operators for enhancement

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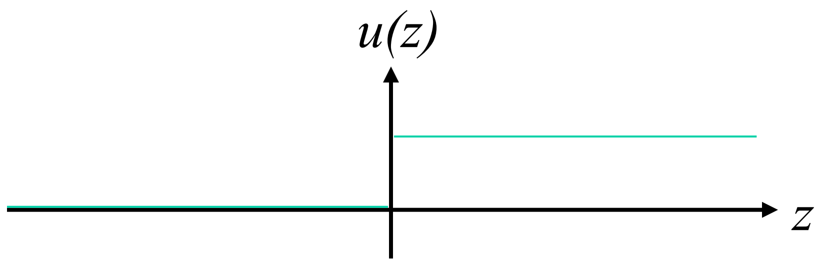
$$u(z) = \begin{cases} 1, z > 0 \\ 1/2, z = 0 \\ 0, z < 0 \end{cases}$$



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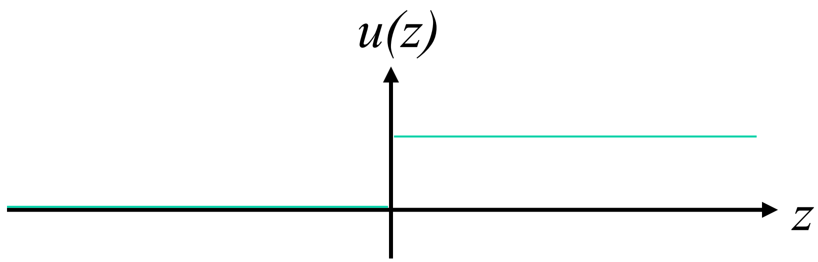

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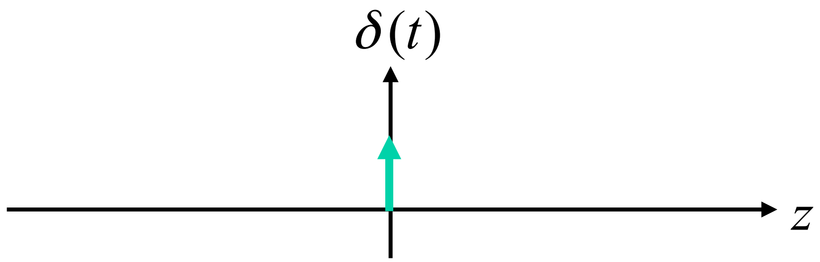
$$u(z) = \int_{-\infty}^{z} \delta(t)dt$$



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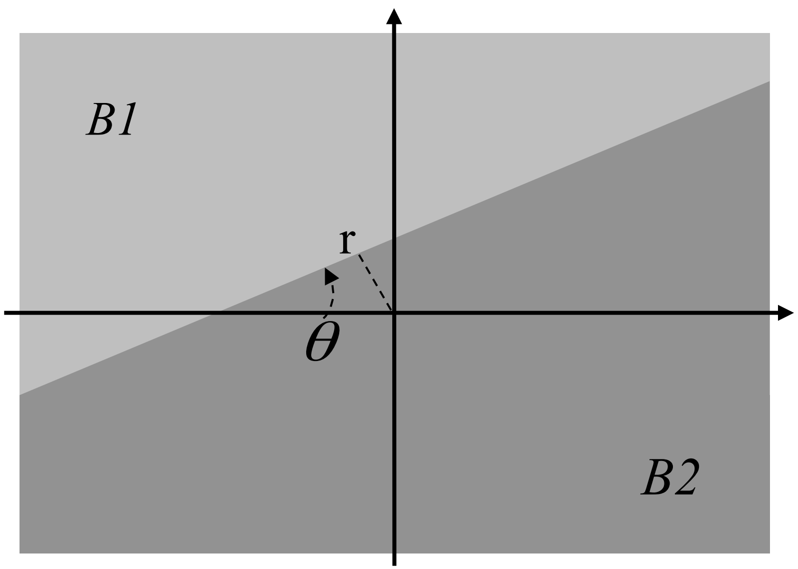
- Suppose that the edge lies along the line

$$x \sin\theta - y\cos\theta + r = 0$$



Interlude: r, θ line parameterization

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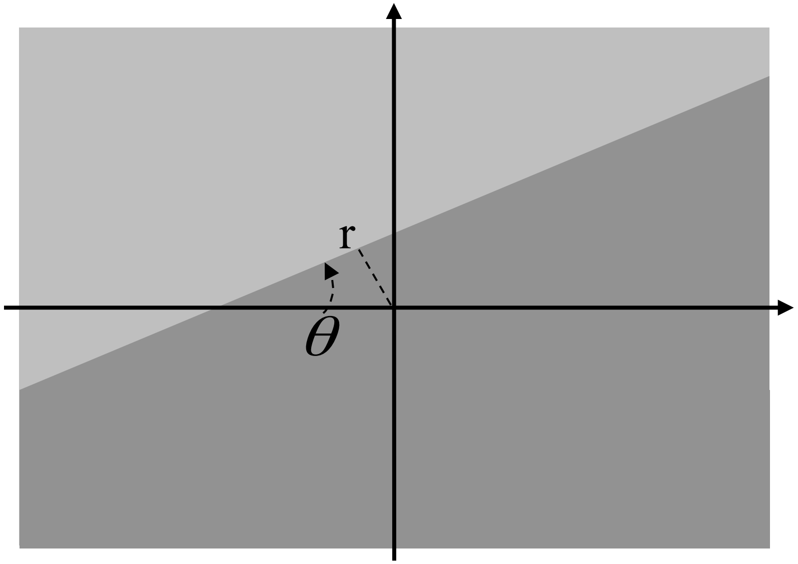

Observations

- The line intersects the x-axis at $-r/\sin\theta$
- The line intersects the y-axis at $r/\cos\theta$
- The closest point on the line to the origin is $(-r\sin\theta, r\cos\theta)$

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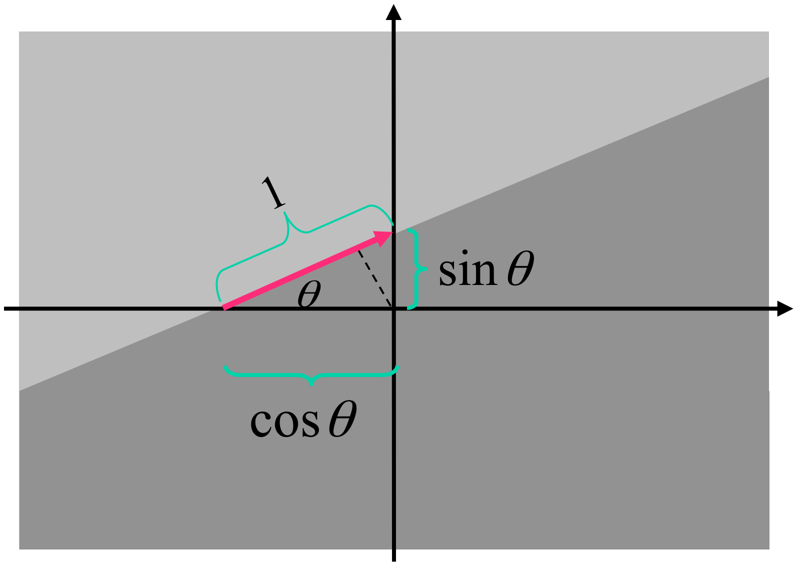

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$$x_0 = -r\sin\theta + s\cos\theta$$

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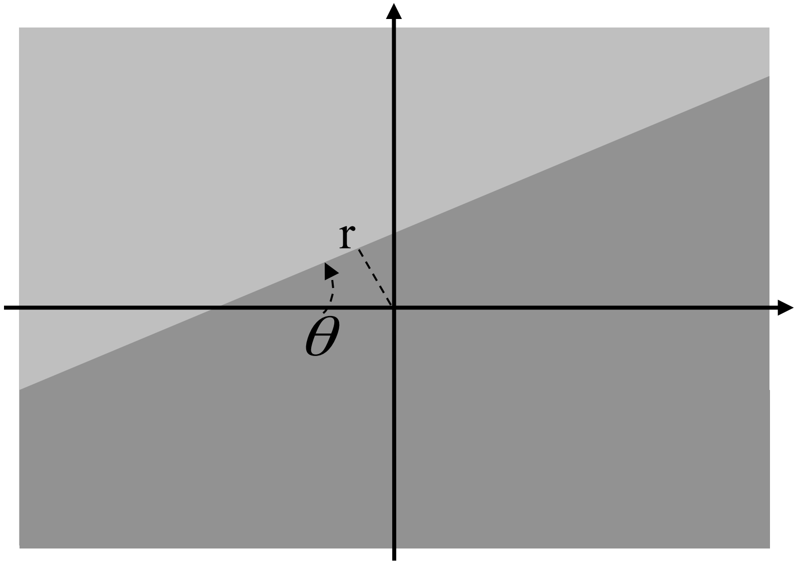

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- Given a point (x,y), find the nearest point (x0,y0) on the line and its distance.
- We define a distance

$$d^2 = (x - x_0)^2 + (y - y_0)^2$$

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- Substituting our parametric expressions for (x0,y0) we obtain

$$d^2 = (x^2 + y^2) + r^2 + 2r(x\sin\theta - y\cos\theta) - 2s(x\cos\theta + y\sin\theta) + s^2$$

- Differentiating WRT s and setting to zero leads to

$$s = x\cos\theta + y\sin\theta$$

- This result can be substituted back into the parameteric equations for (x0,y0).
- To find the distance to the line we compute the differences

$$x - x_0 = \sin\theta(x\sin\theta - y\cos\theta + r)$$

$$y - y_0 = -\cos\theta(x\sin\theta - y\cos\theta + r)$$

- And enter into the distance formula to yield

$$d^2 = (x\sin\theta - y\cos\theta + r)^2$$

- We conclude that the distance of a point from the line is given by the expression of the line itself!

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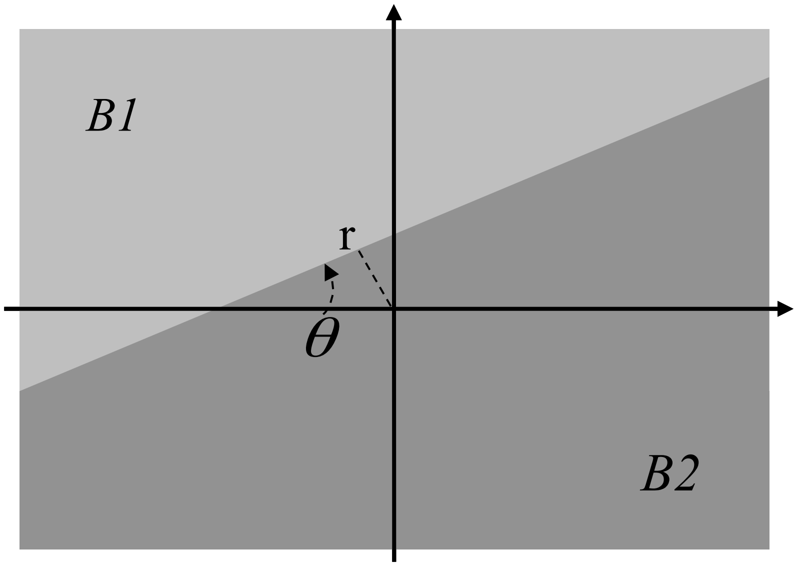
$$E(x, y) = B1 + (B2 - B1)u(x\sin\theta - y\cos\theta + r)$$



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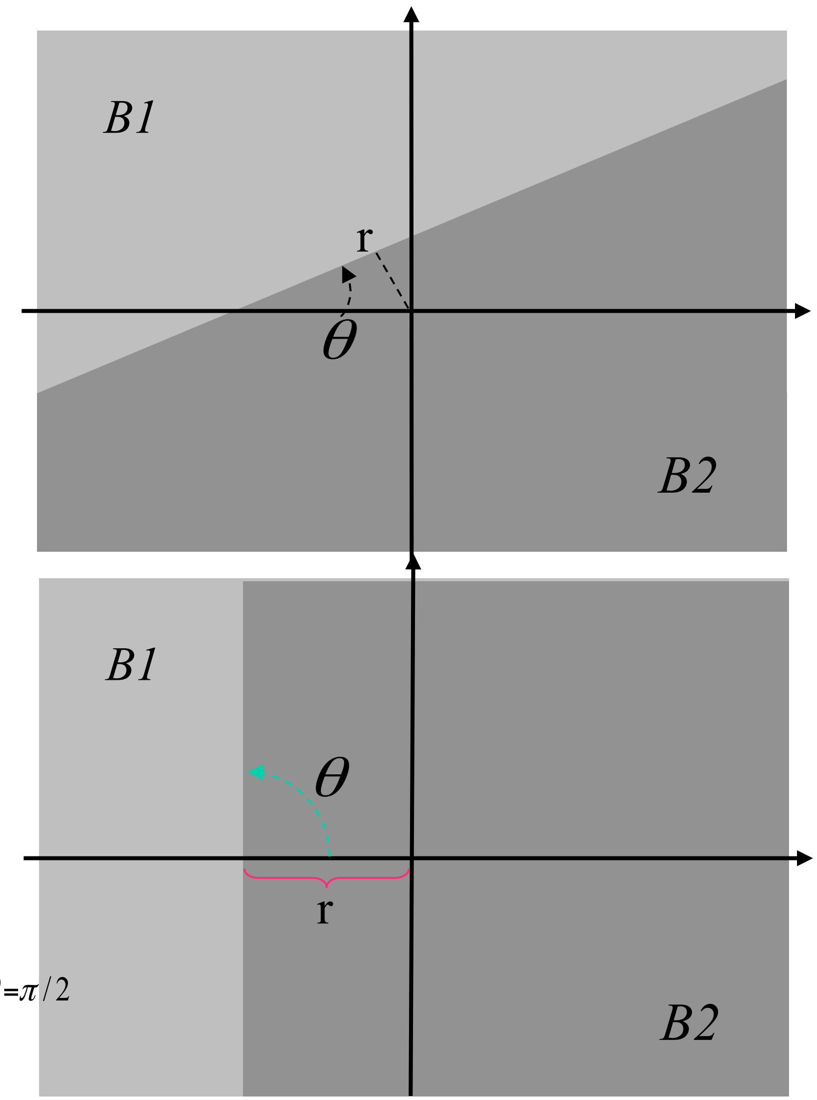

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$$E(x,y) = B1 + (B2 - B1)u(x\sin\theta - y\cos\theta + r)\big|_{\theta=\pi/2}$$
$$= B1 + (B2 - B1)u(x + r)$$

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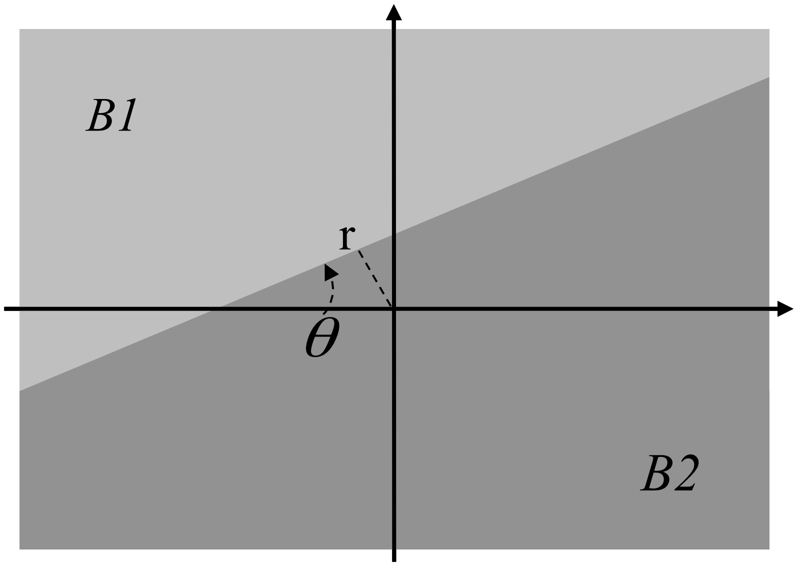

Edge detection: Differential operators for enhancement

The gradient

- Considering our model of the brightness

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$$E(x, y) = B1 + (B2 - B1)u(x\sin\theta - y\cos\theta + r)$$

- We can calculate the partial derivatives

$$\frac{\partial E}{\partial x} = \sin\theta(B2 - B1)\delta(x\sin\theta - y\cos\theta + r)$$

$$\frac{\partial E}{\partial y} = -\cos\theta(B2 - B1)\delta(x\sin\theta - y\cos\theta + r)$$

Recall: The "chain rule", let

$h(x)=g[f(x)]$

then

$h'(x)=g'[f(x)]\,f'(x)$

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- We define the brightness gradient as

$$\nabla E = \left(\frac{\partial E}{\partial x}, \frac{\partial E}{\partial y}\right)^{\mathrm{T}}$$

and note that it points along the direction of the edge transition.

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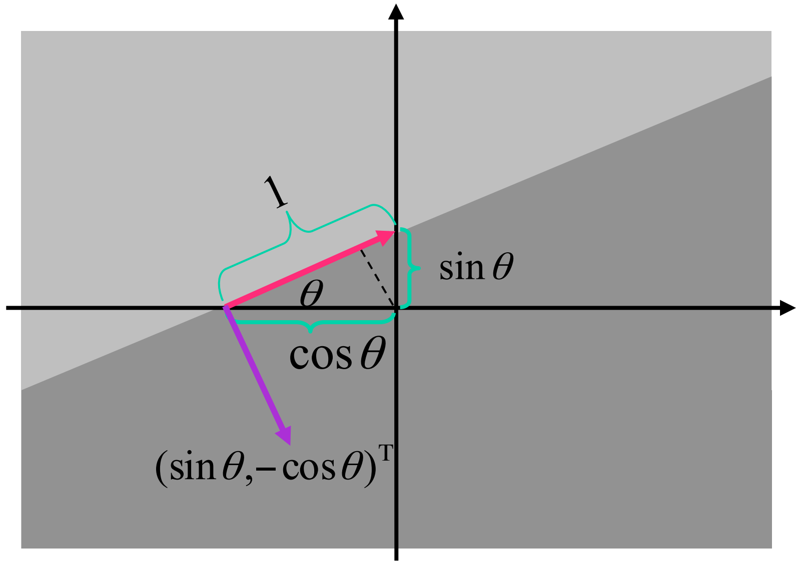

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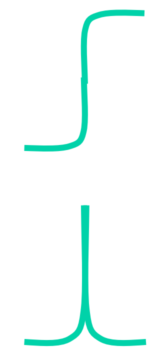

- Calculating the squared gradient

$$\left(\frac{\partial E}{\partial x}\right)^2 + \left(\frac{\partial E}{\partial y}\right)^2 = [(B2 - B1)\delta(x\sin\theta - y\cos\theta + r)]^2$$

we see that it has magnitude proportional to the brightness jump as we cross the step.

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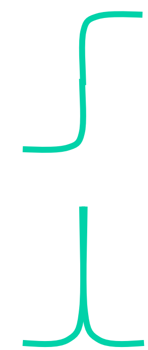

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- Remarks:
  - The response of this operator is independent of the edge orientation.
  - Calculation of the squared gradient is a nonlinear operation.

Edge detection: Differential operators for enhancement

The Laplacian

- Now let us consider the second (partial) derivatives

$$\frac{\partial E}{\partial x} = \sin\theta (B2 - B1)\delta(x\sin\theta - y\cos\theta + r)$$

$$\frac{\partial E}{\partial y} = -\cos\theta (B2 - B1)\delta(x\sin\theta - y\cos\theta + r)$$

$$\frac{\partial^2 E}{\partial x^2} = \sin^2\theta (B2 - B1)\delta'(x\sin\theta - y\cos\theta + r)$$

$$\frac{\partial^2 E}{\partial x \partial y} = -\sin\theta\cos\theta (B2 - B1)\delta'(x\sin\theta - y\cos\theta + r)$$

$$\frac{\partial^2 E}{\partial y^2} = \cos^2\theta (B2 - B1)\delta'(x\sin\theta - y\cos\theta + r)$$

where we make use of the notation δ' for the unit doublet, the derivative of the unit impulse (introduced previously).

Edge detection: Differential operators for enhancement

The Laplacian

- Keeping in mind that we have

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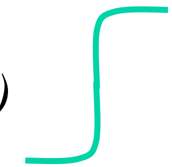
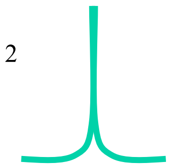
$E(x,y)$



- We define the Laplacian of the image as

$$\nabla^2 E = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = (B2 - B1)\delta'(x\sin\theta - y\cos\theta + r)$$

$$\left(\frac{\partial E}{\partial x}\right)^2 + \left(\frac{\partial E}{\partial y}\right)^2$$



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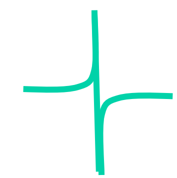
$\nabla^2 E$



Edge detection: Differential operators for enhancement

The Laplacian

- Keeping in mind that we have

$$\frac{\partial^2 E}{\partial x^2} = \sin^2\theta(B2 - B1)\delta'(x\sin\theta - y\cos\theta + r)$$

$$\frac{\partial^2 E}{\partial y^2} = \cos^2\theta(B2 - B1)\delta'(x\sin\theta - y\cos\theta + r)$$

- We define the Laplacian of the image as

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- We note that (like the squared gradient)
  - The response of the operator is independent of the edge orientation.

$E(x,y)$

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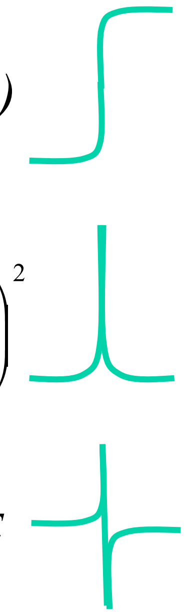

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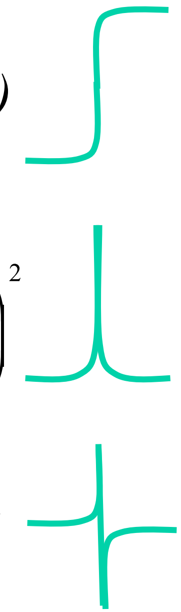
- Apparently, this operation will show a "zero-crossing" as we cross an edge.
- We note that (like the squared gradient)
  - The response of the operator is independent of the edge orientation.
- We note that (unlike the squared gradient)
  - The Laplacian is linear
  - The Laplacian preserves the sign of intensity change across the edge.

$E(x,y)$

$$\left(\frac{\partial E}{\partial x}\right)^2 + \left(\frac{\partial E}{\partial y}\right)^2$$

$\nabla^2 E$



Edge detection: Discrete approximations

The gradient

- Considering a 2x2 group of pixels

$E_{i,j+1}$	$E_{i+1,j+1}$
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Edge detection: Discrete approximations

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$E_{i,j+1}$	$E_{i+1,j+1}$
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- Using finite differences, we can then estimate the derivatives at the center of this group as

$$\frac{\partial E}{\partial x} \approx \frac{1}{2\varepsilon}[(E_{i+1,j+1} - E_{i,j+1}) + (E_{i+1,j} - E_{i,j})]$$

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$$\left(\frac{\partial E}{\partial x}\right)^2 + \left(\frac{\partial E}{\partial y}\right)^2 \approx \frac{1}{2\varepsilon}[(E_{i+1,j+1} - E_{i,j})^2 + (E_{i,j+1} - E_{i+1,j})^2]$$

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- Performing this calculation over an image of interest, we obtain large values where the image brightness is changing rapidly.
- We write the results in a new image array, in which the edges are strongly emphasized.

Edge detection: Discrete approximations

The Laplacian

- Considering a 3x3 group of pixels

$E_{i-1,j+1}$	$E_{i,j+1}$	$E_{i+1,j+1}$
$E_{i-1,j}$	$E_{i,j}$	$E_{i+1,j}$
$E_{i-1,j-1}$	$E_{i,j-1}$	$E_{i+1,j-1}$

Edge detection: Discrete approximations

The Laplacian

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- Using finite differences, we estimate the Laplacian at the center of this group using

$$\frac{\partial^2 E}{\partial x^2} \approx \frac{1}{\varepsilon^2}(E_{i-1,j} - 2E_{i,j} + E_{i+1,j})$$

$$\frac{\partial^2 E}{\partial y^2} \approx \frac{1}{\varepsilon^2}(E_{i,j-1} - 2E_{i,j} + E_{i,j+1})$$

Edge detection: Discrete approximations

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$$\frac{\partial^2 E}{\partial y^2} \approx \frac{1}{\varepsilon^2}(E_{i,j-1} - 2E_{i,j} + E_{i,j+1})$$

to yield

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} \approx \frac{4}{\varepsilon^2}\left[\frac{1}{4}(E_{i-1,j} + E_{i,j-1} + E_{i+1,j} + E_{i,j+1}) - E_{i,j}\right]$$

Edge detection: Discrete approximations

The Laplacian

- We notice that our discrete Laplacian operation

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essentially computes a average of the surrounding values and subtracts that of the center.

- We further note that this operation can be mapped onto our local 3x3 mask as

$$\frac{1}{\varepsilon^2}\begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 1 & -4 & 1 \\ \hline 0 & 1 & 0 \\ \hline\end{array}$$

- Indeed, we can use this mask as the (discrete) PSF in a (discrete) convolution to perform the necessary calculations.

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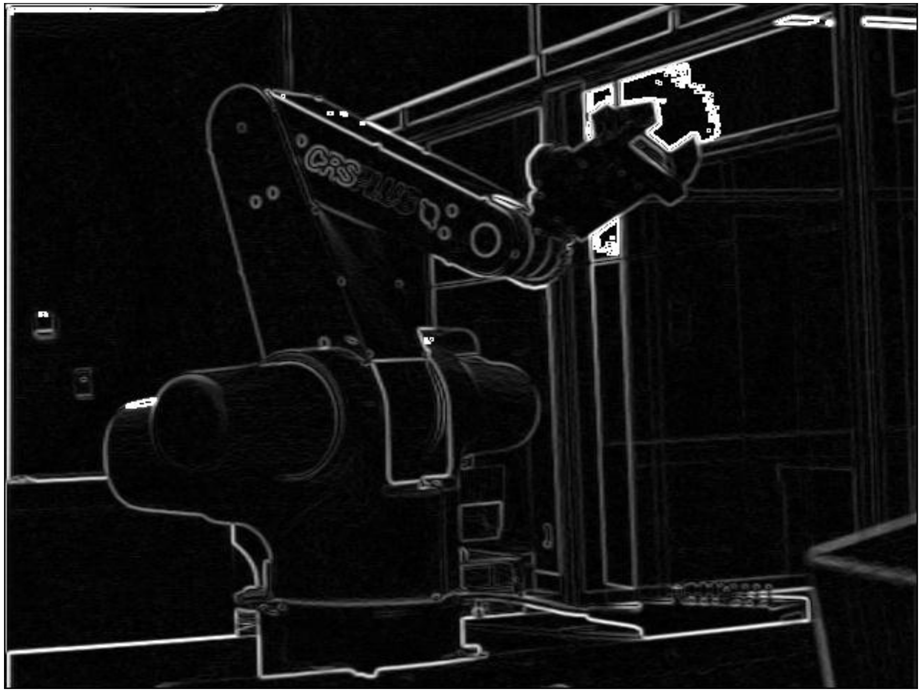
- Indeed, we can use this mask as the (discrete) PSF in a (discrete) convolution to perform the necessary calculations.
- Recall: Previously we noted that
  - the Laplacian is linear
  - And (more generally) differentiation is LSI

Edge detection: Example



Edge detection: Noise suppression

Local operators and noise

- In practice, application of the discrete operations that we have formulated can lead to poor results.
- Recalling that differentiation accentuates high frequency components of the image, we expect that (high frequency) noise will be accentuated as are the edges of interest.
- Our recourse is to rely on the observation that the edges of interest will (typically) have frequency components across a wider range of frequencies (especially) lower frequencies than the noise.

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- If this is the case, a useful noise suppression is to convolve the image with a Gaussian PSF

$$h(x,y) = \frac{1}{2\pi\sigma^2}\exp\left[-\frac{1}{2}\left(\frac{x^2+y^2}{\sigma^2}\right)\right]$$

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- Interestingly, recalling that
  - Derivatives can be implemented as convolutions
  - Convolution is associative

  we choose to combine the operations of noise suppression and smoothing via application of the PSFs

$$h_x(x,y) = -\frac{x}{2\pi\sigma^4} \exp\left[-\frac{1}{2}\left(\frac{x^2+y^2}{\sigma^2}\right)\right]$$

$$h_y(x,y) = -\frac{y}{2\pi\sigma^4} \exp\left[-\frac{1}{2}\left(\frac{x^2+y^2}{\sigma^2}\right)\right]$$

Edge detection: Noise suppression

Toward multiresolution analysis

- In applying the operators in practice,

$$h_x(x,y) = -\frac{x}{2\pi\sigma^4}\exp\left[-\frac{1}{2}\left(\frac{x^2+y^2}{\sigma^2}\right)\right]$$

$$h_y(x,y) = -\frac{y}{2\pi\sigma^4}\exp\left[-\frac{1}{2}\left(\frac{x^2+y^2}{\sigma^2}\right)\right]$$

  we frequently will need to select values of the standard deviation so that the resulting PSF has a large spatial support.
- More generally, we may choose to incorporate the notion of multiresolution processing and detect edges using a range of values for the standard deviation.

Edge detection: Example

Edge detection: Localization

A matter of thresholding

- Having enhanced the edge loci (hopefully while ameliorating the effects of noise).
- We must localize the edges per se.
- In essence, this comes down to selecting a threshold for accepting an (enhanced) image value as corresponding to an edge (as opposed to noise).
- For the gradient magnitude, we seek a decision point above which we will declare a value as marking an edge location.
- For the Laplacian, we seek a transition magnitude across the zero-crossing above which we will declare a value as marking an edge location.
- Remark: Having good a priori models of what is an edge and what is noise in a particular situation can provide a principled basis for threshold selection.

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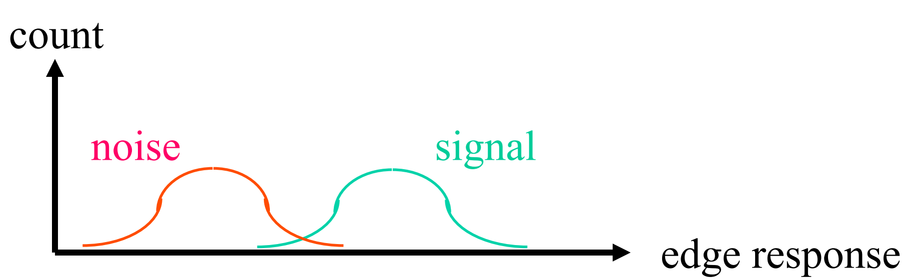

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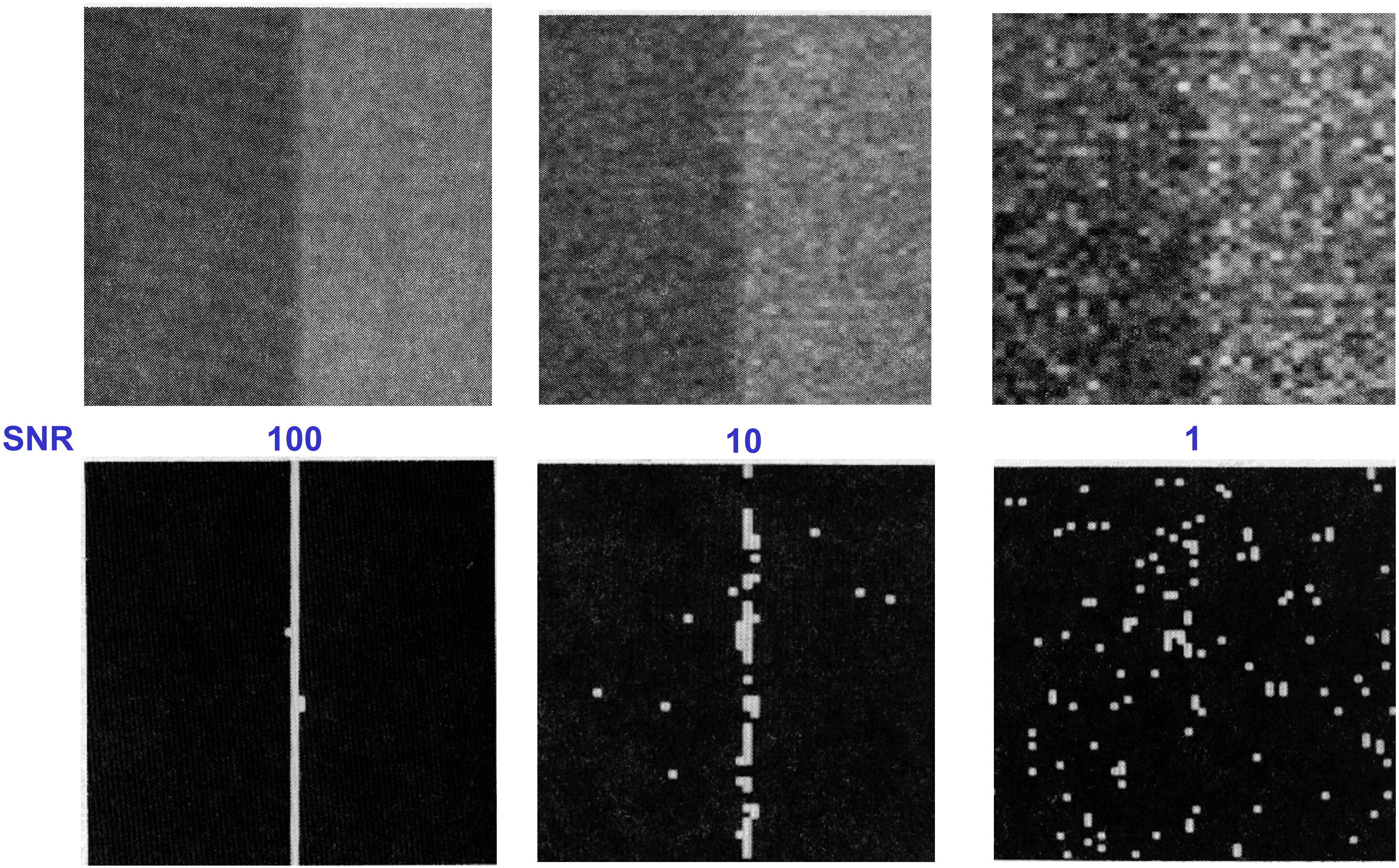


Edge detection: Additional examples presented in lecture

Edge detection: Recapitulation

Model
- Edges in the image appear as light/dark transitions; typically with physical meaning.
- The ideal step edge.
- But as corrupted by noise.

3 step process
- Suppress noise.
- Enhance edges.
- Locate edges.

Case studies
- (Squared) gradient.
- Laplacian.
- Lots of comparative examples.

Outline

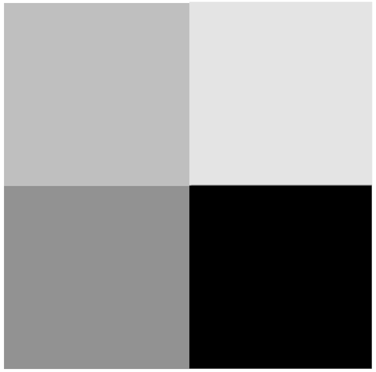
- Introduction
- Edge detection
- Corner detection
- Hough transform
- Deformable templates

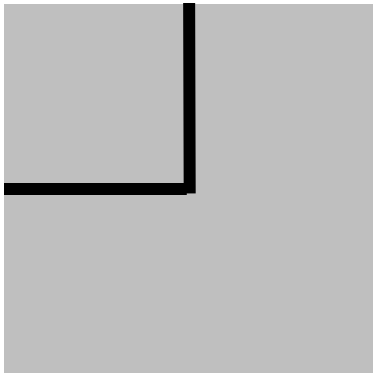
Corner detection: Basics

What is an corner

- A corner is an image location where two distinct image orientations occur in a local region.
- Physically, image corners tend to arise for similar reasons as edges (e.g., changes of reflectance, surface orientation).
- Corners are of interest for two main reasons

1. Corners provide constrain 2 degrees of freedom in a pattern's location.
2. Corners tend to persist across changes in viewpoint.





Corner detection: Differential analysis

The gradient

- Given that we are concerned with local measures of orientation, one approach is to calculate the local spatial derivatives Ex and Ey, using subscript notation for partial derivatives

Corner detection: Differential analysis

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- We choose to accumulate these measures over a neighorhood via summation and construct the matrix

$$\mathbf{C} = \begin{pmatrix} \sum E_x^2 & \sum E_x E_y \\ \sum E_x E_y & \sum E_y^2 \end{pmatrix}$$

- We appeal to this matrix because it encapsulates the local orientation structure as captured by the gradients.

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- To see this, note that we could choose a vector (x, y) which maximizes

$$\sum (E_x x + E_y y)^2 = \sum [(E_x, E_y) \bullet (x, y)]^2$$

as representing the local gradient direction.

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- We can rewrite this as expression as

$$\begin{aligned}\sum (E_x x + E_y y)^2 &= \sum (E_x^2 x^2 + 2E_x E_y xy + E_y^2 y^2) \\ &= \left(\sum E_x^2\right)x^2 + 2\left(\sum E_x E_y\right)xy + \left(\sum E_y^2\right)y^2 \\ &= x\left(\sum E_x^2\right)x + 2x\left(\sum E_x E_y\right)y + y\left(\sum E_y^2\right)y \\ &= (x, y)\begin{pmatrix} \sum E_x^2 & \sum E_x E_y \\ \sum E_x E_y & \sum E_y^2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}\end{aligned}$$

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  which brings us back to our matrix of concern.

Corner detection: Differential analysis

Eigenvalues

- Because the matrix

$$C = \begin{pmatrix} \sum E_x^2 & \sum E_x E_y \\ \sum E_x E_y & \sum E_y^2 \end{pmatrix}$$

is symmetric, we can diagonalize it by a rotation of the coordinates axes to yield a form

$$\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

where λ_1, λ_2 are the eigenvalues of the matrix.

Corner detection: Differential analysis

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$$\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}; C = R \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} R^{-1}$$

Remark: Columns of **R** are the Eigenvectors of **C**.

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Corner detection: Differential analysis

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$$\det\begin{pmatrix} \sum E_x^2 & \sum E_x E_y \\ \sum E_x E_y & \sum E_y^2 \end{pmatrix} = \sum E_x^2 \sum E_y^2 - \left(\sum E_x E_y\right)^2 \geq 0$$

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2. If the region contains and ideal step edge, then there is only one gradient direction: $\lambda_1 > 0, \lambda_2 = 0$
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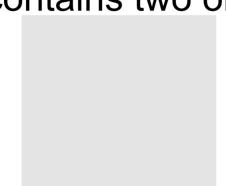

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$$\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

where λ_1, λ_2 are the eigenvalues of the matrix.

- Remark: Owing to the fact that our matrix is positive definite (e.g., the determinant is >0), we are guaranteed that both of the eigenvalues are positive (or perhaps 0 for degeneracies).
- Consider three cases

1. If the region of interest is perfectly uniform, then the gradients are identically zero: $\lambda_1 = \lambda_2 = 0$
2. If the region contains and ideal step edge, then there is only one gradient direction: $\lambda_1 > 0, \lambda_2 = 0$
3. If the region contains two orientations, then there are two gradient directions: $\lambda_1 \geq \lambda_2 > 0$

Corner detection: Differential analysis

Eigenvalues

- Because the matrix

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- In conclusion
  - The eigenvalues capture edge strength.
  - The eigenvectors capture edge direction.

Corner detection: Differential analysis

What we have learned

- For the summed gradient matrix, **C**
  - The eigenvectors capture edge direction.
  - The eigenvalues capture edge strength.

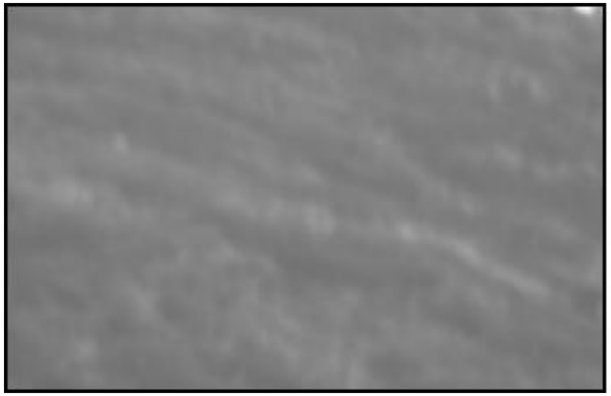
Resulting approach

- Detection of corners and lines
  - For each point in an image of interest
  - Construct the 2x2 summed image gradient matrix, **C**
  - Calculate the eigenvalues of **C**
  - If the eigenvalues are similar (nonzero) magnitude, then a corner is marked.
  - Also, when only one eigenvalues is nonzero, then a line can be marked.

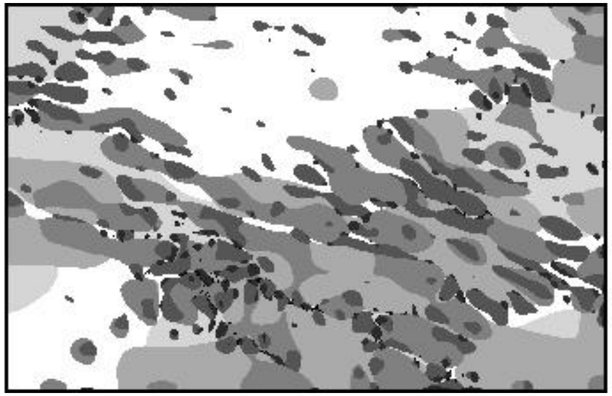
Corner detection: Exploiting local orientation estimates

An alternative image measurement

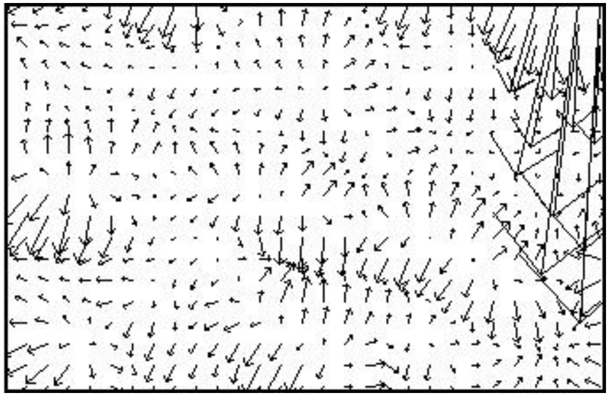
- Previously, we developed the ability to decompose images according to their local orientation structure
  - E.g., via convolution with Gabor filters
- As an application, we noted the abililty to determine the locally dominant orientation and its magnitude
  - E.g., by scanning across the oriented bandpass decomposition for (locally) largest magnitudes



Source image
(natural terrain)



Locally dominant scale
(darker intensity for finer scale)



Locally dominant orientation
(shown as normal vector)

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- We can exploit that analysis in the present context
  - Rather than construct our corner (and line) detection matrix with image gradients
  - Use the locally dominant orientation magnitudes as projected on the coordinate axes.
- Let
  - The dominant orientation be recovered as $(\cos w, \sin w)$
  - The corresponding response magnitude be given as r
  - Then replace Ex with $r\,(\cos w, \sin w).(1,0) = r \cos w$
  - And replace Ey with $r\,(\cos w, \sin w).(0,1) = r \sin w$
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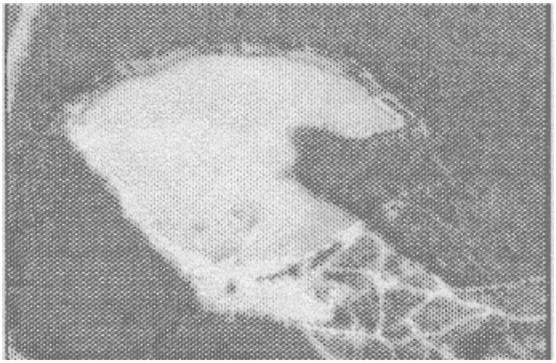
$$\begin{pmatrix} \sum E_x^2 & \sum E_x E_y \\ \sum E_x E_y & \sum E_y^2 \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} \sum r^2 \cos^2 w & \sum r^2 \cos w \sin w \\ \sum r^2 \cos w \sin w & \sum r^2 \sin^2 w \end{pmatrix}$$

Corner/line detection: Example



Source image

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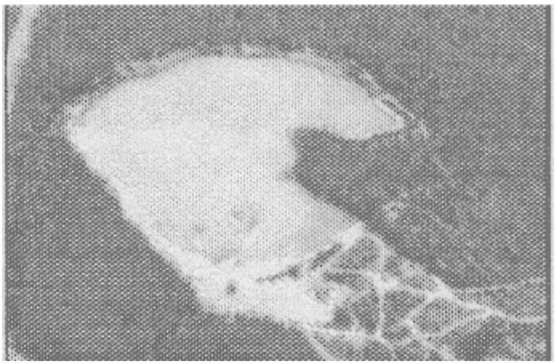

Source image

Filtered response magnitude at 4 orientations

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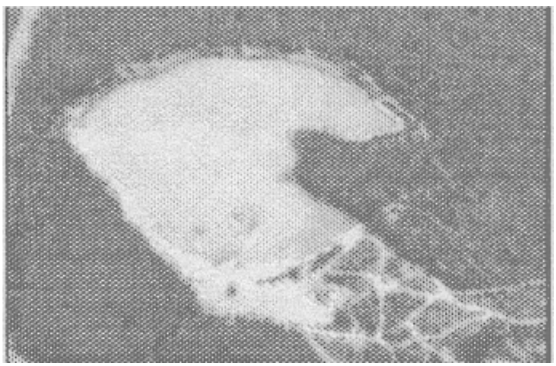

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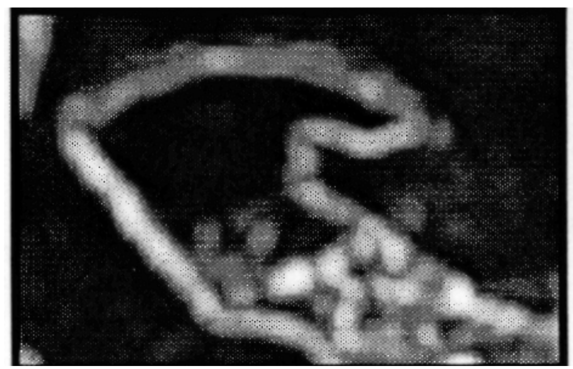

Detected lines

Detected corners

Filtered response magnitude at 4 orientations

Corner detection: Recapitulation

Model

- Image loci where multiple orientations are present.
- Local orientation structure captured by summed gradient matrix.
- Approach also captures line structure.

3 step process

- Recover local estimates of image orientation structure: direction and magnitude
- Accumulate local measures of orientation structure into summed gradient matrix.
- Perform eigenvalue decomposition.

Case studies

- Image gradient based measurements
- More general oriented filtering based measurements
- Natural image example.

Outline

- Introduction
- Edge detection
- Corner detection
- **Hough transform**
- Deformable templates

Detecting contour features: Beyond simplest features

Motivation

- So far, the features of interest (edges, corners) have been defined on a purely local basis.
- Now consider configurations of contours that correspond to more complicated geometries
  - Extended lines
  - Circles
  - Simply parameterized objects
- Two types of approach covered

1. First extract edge features; then fit the model.
2. Fit the model more directly to an (enhanced) image.

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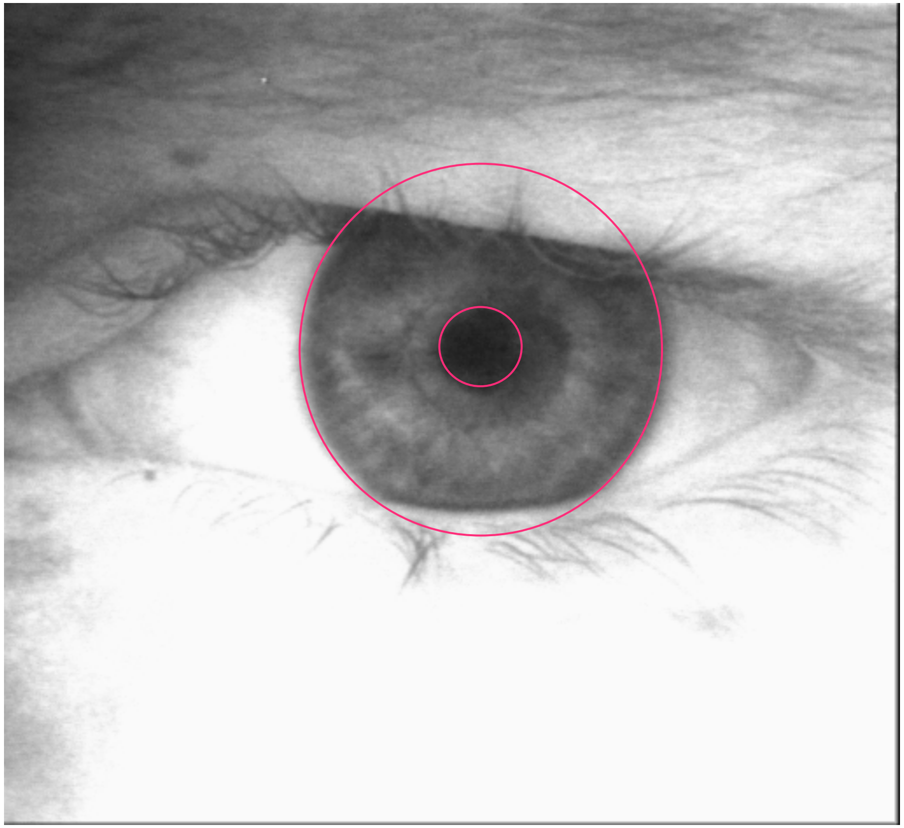

Hough transform: Introduction

Basic idea

- The Hough transform was introduced to detect patterns of points in binary images.
- It thus corresponds to the class of techniques that assume edge (or some other primitive detection) already has marked points of interest in an image.
- The key idea:
  - transform a potentially difficult problem: Detection of a relatively complex pattern in the image domain
  - into a simpler problem of peak detection in the space of the pattern's parameters

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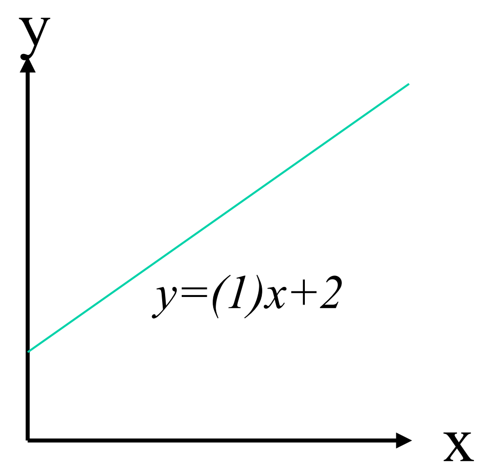
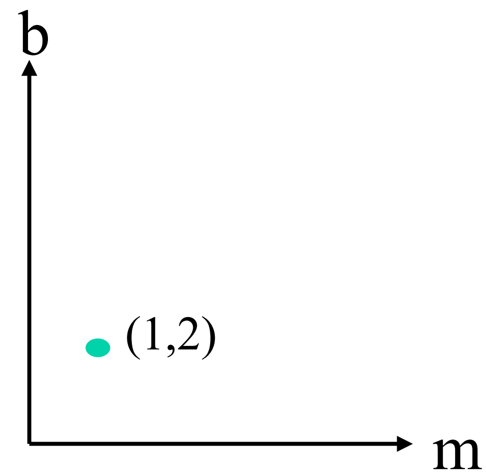
  - transform a potentially difficult problem: Detection of a relatively complex pattern in the image domain
  - into a simpler problem of peak detection in the space of the pattern’s parameters

Example

- Detection of lines
  - Suppose we represent a line as $y = mx + b$
  - Move from the space of image position (x,y)
  - To the space of line parameters (m,b).



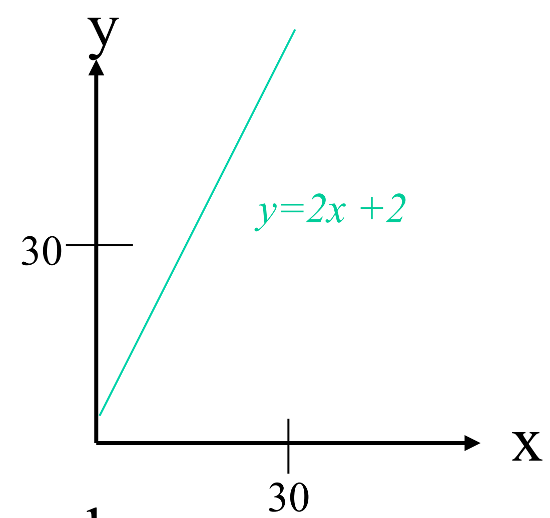


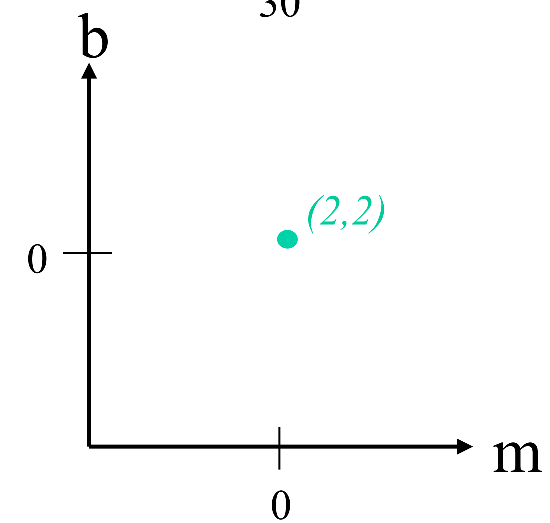
Hough transform: Line detection

A closer look (in 2 parts)

1. Transform line detection into line intersection

- Any line $y = mx + b$ is uniquely identified by a parameter pair (m,b).
- The line is represented by a point in the (m,b) plane (parameter space).





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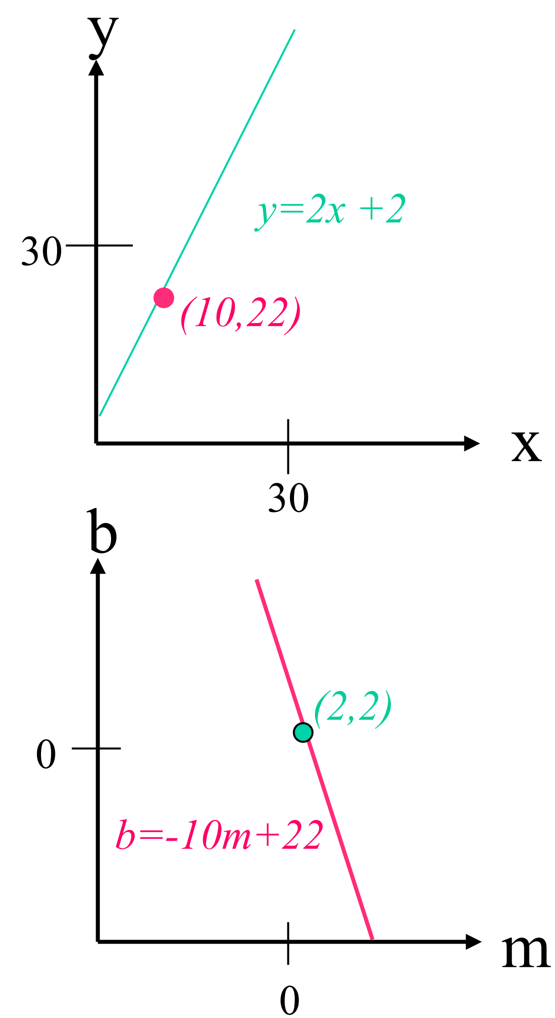

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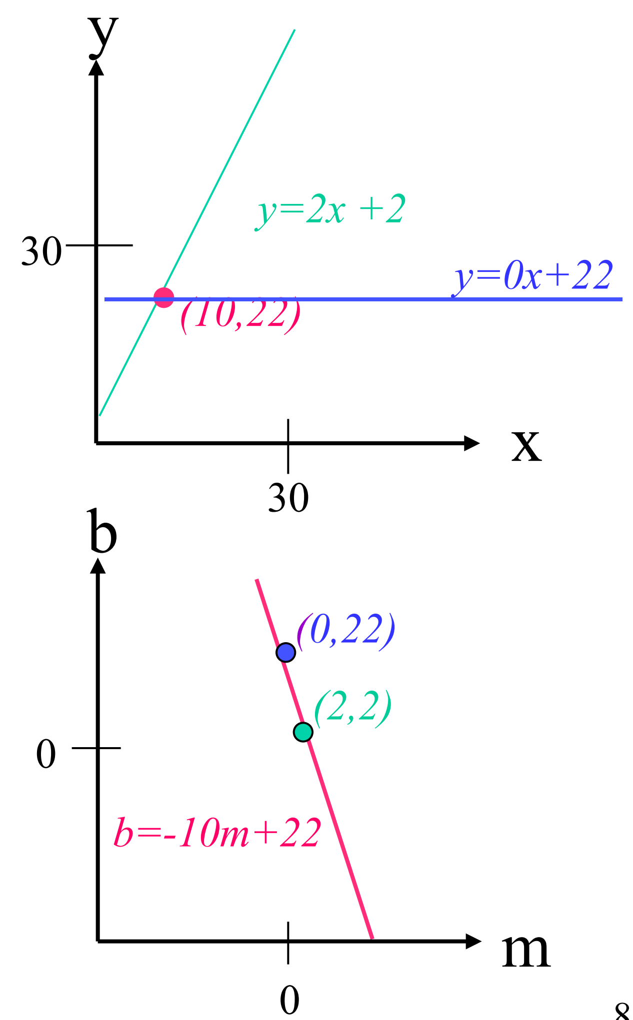

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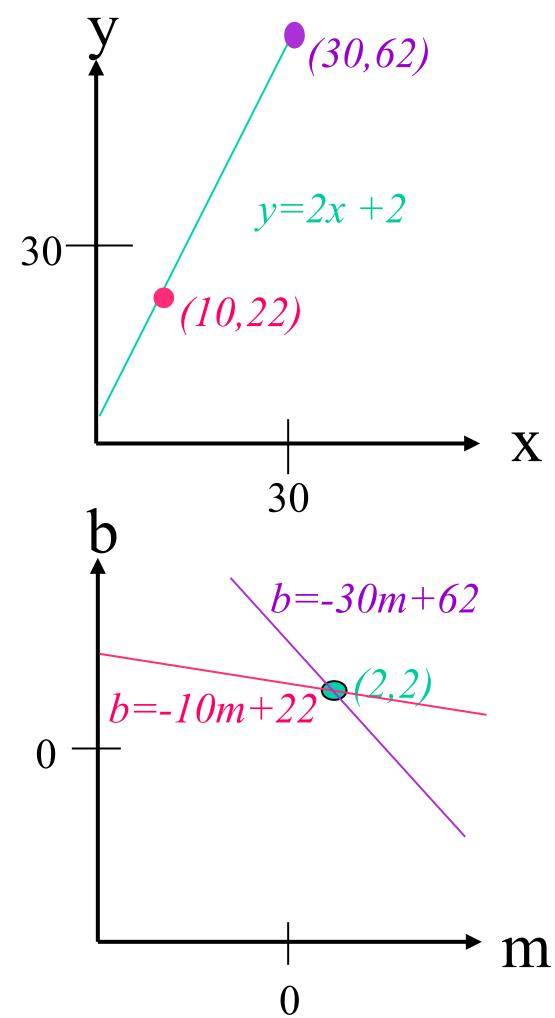
- So, a line defined by N collinear image points is identified in parameter space by the intersection of the N associated lines in parameter space.

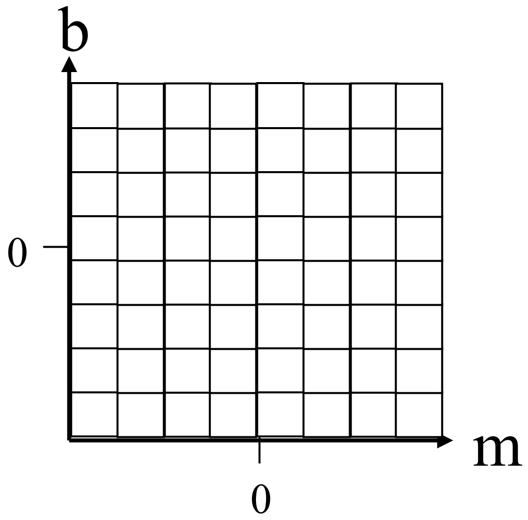


Hough transform: Line detection

A closer look (in 2 parts)

2. Transform line intersection into peak detection

- Divide the *(m,b)*-plane into a finite grid of cells.
- Associate a counter *c(m,b),* with each cell; initialize it to *0*.



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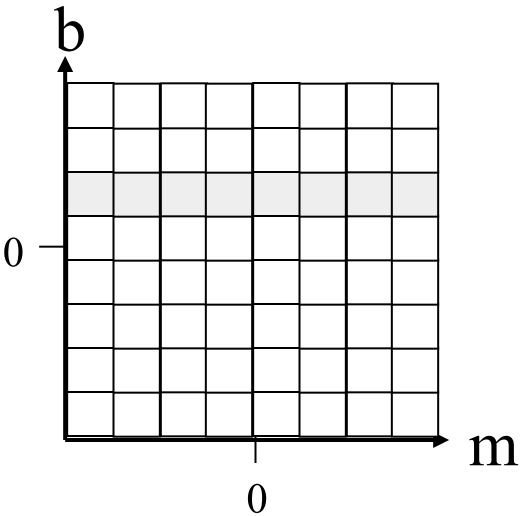

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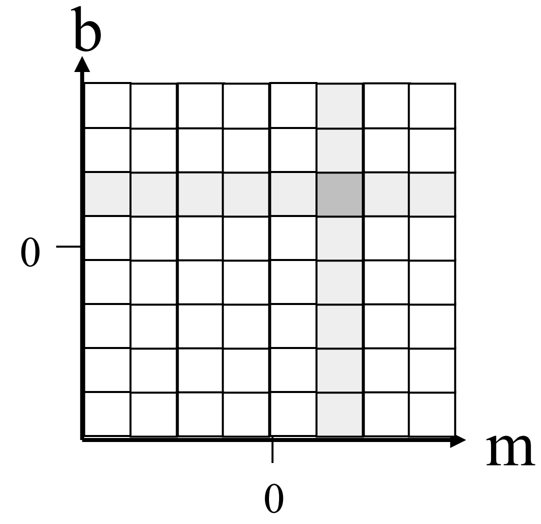

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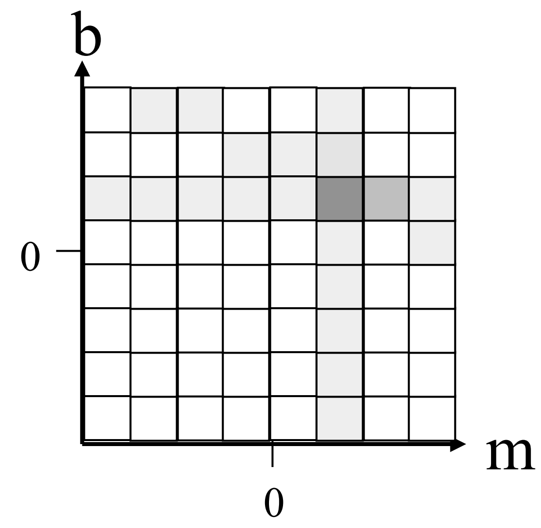

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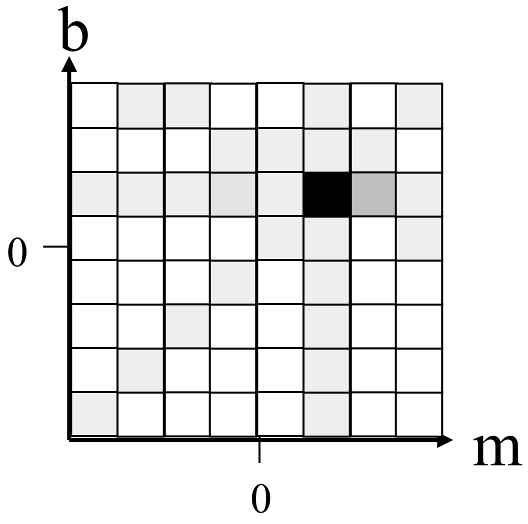
- So, the line is identified with the parameters corresponding to the largest count, *c(m,b)*.



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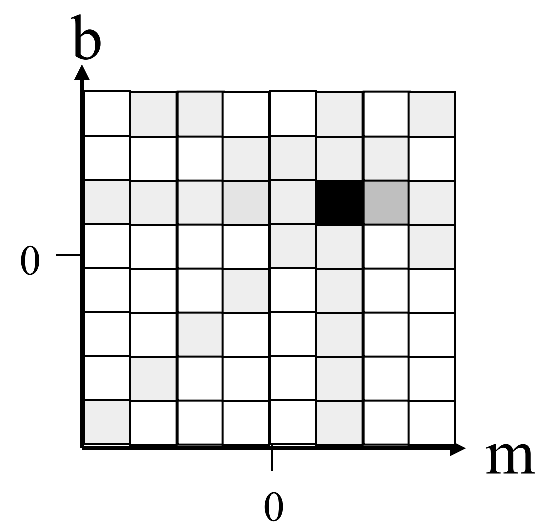

Recap: The 2 parts

1. Transform line detection into line intersection
2. Transform line intersection into peak detection

Hough transform: Line detection

A few practical considerations

- In theory, the parameter values can take on any real value.
  - Must discretize while weighting precision vs. storage/processing requirements.
- Real images will contain pixels "incorrectly" marked as edges due to noise
  - Must select a threshold on a minimally acceptable value for $C(m,b)$.
- There may be multiple lines present in an image
  - The Hough can simultaneously detect all of these by returning all (m,b) pairs whose counter exceeds the threshold.

Hough transform: Line detection

Procedure

- Input: Binary image, $I(i,j)$ pixels marked 1 of edge has been detection; else 0.
- Output: (m,b) detected line parameters.

1. Discretize the parameter space (m,b) using sampling intervals dm, db, which yield precision suited to application, yet reasonable storage requirements; let the resulting number of values for b and m be B and M, respectively.
2. Let $C(m,b)$ be an integer array of counters; initialize all elements to 0.
3. For each pixel $I(i,j)=1$,

   For each $m=1..M$

   i) Let $b' = j - m\,i$

   ii) Find the index b closest to b'.

   iii) Increment $C(m,b)$ by 1.
4. Return all (m,b) where $C(m,b) > t$; t a threshold value.

Hough transform: Generalization

The generalized Hough transform

- We start by assuming that we are given a set of edge points $(x_i, y_i), i = 1..n$
- Suppose we represent a contour geometry of interest as a parametric expression of the form

$$g(x_i, y_i, \mathrm{p}) = 0$$

with $\mathbf{p}$ a parameter vector.

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e.g., for line: *$mx+b-y=0$*

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- Let the characteristic function of g be

$$h(x_i, y_i, \mathbf{p}) = \begin{Bmatrix} 1, g(x_i, y_i, \mathbf{p}) = 0 \\ 0, \text{otherwise} \end{Bmatrix}$$

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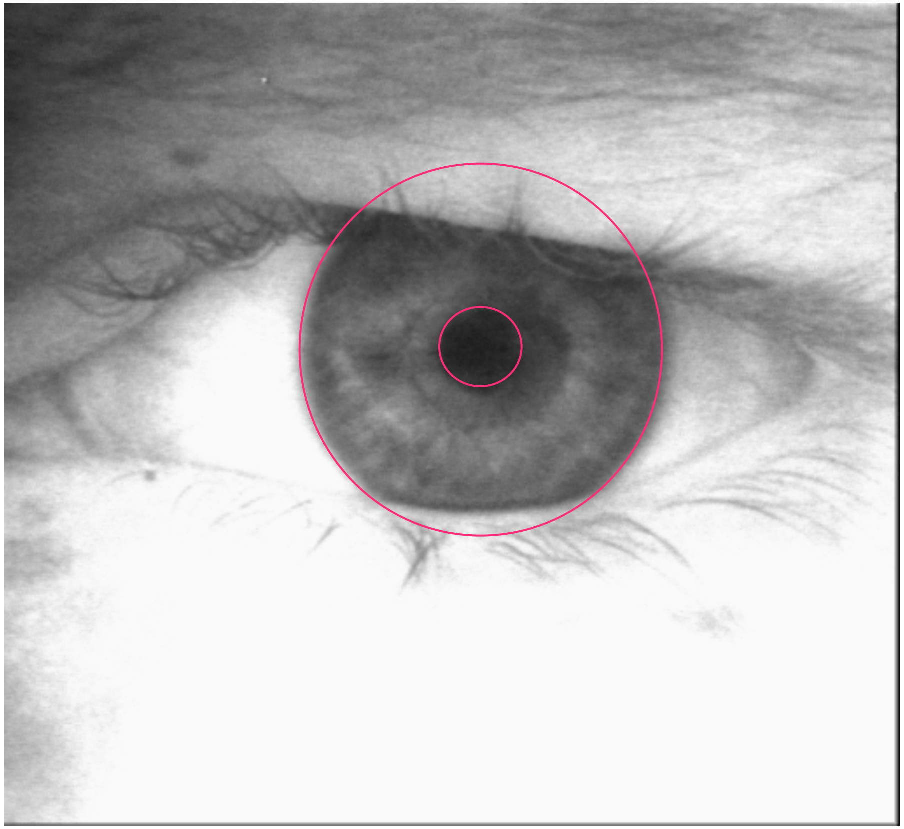
Example

- For a circle, we might let $\mathbf{p}^{\mathrm{T}} = (x_c, y_c, r)$

$$g(x_i, y_i, x_c, y_c, r) = (x_i - x_c)^2 + (y_i - y_c)^2 - r^2$$

Hough transform: Examples



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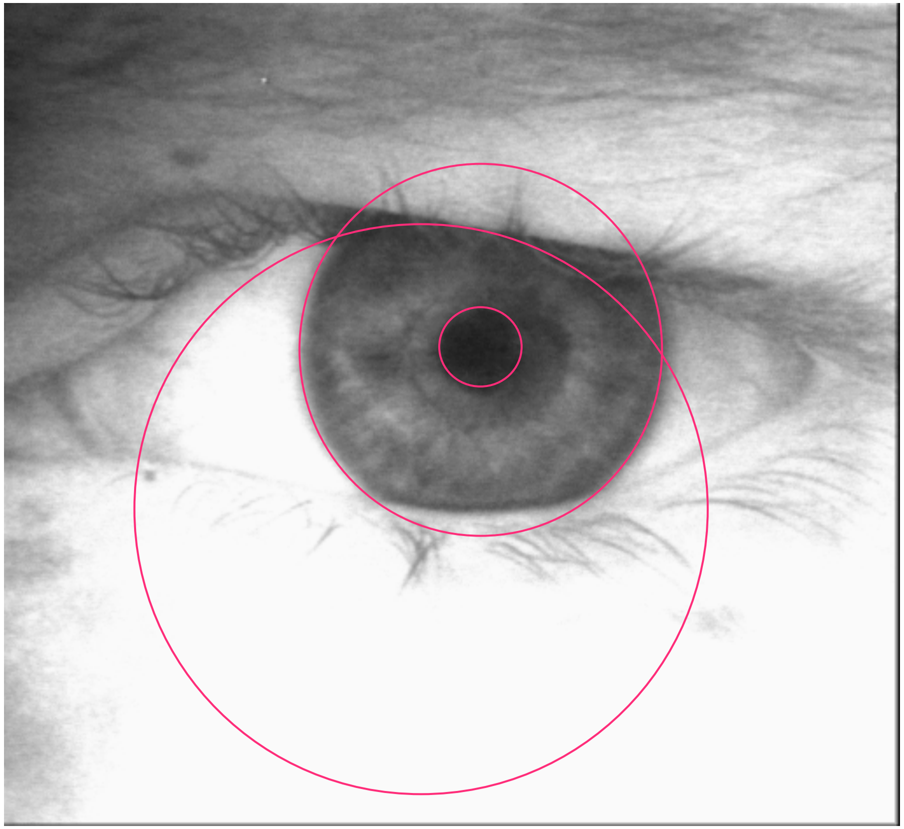

Hough transform: Remarks

Pluses

- The Hough can convert difficult model fitting problems into a simple histogramming operation.
- For a given image, it allows for simultaneous detection of multiple instances of a model.
- It is robust, to outliers in the data (marked edge pixels).

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Minuses

- The required space can be large for even a moderate number of parameters, if precision is required.
- The time required in peak search grows rapidly with the number of parameters.
- In depending on detected edge pixels, it has no recourse to the original image data.

Outline

- Introduction
- Edge detection
- Corner detection
- Hough transform
- **Deformable templates**

Deformable templates: Basic idea

Motivation

- Recent thinking has questioned the approach of fitting contour models in two discrete stages (first extract generic interesting points, then fit the model).
- At issue is the fact that much of the image data is discarded without knowing exactly what it will be used for.
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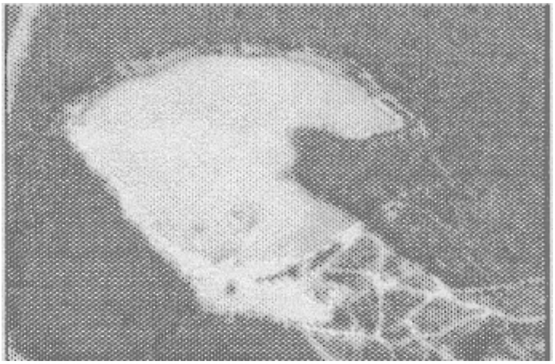

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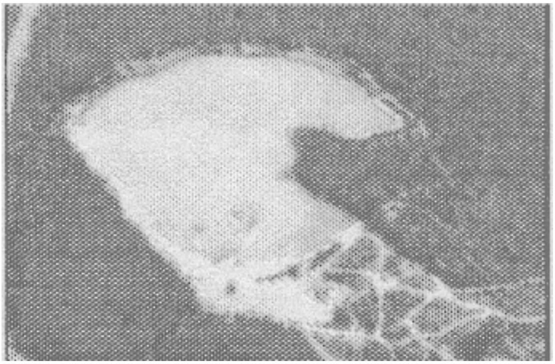

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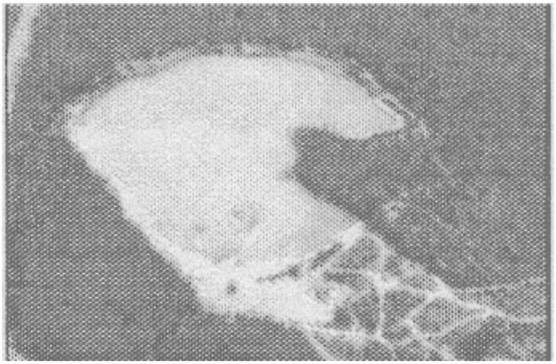

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1. Enhance the image to make those portions most likely to be important stand out.
    - e.g., high contrast contours, i.e., strong light/dark transitions.
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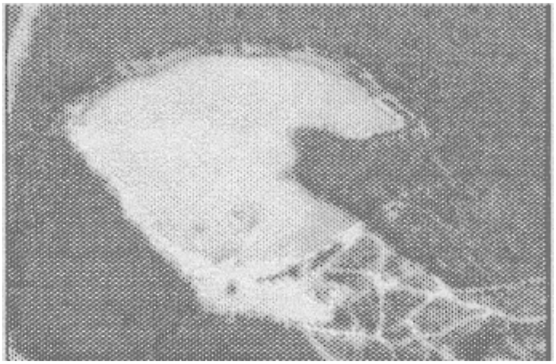

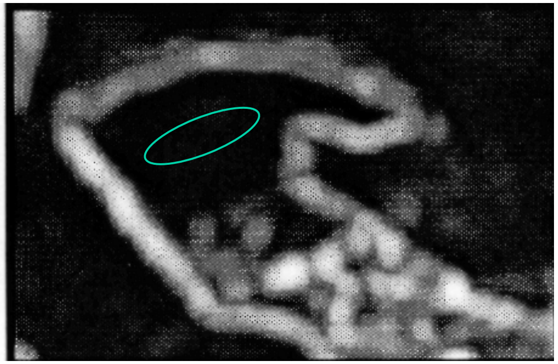

Deformable templates: Basic idea

Motivation

- Recent thinking has questioned the approach of fitting contour models in two discrete stages (first extract generic interesting points, then fit the model).
- At issue is the fact that much of the image data is discarded without knowing exactly what it will be used for.
- Instead, consider fitting a parameterized model more directly to the image data.
- Again, a two step process results, but without a hard up front decision about which parts of the data are relevant

1. Enhance the image to make those portions most likely to be important stand out.
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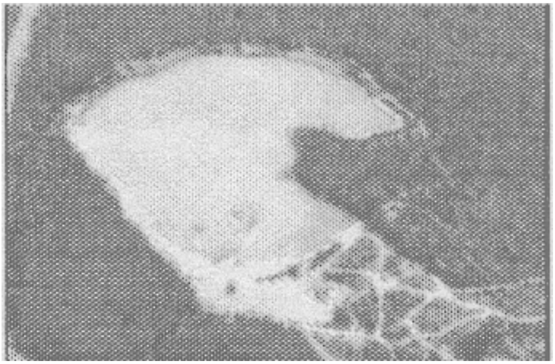

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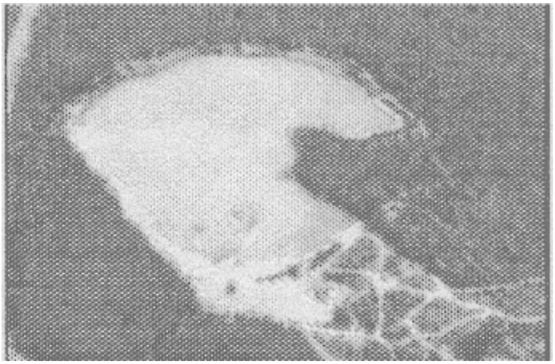

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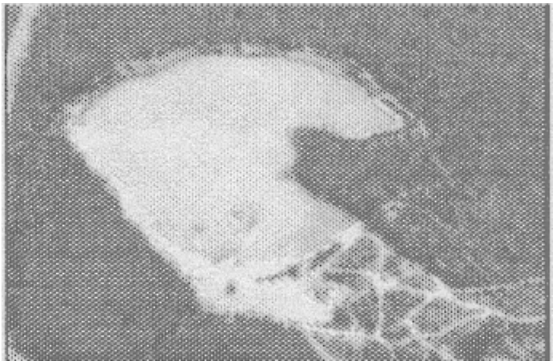

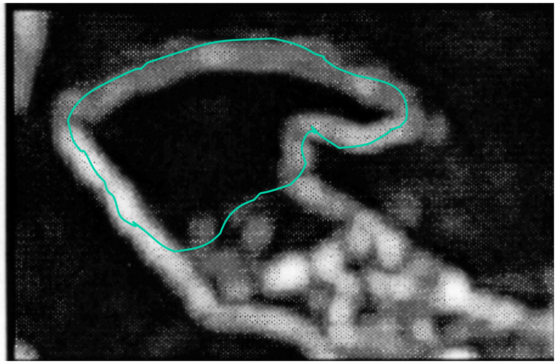

Deformable templates: Formalization

Template model

- For illustration, we consider a simple contour model, that of a parabolic segment.

$$x(s) = as^2 + bs + c$$

$$y(s) = ds^2 + es + f$$

  with s varying from 0 to 1.

- This contour model varies in shape (deforms) as we vary the parameters a, b, c, d, e, f.
- We will seek to vary the parameters so that the final shape lies along high contrast image points.

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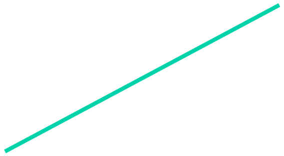

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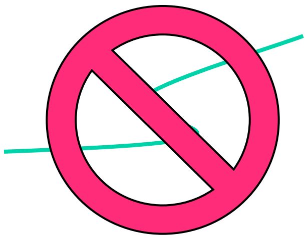

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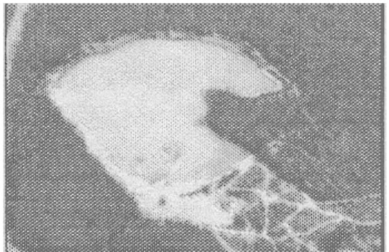
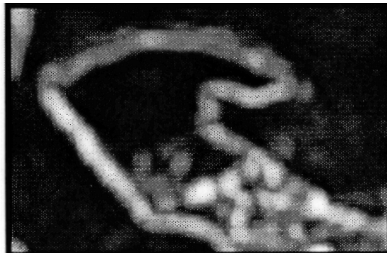
Energy image

- We seek to enhance those portions of the image that have high contrast contours.
- Various approaches could be considered (e.g., perhaps most simply, image gradient magnitude).
- We choose to use an image derived by taking the magnitude of the strongest response in our oriented bandpass image decomposition (as derived from Gabor filters).
- We refer to this image, I, as the energy image.
  - It has most energy along high contrast contours.





Deformable templates: Formalization

Fitting the model

- The model is initialized with a starting set of parameter values, perhaps provided by hand.
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  - Let

$$I_{total} = \left[\int_0^1 I(x,y)\,ds \right]$$

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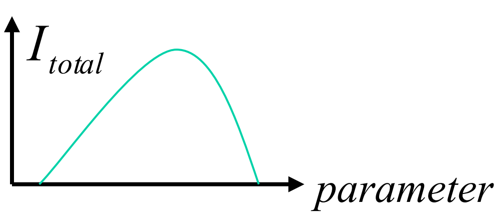

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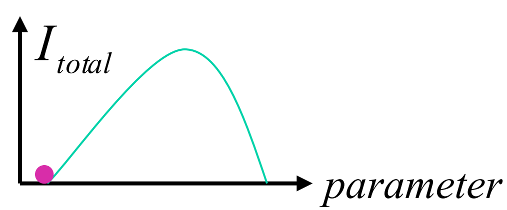

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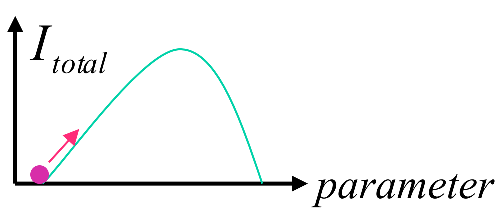

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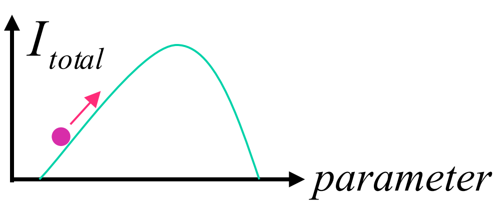

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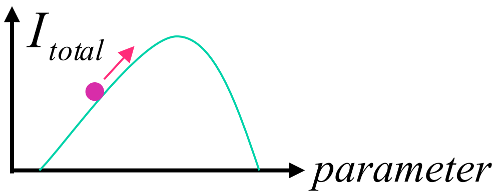

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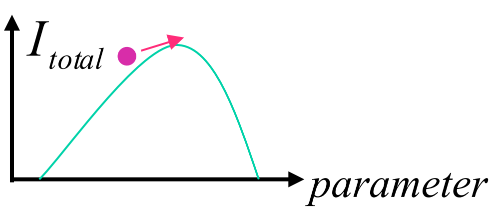

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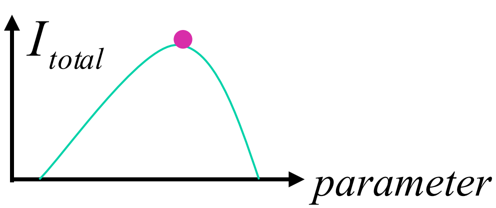

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Fitting the model (cont.)

- For our specific template parameterization

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at each iteration we increment, e.g., a according to

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and applying the chain rule yields

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And similarly for d, e and f.

Deformable templates: Key components

Template model

- Parameterized contour model.
- For example, parabolic segment.

Image representation

- Energy image that enhances high contrast (strong light/dark transition) loci in the image
- For example, maximal magnitude of response in oriented bandpass image representation.

Fitting the model

- The model is initialized with a starting set of parameter values, perhaps provided by hand.
- Adjust via gradient ascent the template parameters so that it lies along the largest values in the energy.

Deformable templates: Examples

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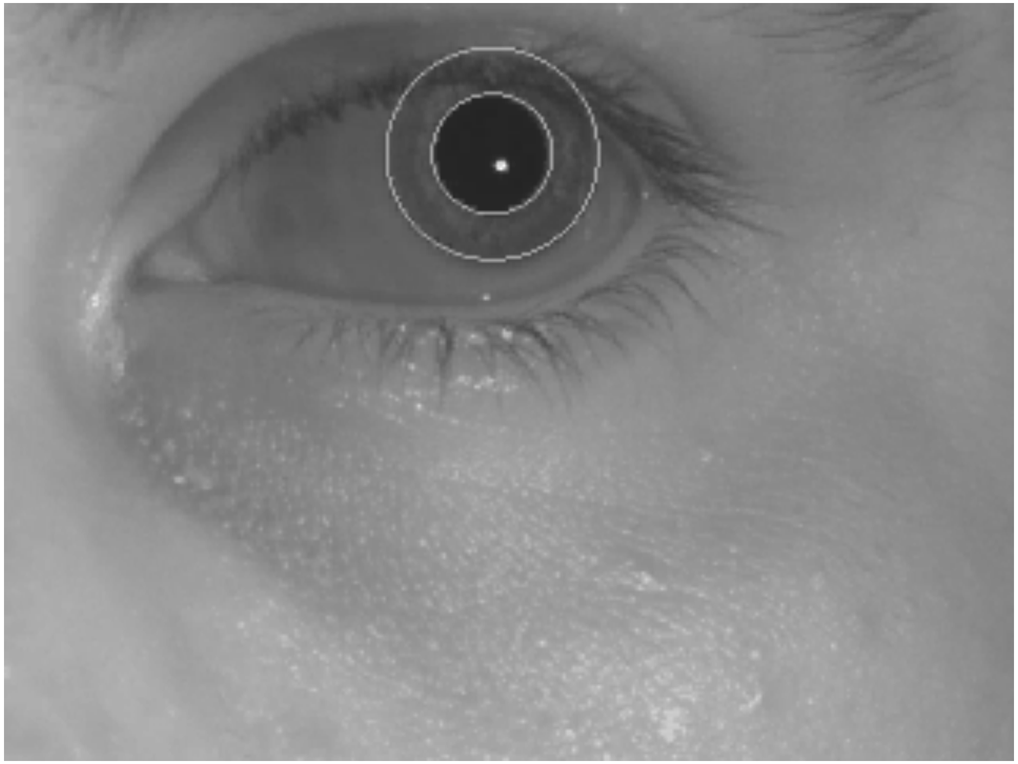

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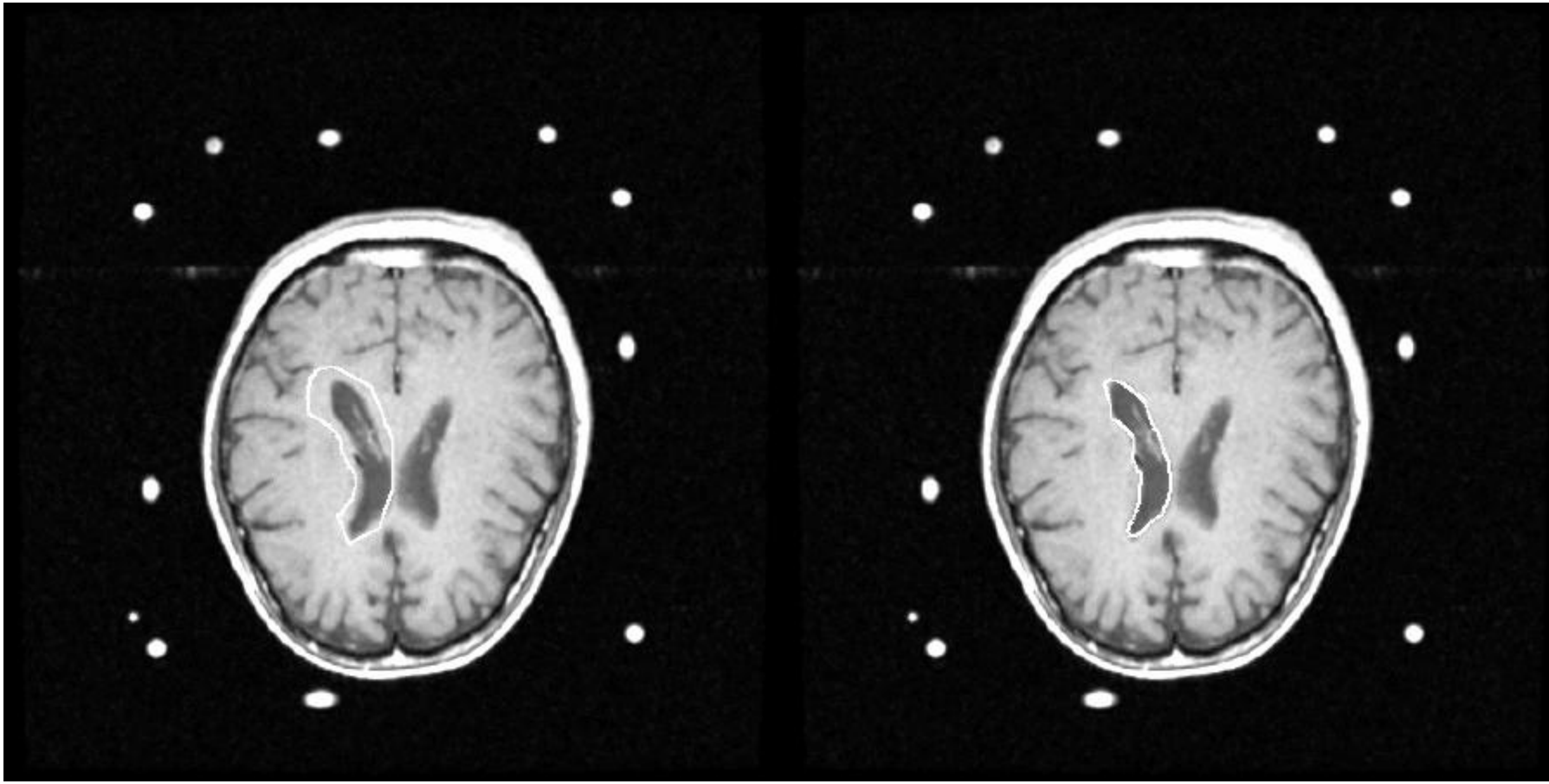

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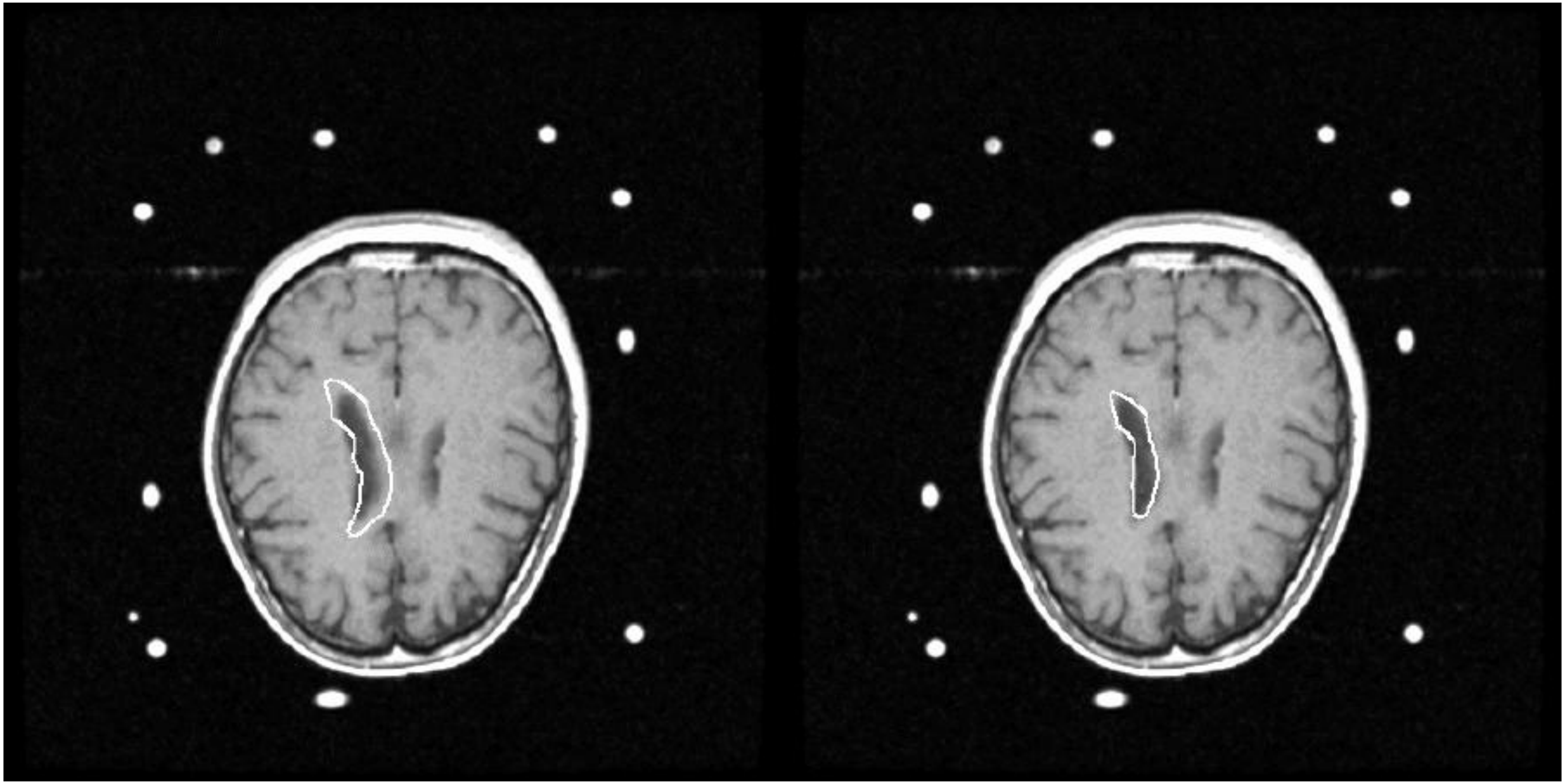

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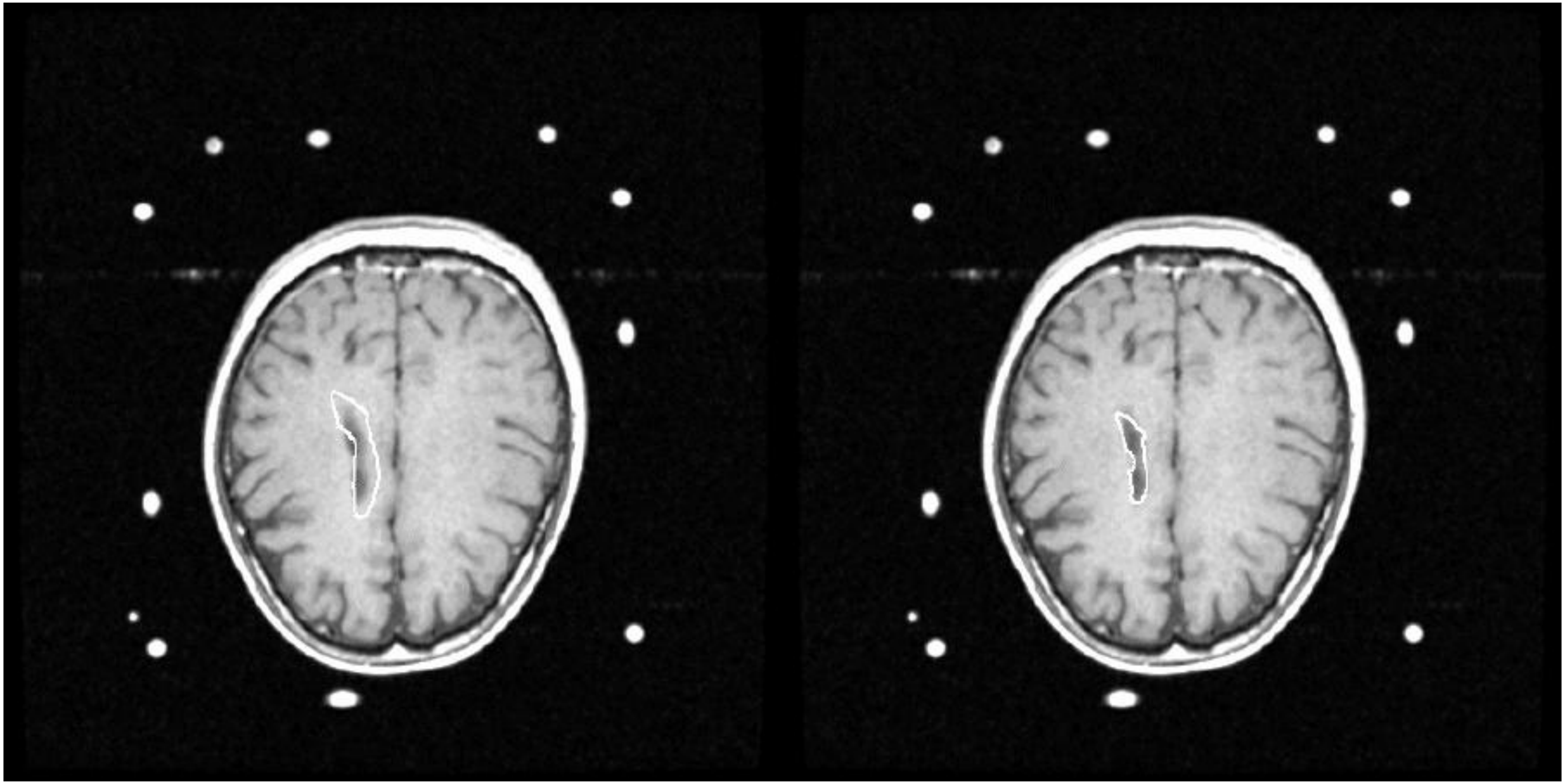

Deformable templates: Remarks

Pluses

- Make use of all the available image data.
- Can accommodate fairly complex contour models.
- Can be intuitive in HCI.

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- Can accommodate fairly complex contour models.
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Minuses

- The iterative fit can be slow to converge.
- Prone to local minima.
- Model construction can require careful design.

Summary

- **Introduction**
- **Edge detection**
- **Corner detection**
- **Hough transform**
- **Deformable templates**