CSE 4422/5323 Computer Vision

Unit 1: Image Formation

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Outline

- Introduction
- Basic optics
- Basic radiometry
- Geometric image formation
- Image acquisition
- Our visual world

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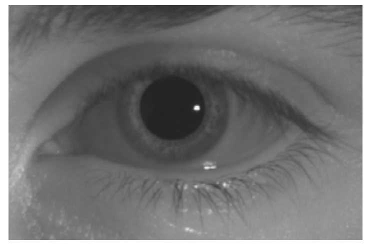
Introduction: Overview

Motivation

- To fully understand information recovery from images
- it is necessary to understand how images are formed.

Key questions

- What determines where a 3D scene point will appear in an image?
- With what intensity will the point be imaged?



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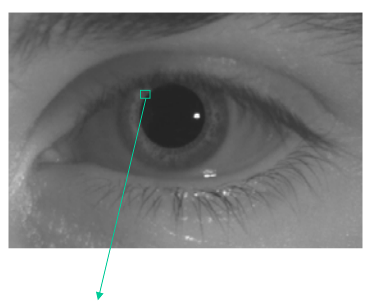
Introduction: Overview

Major image types used in computer vision

- Intensity images
- Range images

Any digital image is just a numerical array

- Exact relationship of image to physical world depends on the image formation process.
- Information in the image is implicit and must be recovered through processing.



128	123	123	131	124	68	68	70
122	124	138	139	89	72	68	70
121	126	135	136	75	69	69	69
125	127	130	131	80	79	75	70
125	126	255	132	75	78	75	75
126	125	130	80	75	72	75	74
125	126	127	80	79	77	76	75
126	127	127	79	78	78	77	76

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Introduction: Basic concepts of intensity images

Optical parameters of lens: Characterize sensor optics

- lens type
- focal length
- field of view
- angular apertures

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Photometric parameters: Models of light energy reaching sensor following reflection from surfaces in scene

- type, intensity and direction of illumination
- reflectance properties of visible surfaces
- effects of sensor structure on light reaching photoreceptors

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- type, intensity and direction of illumination
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Geometric parameters: Determine image position at which 3D points are imaged

- type of projection
- position and orientation of camera in space
- geometric distortions from imaging process

Introduction: Basic concepts of intensity images

Parameters specific to digital imaging: Photoreceptors of viewing camera

- physical properties of photosensitive matrix
- discrete nature of photoreceptors
- quantization of intensity

Optics (focus & lens)

Radiometry

Geometry

Image acquisition

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Optics (focus & lens)

Radiometry

Geometry

Image acquisition

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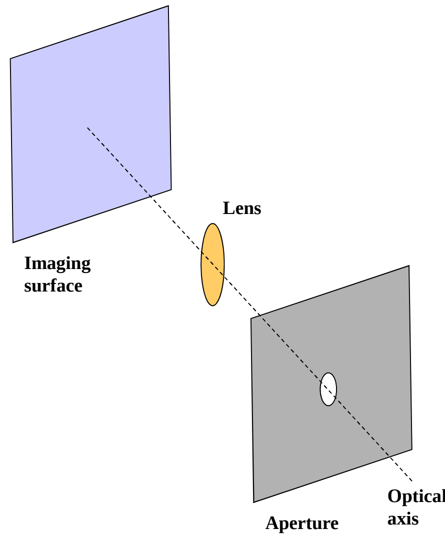
Basic optics: Overview

Fundamentals

- Image formation begins when light rays enter an aperture to impinge on an imaging surface.
- Typically, these rays are reflections of light rays off surfaces in the scene,
- but can also be direct images of light sources in the scene.



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Basic optics: Focus

Being in focus

- Any single point in the world reflects light in (possibly) many directions.
- Many rays reflected by same point may enter camera.
- To obtain sharp images, want all rays from a single scene point, ***P***, to converge on a single image point, ***p***.
- Say the image, ***p***, of ***P*** is in focus.

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Two ways to achieve focus

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Two ways to achieve focus

1. Reduce the camera's aperture to a pinhole.

- Only one ray from a given point enters the camera
- Sharp, undistorted images over wide range of distances.

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2. Introducing an optical system with apertures, lenses…
- Designed to make all rays coming from same 3D point converge to same image point.
- Sharp, undistorted images over a range of exposure times.

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Basic optics: Focus

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2. Introducing an optical system with apertures, lenses…

- Designed to make all rays coming from same 3D point converge to same image point.
- Sharp, undistorted images over a range of exposure times.
- Can be complicated and focus at one distance at a time.

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Basic optics: Thin lens - a simple idealization

Two characterizing elements

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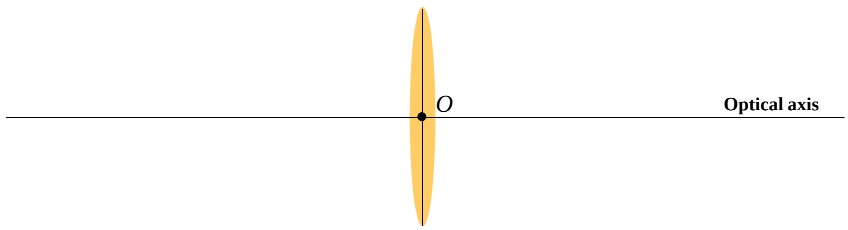
Basic optics: Thin lens - a simple idealization



Two characterizing elements

1. Optical axis going through lens center, O, and perpendicular to its plane.

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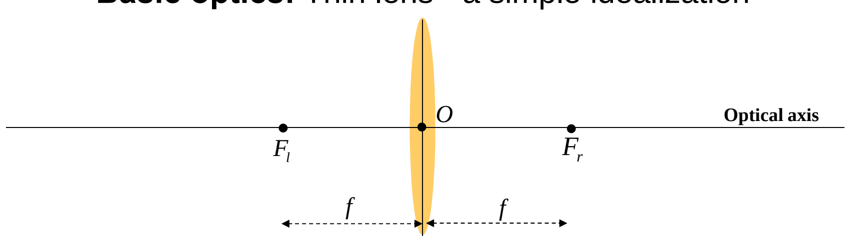
Basic optics: Thin lens - a simple idealization



Two characterizing elements

1. Optical axis going through lens center, O, and perpendicular to its plane.
2. Two points, F_l and F_r, called left and right focus along the optical axis at a distance, f, the focal length, from the lens center.

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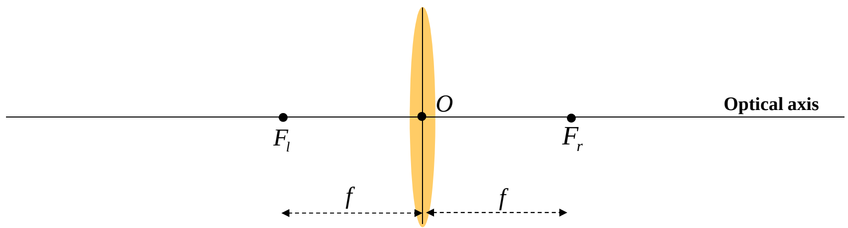

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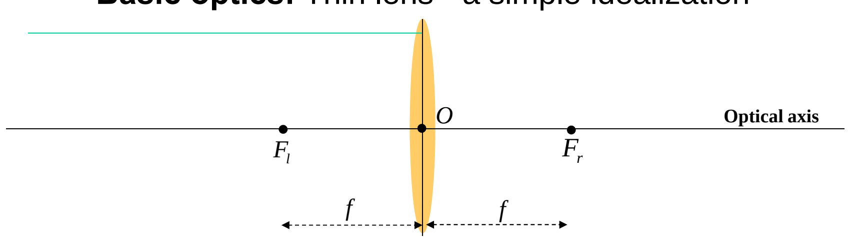

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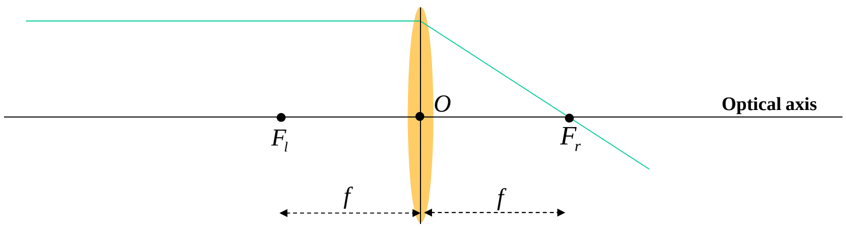

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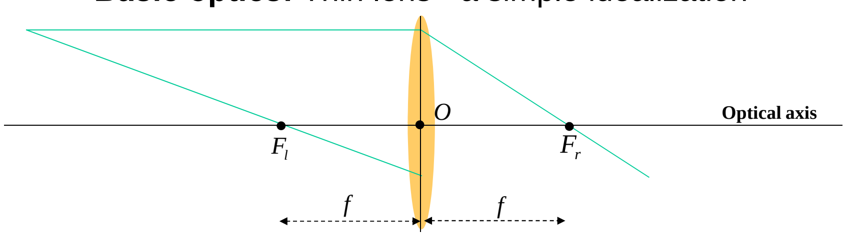

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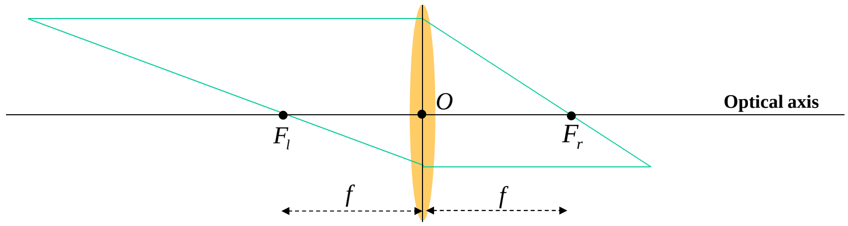

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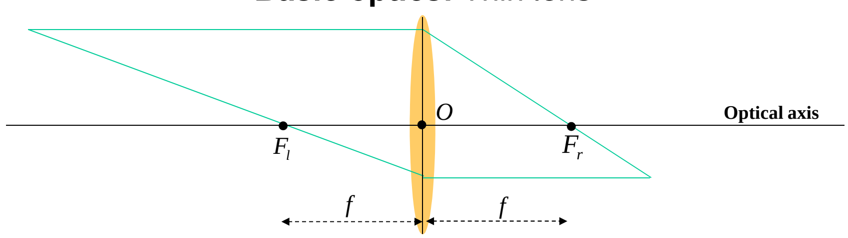

Fundamental equation of thin lens: Derived from the two basic properties.

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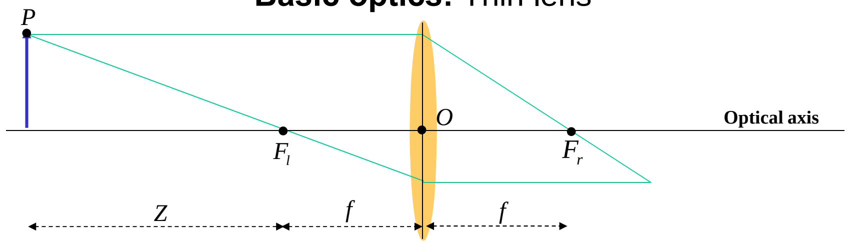

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- Consider a point P at a distance $Z + f$ from the lens along the optical axis.

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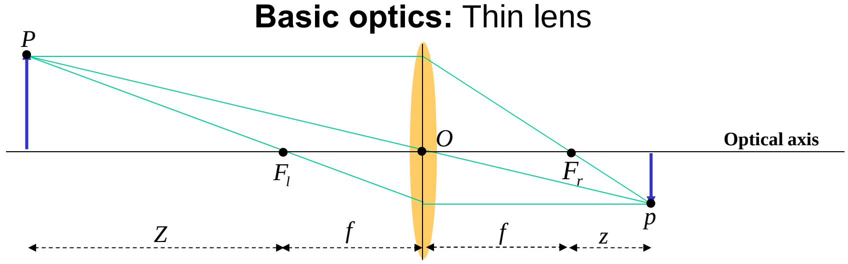
Basic optics: Thin lens



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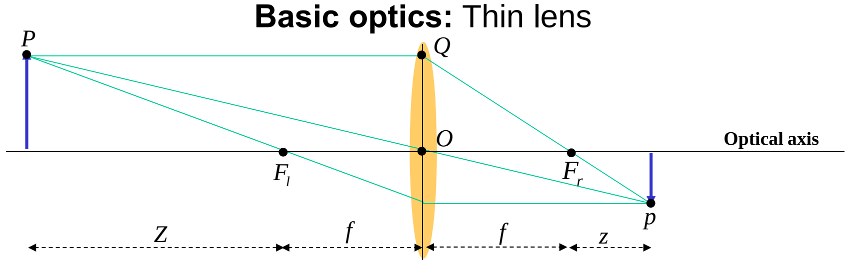

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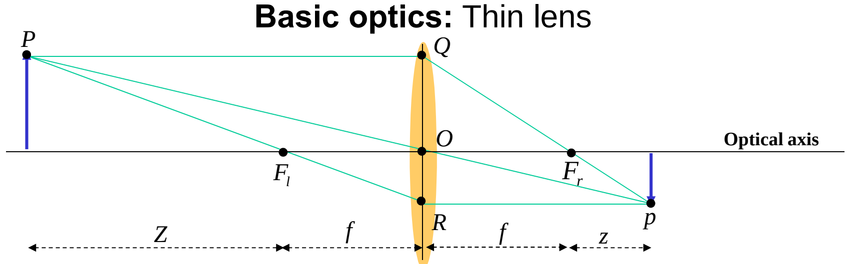

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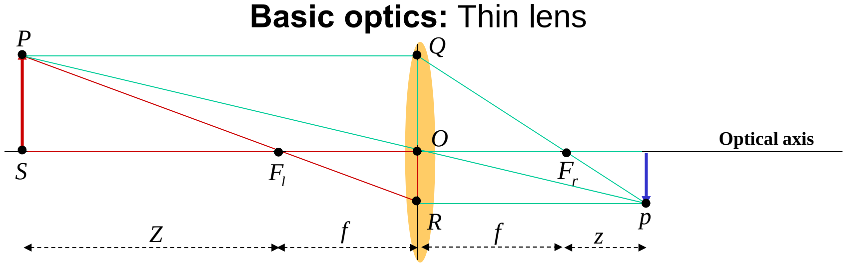

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- From similar triangles PF_lS & RF_lO

$$\frac{Z}{f} = \frac{PS}{OR}$$

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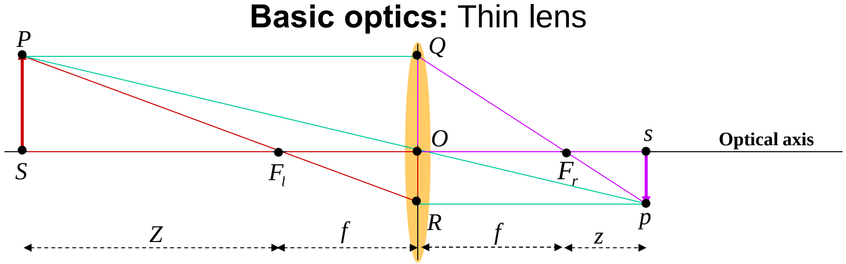
Basic optics: Thin lens



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- From similar triangles PF_lS & RF_lO and psF_r & QOF_r we have

$$\frac{Z}{f} = \frac{PS}{OR} \;\&\; \frac{QO}{sp} = \frac{f}{z}$$

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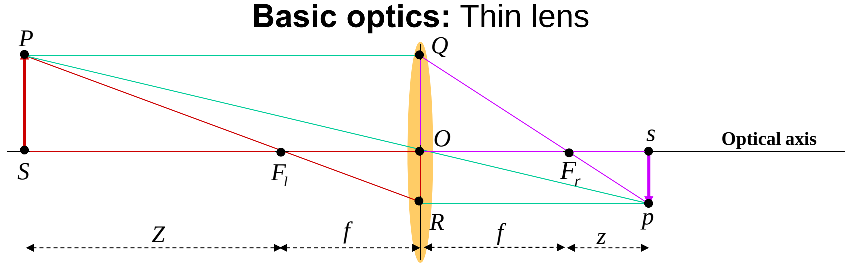
Basic optics: Thin lens



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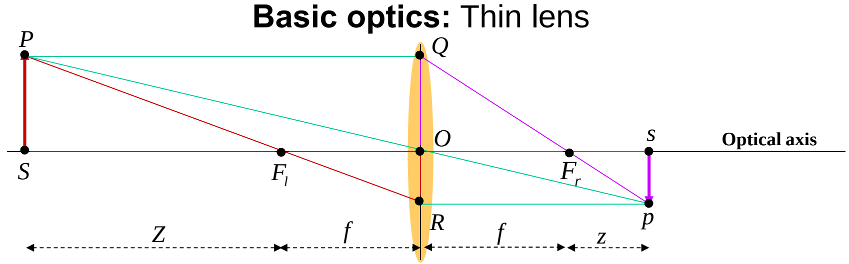

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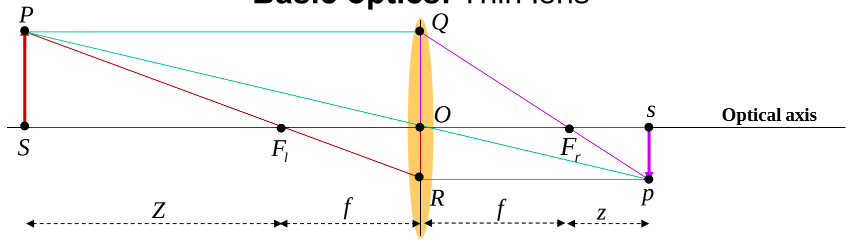

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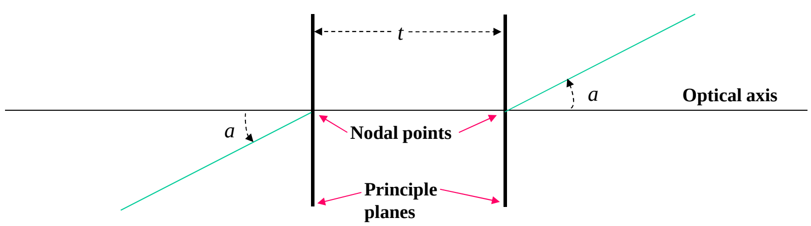
- From similar triangles PF_lS & $RF_l\,O$ and psF_r & QOF_r we have

$$\frac{Z}{f} = \cancel{\frac{PS}{OR}} = \cancel{\frac{QO}{sp}} = \frac{f}{z}$$

- Letting $\hat{Z} = Z + f$ and $\hat{z} = z + f$ yields

$$\frac{1}{\hat{Z}} + \frac{1}{\hat{z}} = \frac{1}{f}$$

Basic optics: Thick lens - a more realistic model



Motivation

- Any simple lens will have number of optical defects.
- For better imaging it is customary to combine several simple lenses by aligning their optical axes to yield a compound lens.
- The thick lens provides a reasonable model of such systems.

Two basic characterizing elements

1. A pair of principle planes parallel to the common optical axis.
2. A pair of nodal points, separated by a distance t – the thickness, where the planes intersect the optical axis.

Fundamental properties

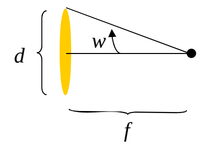
- A ray entering at one nodal point exits at the other without changing direction.
- Produces the same projection as an ideal thin lens, but with an additional offset, t.
- A thin lens is a thick lens where the two nodal points coincide.

Basic optics: Additional considerations

Field of view

- An angular measure of the portion of 3D space seen be the camera.
- Let d be the effective diameter of the lens, that portion reachable by light rays.
- Define the field of view, w, as half the angle subtended by the lens diameter as seen from the focus

$$\tan w = \frac{d}{2f}$$



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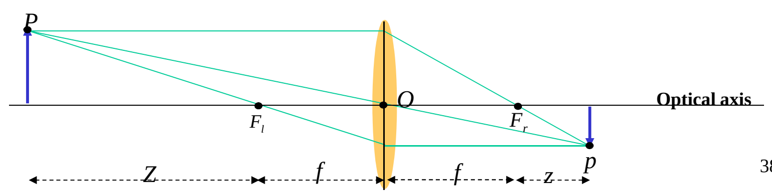
Depth of field

- The fundamental equation of the thin lens tells us that points at a distance Z will be focused at z.



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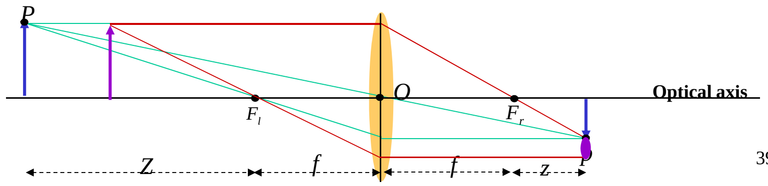
Depth of field

- The fundamental equation of the thin lens tells us that points at a distance Z will be focused at z.
- Other points will be imaged as (small) circles.
- These other points will have been focused at the apex of a cone at a different distance that will cut the image plane in circles.



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Aberrations

- spherical aberration: defocusing of nonparaxial rays
- chromatic aberration: differential defocusing as function of wavelength of light
- vignetting: loss of image intensity near periphery as complex aperture occlude light
- Remark: Minimization of aberrations becomes more difficult as lens aperture increases
  ➡ Trade-off between light gathering power and image quality.

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Optics (focus & lens)

Radiometry

Geometry

Image acquisition

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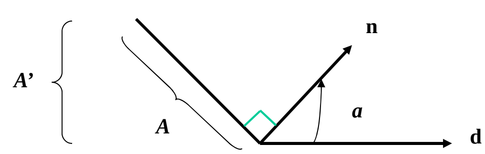
Basic radiometry: Surface reflectance

Prelude: Foreshortening and lengthening



- How does the length A (of the surface element) relate to the length A' (the viewed projection of the surface element)?

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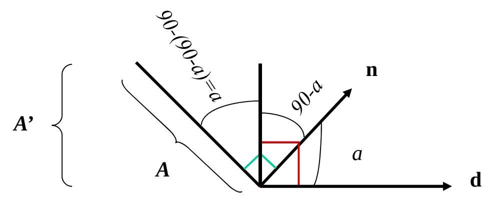
Basic radiometry: Surface reflectance

Prelude: Foreshortening and lengthening



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Basic radiometry: Surface reflectance

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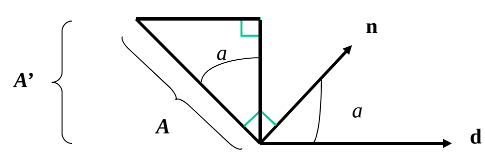

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- cos(a) = base/hypotenuse = A'/A

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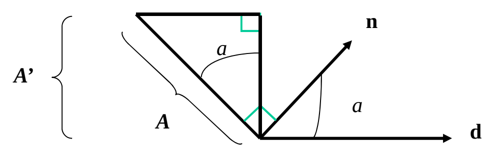
Basic radiometry: Surface reflectance

Prelude: Foreshortening and lengthening



- How does the length A (of the surface element) relate to the length A' (the viewed projection of the surface element)?
- $\cos(a)$ = base/hypotenuse = A'/A
- $A' = A\cos(a)$, i.e., foreshortening
- $A = A'/\cos(a)$, i.e., for a given viewing length, A', more of the surface length, A, is seen as a increases (and hence $\cos(a)$ decreases).

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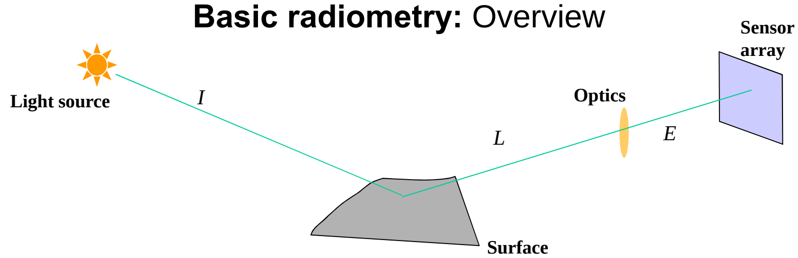
Basic radiometry: Overview



Radiometry is concerned with relations between

- amounts of light energy emitted from light sources, I,
- reflected from surfaces, L,
- and registered by sensors, E.

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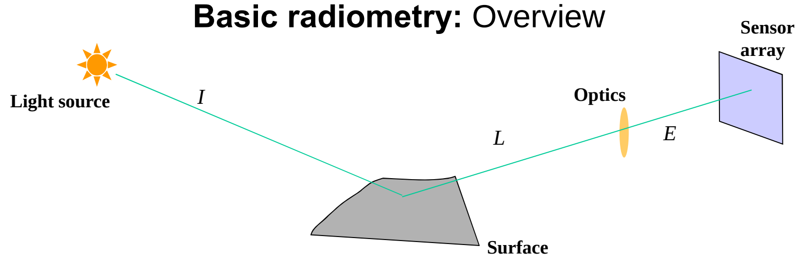
Basic radiometry: Overview



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Two purposes in study

1. Modeling how much of illuminating light is reflected from surfaces.
2. Modeling how much of reflected light reaches the sensor array.

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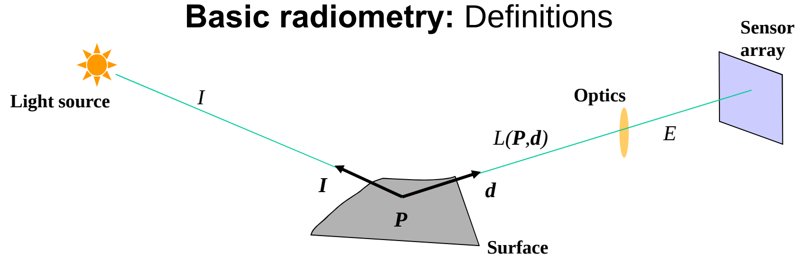
Basic radiometry: Definitions

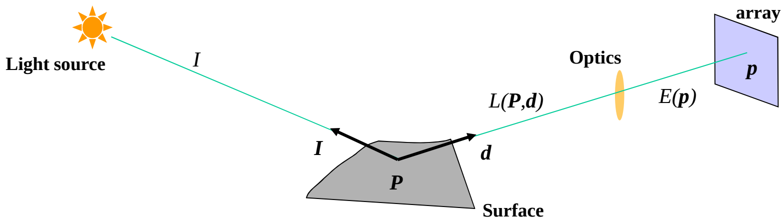


Scene radiance

- The power of light, per unit area, emitted at each point, $\boldsymbol{P}$, of a surface in 3D space in a given direction, $\boldsymbol{d}$.
- Units of power per unit foreshortened area emitted into a unit solid angle $(W.m^{-2}.sr^{-1})$
- Denote as $L(\boldsymbol{P},\boldsymbol{d})$.

48

Basic radiometry: Definitions



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- Units of power per unit foreshortened area emitted into a unit solid angle $(W.m^{-2}.sr^{-1})$
- Denote as $L(\boldsymbol{P},\boldsymbol{d})$.

Image irradiance

- The power of light, per unit area, at each point, ***p***, of the image plane.
- Units of power per unit area $(W.m^{-2})$
- Denote as $E(\boldsymbol{p})$.

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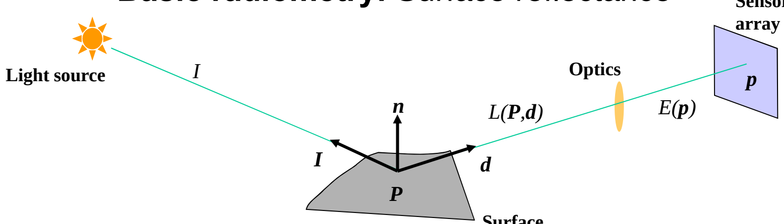
Basic radiometry: Surface reflectance

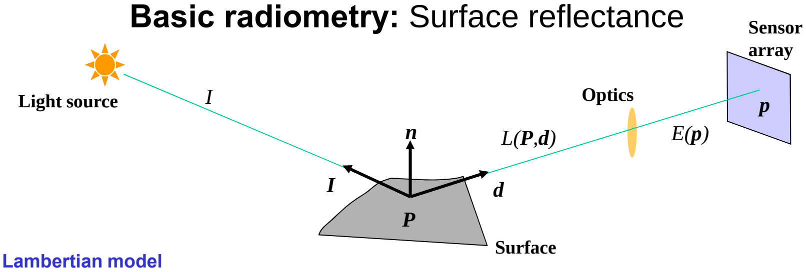


Lambertian model

- Assumes each surface point appears equally bright from all view directions.
  - An approximation for the appearance of matte surfaces.
- Let
  - ***I*** be the direction and amount of incident light
  - ***n*** be the unit surface normal at ***P***
  - $1>r>0$ be the surface albedo, a material property, giving ratio of reflected to incident light
- Scene radiance is given as $L = r\,\boldsymbol{I}.\boldsymbol{n}$
- Basis of derivation
  1. The amount of light reaching a surface is proportional to the cosine of the angle between the illumination direction and the surface normal.
  2. The amount of light reflected in a given direction is proportional to the cosine of the angle, a, between that direction and the surface normal
  3. But, the surface's area seen from that direction is inversely proportional to $cos(a)$; so, view direction effects cancel.

50

Basic radiometry: Surface reflectance



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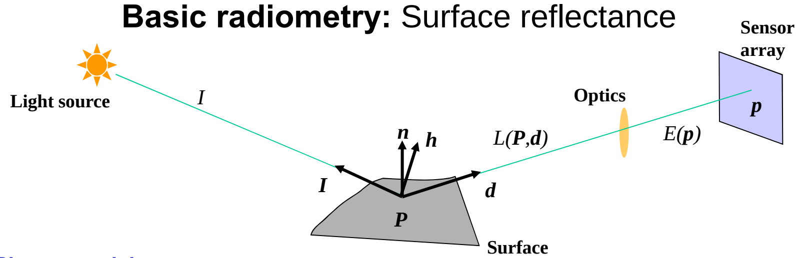
51

Basic radiometry: Surface reflectance



Phong model

- Augments the Lambertian model by including a specular component.
  - An approximation for the appearance of (partially) shiny surfaces.
- Let
  - $\mathbf{h}$ be the bisectrix of the view and illuminant directions
  - e be the specular exponent that deterimines how "tight" specularities are
  - $0<=a<=1$ be the weighting between matte and specular effects
- Scene radiance is given as $L = \max\{0, r[a(n.I)+(1-a)(n.h)^e]\}$
- Basis of derivation is that specular component is mirror-like.

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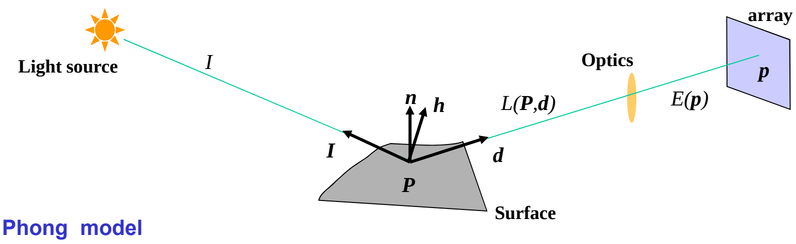
Basic radiometry: Surface reflectance



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Remarks

- Both the Lambertian and Phong models are essentially phenomenological models.
- Physical optics literature presents more detailed and rigorous models.
- Both phenomenological and physical models have been employed successfully in computer vision.

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Basic radiometry: Relating radiance and irradiance

Goal

- Derive the relationship between light reflected by surface and light registered by sensor.
- Assume thin lens optical model.

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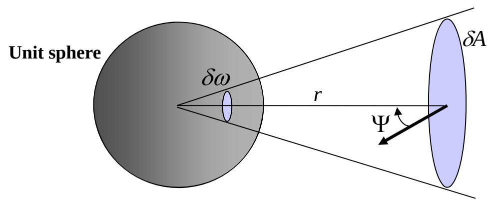
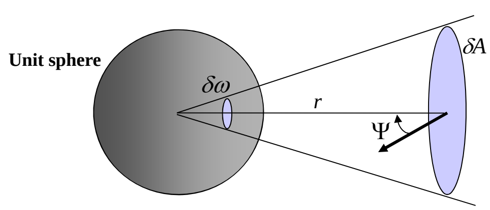
Basic radiometry: Relating radiance and irradiance



Review: Solid angle

- The solid angle of a cone of directions is the area cut out by the cone on the unit sphere centered on the cone's vertex.
- Consider a small planar patch and let
  - δA be its area
  - r be Its distance from the origin (sphere center)
  - Ψ be the angle between its normal and the ray to the origin

Basic radiometry: Relating radiance and irradiance



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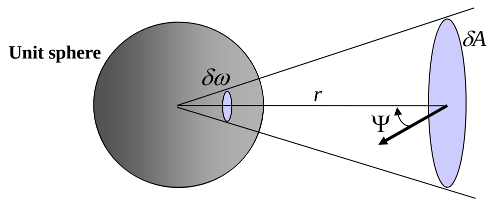
- The solid angle is then given as

$$\delta A$$

Basic radiometry: Relating radiance and irradiance



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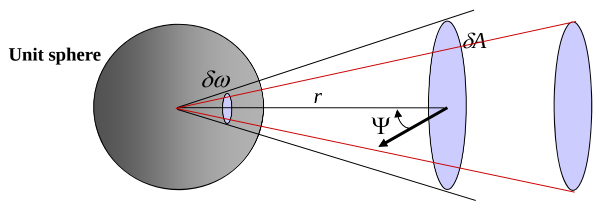
$$\delta A \cos\Psi$$

- Note that the cosine term accounts for foreshortening effects of the patch as seen from the origin.

57

Basic radiometry: Relating radiance and irradiance



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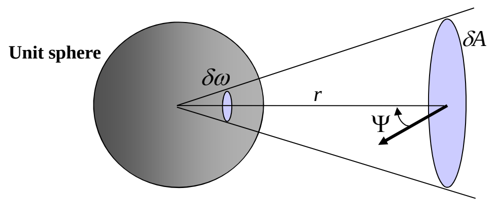

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$$\delta\omega = \frac{\delta A \cos\Psi}{r^2}$$

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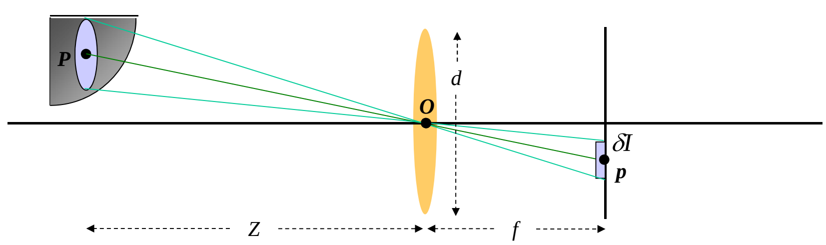
59

Basic radiometry: Relating radiance and irradiance



Fundamental equation of radiometric image formation

- Image irradiance, E, at a point, $\boldsymbol{p}$, is defined as the ratio between the power of light over a small image patch, $\delta\Pi$, and the area of the small image patch, δI ,

$$E = \frac{\delta\Pi}{\delta I}$$

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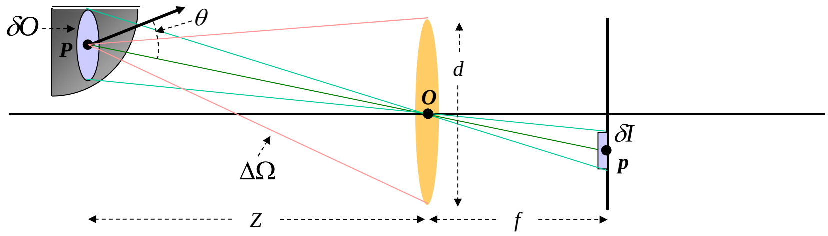
Basic radiometry: Relating radiance and irradiance



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- Evaluate $\delta\Pi$, let
  - δO be the area of a small surface patch about $\boldsymbol{P}$
  - L be the scene radiance at $\boldsymbol{P}$ in direction of lens
  - $\Delta\Omega$ be the solid angle subtended by the lens
  - θ be the angle between the normal at $\boldsymbol{P}$ and the principle ray (through lens center)

then

$$\delta\Pi = \delta O L \Delta\Omega \cos\theta$$

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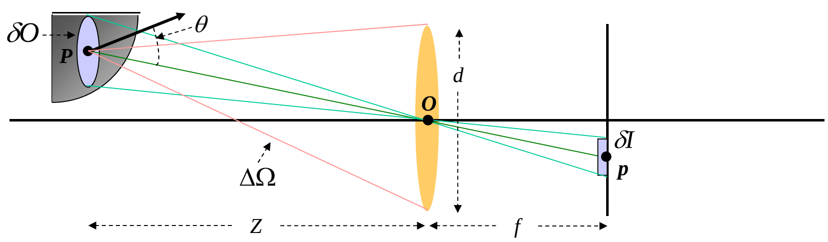
Basic radiometry: Relating radiance and irradiance



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62

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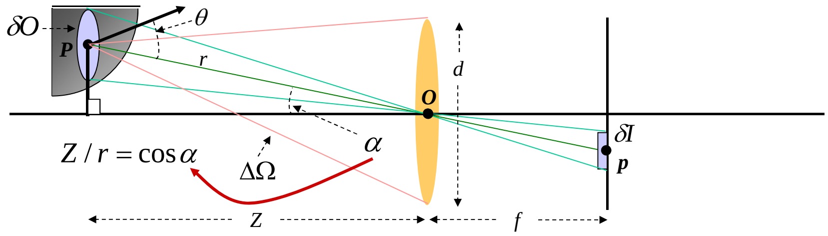

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- Evaluate $\Delta\Omega$ via the solid angle formula, let
  - $\delta A = \pi d^2 / 4$ be the lens area
  - $\Psi = \alpha$ be the angle between the principle ray and the optical axis
  - $r = Z / \cos\alpha$ be the distance of $\boldsymbol{P}$ from the lens center

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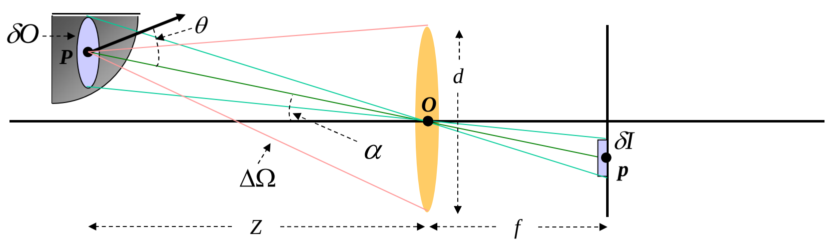

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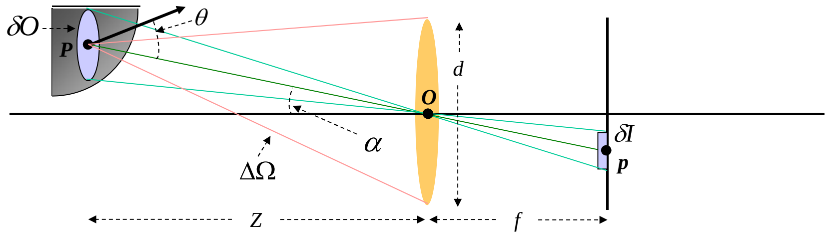
  then

$$\Delta\Omega = \frac{\delta A}{r^2}\cos\Psi = \frac{\pi}{4}d^2\cos\alpha\frac{\cos^2\alpha}{Z^2}$$

Basic radiometry: Relating radiance and irradiance



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65

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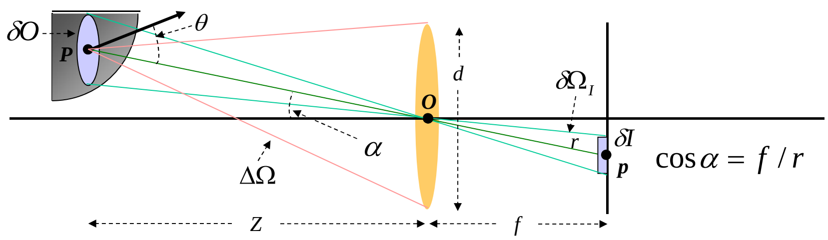

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- Evaluate $\delta O / \delta I$
  - For the solid angle $\delta\Omega_I$ subtended by small image patch δI

$$\delta\Omega_I = \frac{\delta A}{r^2}\cos\Psi = \frac{\delta I}{(f/\cos\alpha)^2}\cos\alpha$$

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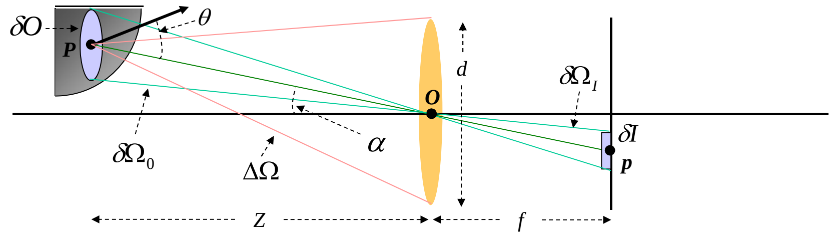
Basic radiometry: Relating radiance and irradiance



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- Evaluate $\delta O / \delta I$
  - For the solid angle $\delta\Omega_O$ subtended by small image patch δO

$$\delta\Omega_O = \frac{\delta A}{r^2}\cos\Psi = \frac{\delta O}{(Z/\cos\alpha)^2}\cos\theta$$

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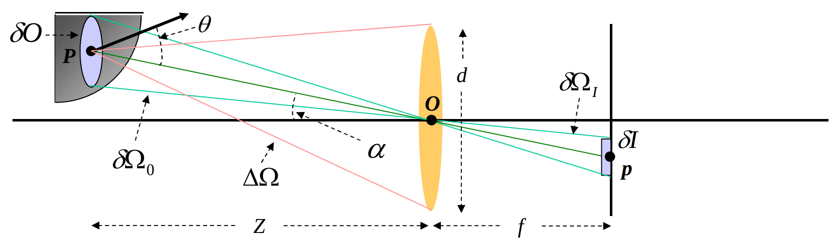
Basic radiometry: Relating radiance and irradiance



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- Evaluate $\delta O / \delta I$
  - We have

$$\delta\Omega_O = \frac{\delta O}{(Z/\cos\alpha)^2}\cos\theta \qquad \delta\Omega_I = \frac{\delta I}{(f/\cos\alpha)^2}\cos\alpha$$

  - Looks like we should divide the left expression by the right expression.

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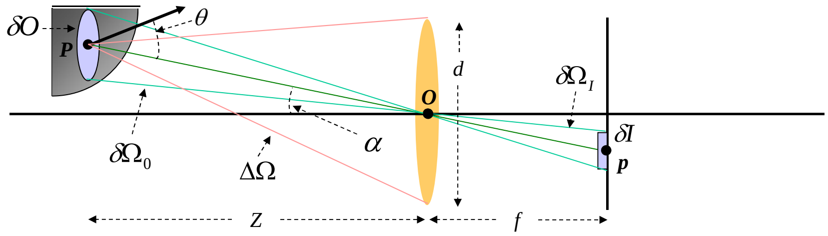
Basic radiometry: Relating radiance and irradiance



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- Evaluate $\delta O / \delta I$
  - Since $\delta\Omega_I = \delta\Omega_O$

$$(\delta\Omega_o)(\delta\Omega_I^{-1}) = 1 = \left(\frac{\delta O\cos\theta}{(Z/\cos\alpha)^2}\right)\left(\frac{(f/\cos\alpha)^2}{\delta I\cos\alpha}\right)$$

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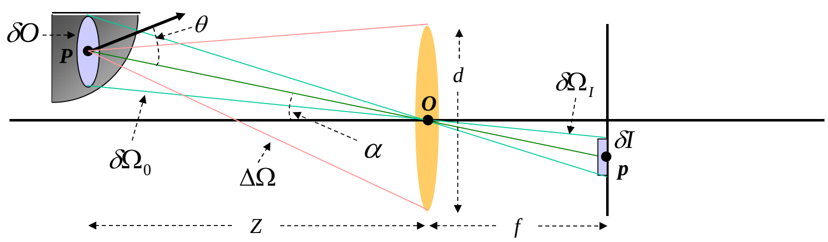
Basic radiometry: Relating radiance and irradiance



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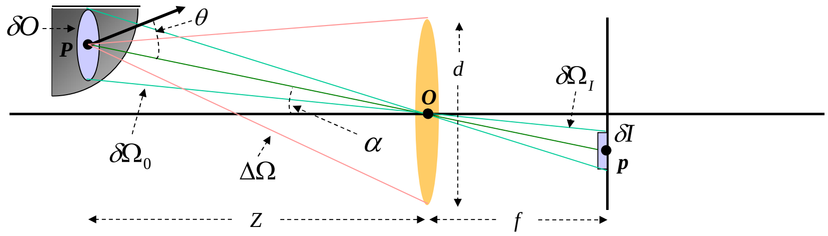
70

Basic radiometry: Relating radiance and irradiance



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$$\Rightarrow \frac{\delta O}{\delta I} = \frac{\cos\alpha}{\cos\theta}\left(\frac{Z}{f}\right)^2$$

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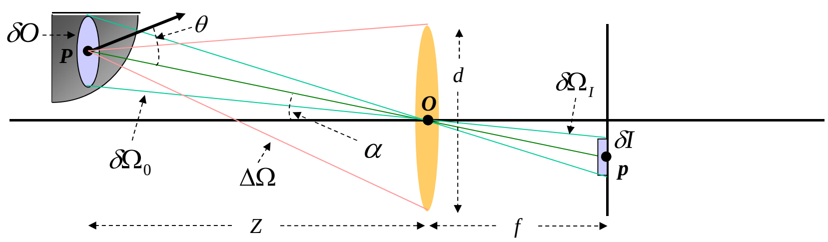
Basic radiometry: Relating radiance and irradiance



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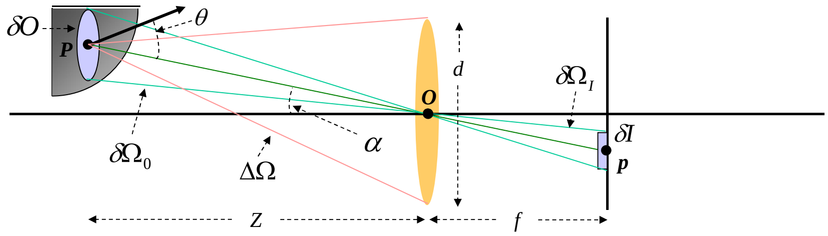
72

Basic radiometry: Relating radiance and irradiance



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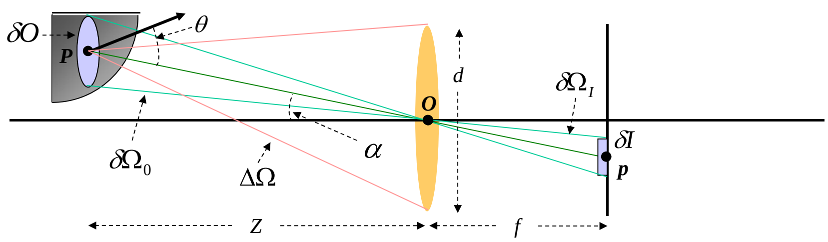
Basic radiometry: Relating radiance and irradiance



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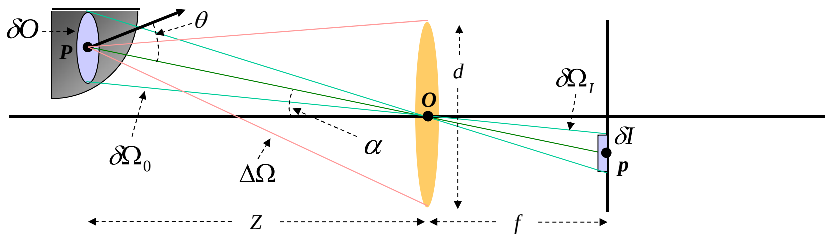

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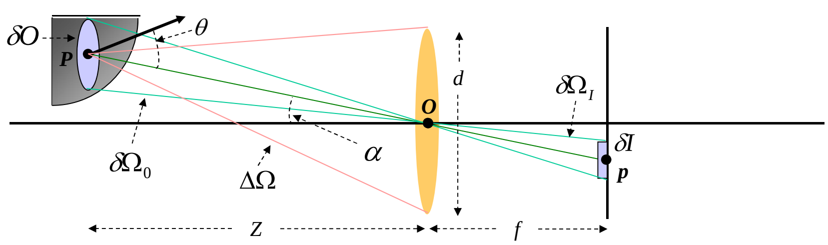
Basic radiometry: Relating radiance and irradiance



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$$E = \frac{\delta\Pi}{\delta I} = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha$$

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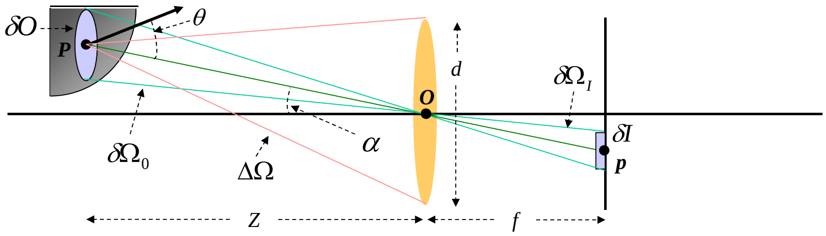
Basic radiometry: Relating radiance and irradiance



Fundamental equation of radiometric image formation

- Image irradiance, E, at a point, $\boldsymbol{p}$, is defined as the ratio between the power of light over a small image patch, $\delta\Pi$, and the area of the small image patch, δI ,

$$E = \frac{\delta\Pi}{\delta I} = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha$$

- Conclusion: The image irradiance at $\boldsymbol{p}$ decreases as the fourth power of the cosine of the angle between the principle ray and the optical axis.
- For small angular aperture, this effect can be neglected

  ➡ Image irradiance can be regarded as uniformly proportional to the scene radiance over the entire image plane,

Optics (focus & lens)

Radiometry

Geometry

Image acquisition

Geometry of image formation: Overview

Goal

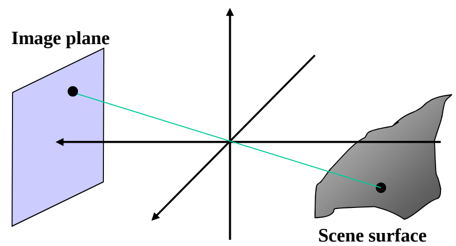
- Relate 3D positions of scene points to their 2D image positions.
- Requires consideration of
1. Geometry of projection
2. Camera models

Fundamental projections

- Perspective
- Weak perspective
- Orthographic (parallel)

Camera models

- Extrinsic parameters
- Intrinsic parameters
- Projections reconsidered



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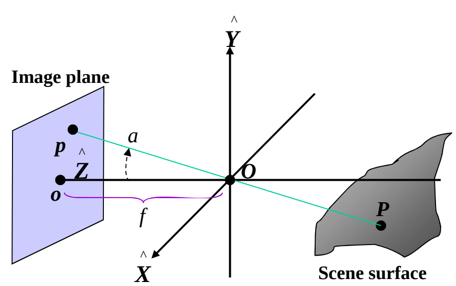
Geometry of image formation: Projections

Perspective

- Define a Cartesian coordinate system at ***O***, the center of projection.
- Let the optical axis align with the ***Z***-axis
- Let the image plane
  - be parallel to the ***XY***-plane
  - at a distance *f*, the focal length, along the optical axis
  - piercing the optical axis at ***o***, the principle point.
- Consider the projection of a scene point ***P***$=(X,Y,Z)$ into the image point ***p***$=(x,y,f)$.
- Fundamental assumption: ***P*** and ***p*** are collinear (pinhole model).
- Let the ray ***Pp*** make and angle *a* with the optical axis



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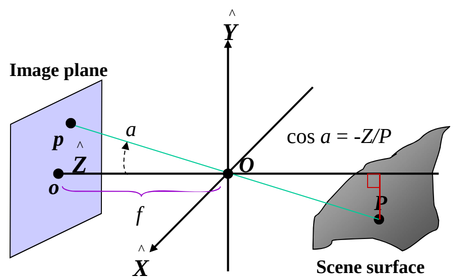
Geometry of image formation: Projections

Perspective

- The length of ***P*** is

  $P = -Z \sec a = -(\boldsymbol{P}.\hat{\boldsymbol{Z}}) \sec a$



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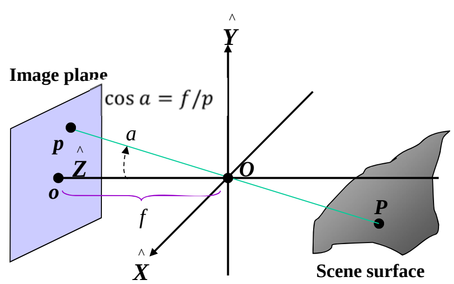
Geometry of image formation: Projections

Perspective

- The length of ***P*** is

  $P = -Z \sec a = -(\boldsymbol{P}.\hat{\boldsymbol{Z}}) \sec a$
- The length of ***p*** is

  $p = f \sec a$



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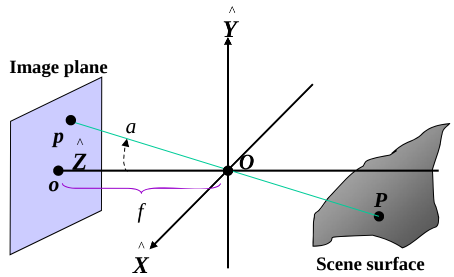
Geometry of image formation: Projections

Perspective

- The length of $\boldsymbol{P}$ is
  $P = -Z \sec a = -(\boldsymbol{P}.\hat{\boldsymbol{Z}}) \sec a$
- The length of $\boldsymbol{p}$ is
  $p = f \sec a$
- So
  $\frac{1}{f}\boldsymbol{p} = \frac{1}{\boldsymbol{P}.\hat{\boldsymbol{Z}}}\boldsymbol{P}$



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Geometry of image formation: Projections

Perspective

- The length of $\boldsymbol{P}$ is
  $P = -Z \sec a = -(\boldsymbol{P}.\hat{\boldsymbol{Z}}) \sec a$
- The length of $\boldsymbol{p}$ is
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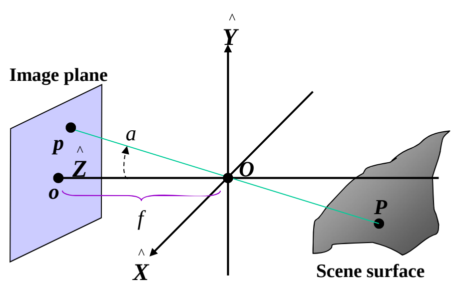
- So
  $\frac{1}{f}\boldsymbol{p} = \frac{1}{\boldsymbol{P}.\hat{\boldsymbol{Z}}}\boldsymbol{P}$
- In component form
  $\frac{x}{f} = \frac{X}{Z}, \frac{y}{f} = \frac{Y}{Z}$
  Or
  $x = f\frac{X}{Z}, y = f\frac{Y}{Z}$



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Geometry of image formation: History

Masaccio 1425

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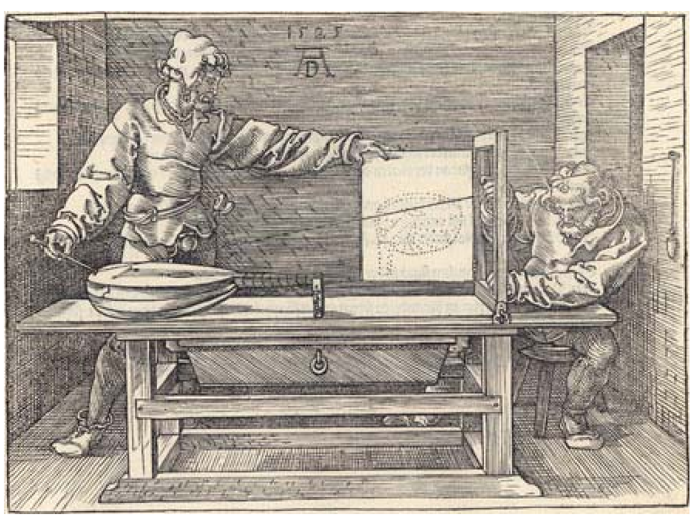
Geometry of image formation: History



Albrecht Durer 1525

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Geometry of image formation: Projections

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- The length of $\boldsymbol{P}$ is
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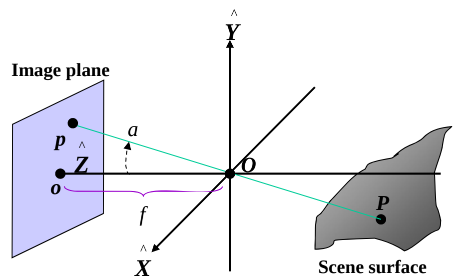

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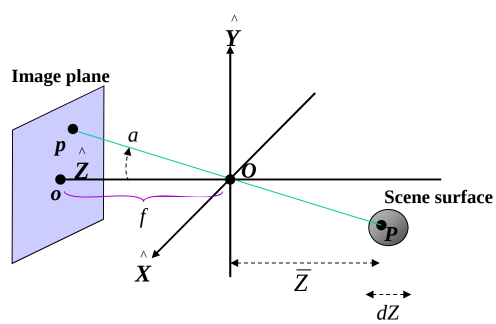
Geometry of image formation: Projections

Weak perspective

- Suppose that the variation of distance along the optical axis, dZ, is small compared to the average distance, $\overline{Z}$
- Then the perspective equations can be approximated as
  $x = f\frac{X}{Z} \approx f\frac{X}{\overline{Z}}, y = f\frac{Y}{Z} \approx f\frac{Y}{\overline{Z}}$



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Geometry of image formation: Projections

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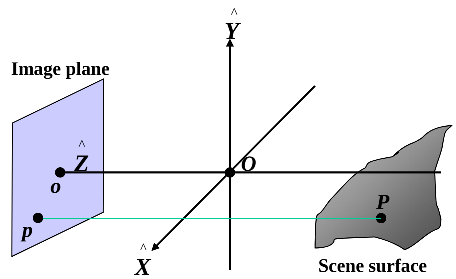
Orthographic

- As a limiting case of perspective, let $f \to \infty$
- Correspondingly, $Z \to \infty$ so that $f / Z \to 1$ and the projection equations become

$$x = X, y = Y$$



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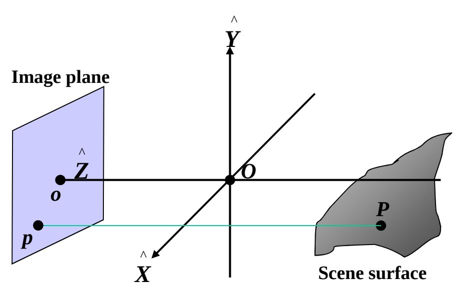

Remark

- Regard weak perspective as an orthographic projection followed by isotropic scaling with $f / \bar{Z}$

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Geometry of image formation: Camera models

Observations

- Thus far we have developed the geometry of image formation in the coordinates of the camera reference frame.
- In many instances, it is desirable to make an explicit correspondence with an external, world reference frame.

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Geometry of image formation: Camera models

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Two basic assumptions

1. The camera reference frame can be located with respect to some other, known, reference frame – the world reference frame.
2. The coordinates of the points in the camera reference frame can be obtained from the image coordinates – the only ones directly available from the image.

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2. The coordinates of the points in the camera reference frame can be obtained from the image coordinates – the only ones directly available from the image.

Definitions

- The extrinsic camera parameters are those that define the location and orientation of the camera reference frame with respect to a known world reference frame.
  - Photogrammetry speaks of exterior orientation.
- The intrinsic camera parameters are those that link the pixel coordinates of an image point with corresponding coordinates in the camera reference frame.
  - Photogrammetry speaks of interior orientation.

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Geometry of image formation: Camera models

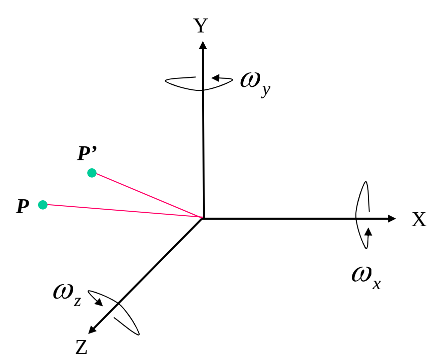
Interlude: 3D rotation

- Given a point $\boldsymbol{P} = (X,Y,Z)^{\mathsf{T}}$ in space
- Its transformation to $\boldsymbol{P'} = (X',Y',Z')^{\mathsf{T}}$ under rotation can be captured as a matrix operation

$$\boldsymbol{P'} = \mathbf{R}(\Omega)\boldsymbol{P}$$

with $\mathbf{R}(\Omega)$ the rotation matrix that captures rotation about the three coordinate axes

$$\boldsymbol{\Omega} = (\omega_x, \omega_y, \omega_z)^{\mathsf{T}}$$



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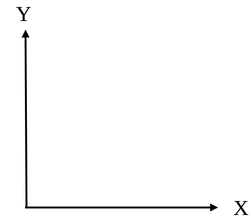
Geometry of image formation: Camera models

Rotation

- In 2D, counter clockwise rotation about the origin by an angle theta is given by the 2x2 matrix

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$



95

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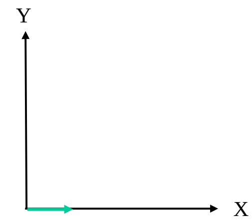
- For example
  - Rotation by 90 deg. Of the unit vector (1,0) yields

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



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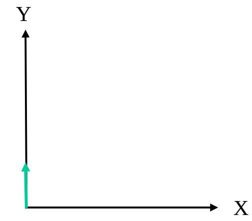

97

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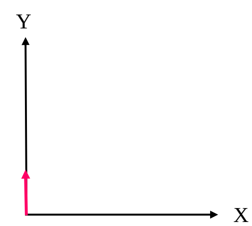

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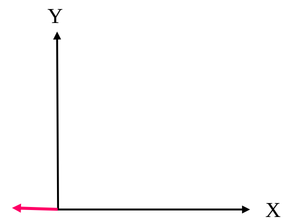


99

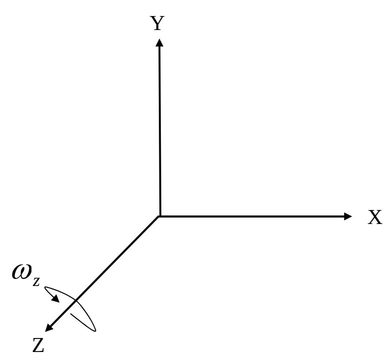
Geometry of image formation: Camera models

Rotation

- In 3D, counterclockwise rotation about the Z-Axis is given via the matrix

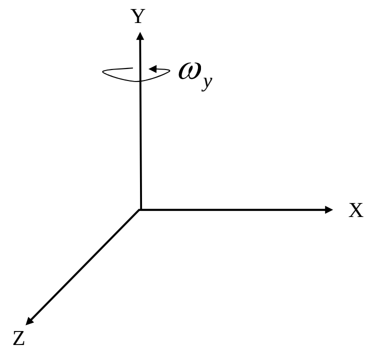
$$\begin{pmatrix} \cos\omega_z & -\sin\omega_z & 0 \\ \sin\omega_z & \cos\omega_z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Rotation is in the XY-plane.



100

Geometry of image formation: Camera models



Rotation

- In 3D, counterclockwise rotation about the Y-Axis is given via the matrix

$$\begin{pmatrix} \cos\omega_y & 0 & \sin\omega_y \\ 0 & 1 & 0 \\ -\sin\omega_y & 0 & \cos\omega_y \end{pmatrix}$$

- Rotation is in the XZ-plane.

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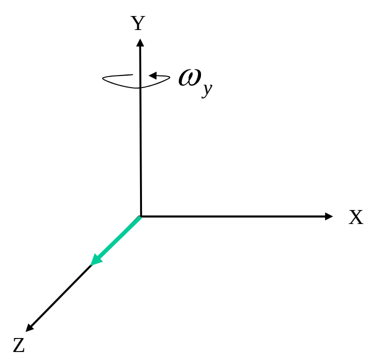
Geometry of image formation: Camera models



Rotation

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$$\begin{pmatrix} \cos\omega_y & 0 & \sin\omega_y \\ 0 & 1 & 0 \\ -\sin\omega_y & 0 & \cos\omega_y \end{pmatrix}$$

- Rotation is in the XZ-plane, e.g., for 90 deg,

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

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Geometry of image formation: Camera models

Rotation

- In 3D, counterclockwise rotation about the Y-Axis is given via the matrix

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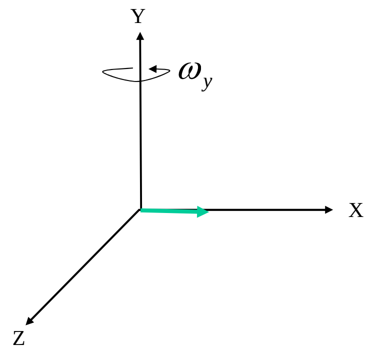


103

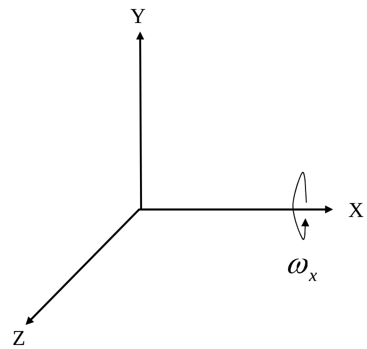
Geometry of image formation: Camera models

Rotation

- In 3D, counterclockwise rotation about the X-Axis is given via the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_x & -\sin\omega_x \\ 0 & \sin\omega_x & \cos\omega_x \end{pmatrix}$$

- Rotation is in the YZ-plane



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Geometry of image formation: Camera models

Rotation

- An arbitrary rotation is then given as

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_x & -\sin\omega_x \\ 0 & \sin\omega_x & \cos\omega_x \end{pmatrix} \begin{pmatrix} \cos\omega_y & 0 & \sin\omega_y \\ 0 & 1 & 0 \\ -\sin\omega_y & 0 & \cos\omega_y \end{pmatrix} \begin{pmatrix} \cos\omega_z & -\sin\omega_z & 0 \\ \sin\omega_z & \cos\omega_z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\omega_y \cos\omega_z & -\cos\omega_y \sin\omega_z & \sin\omega_y \\ \sin\omega_x \sin\omega_y \cos\omega_z + \cos\omega_x \sin\omega_z & -\sin\omega_x \sin\omega_y \sin\omega_z + \cos\omega_x \cos\omega_z & -\sin\omega_x \cos\omega_y \\ -\cos\omega_x \sin\omega_y \cos\omega_z + \sin\omega_x \sin\omega_z & \cos\omega_x \sin\omega_y \sin\omega_z + \sin\omega_x \cos\omega_z & \cos\omega_x \cos\omega_z \end{pmatrix}$$

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Geometry of image formation: Camera models

Rotation

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$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_x & -\sin\omega_x \\ 0 & \sin\omega_x & \cos\omega_x \end{pmatrix} \begin{pmatrix} \cos\omega_y & 0 & \sin\omega_y \\ 0 & 1 & 0 \\ -\sin\omega_y & 0 & \cos\omega_y \end{pmatrix} \begin{pmatrix} \cos\omega_z & -\sin\omega_z & 0 \\ \sin\omega_z & \cos\omega_z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\omega_y \cos\omega_z & -\cos\omega_y \sin\omega_z & \sin\omega_y \\ \sin\omega_x \sin\omega_y \cos\omega_z + \cos\omega_x \sin\omega_z & -\sin\omega_x \sin\omega_y \sin\omega_z + \cos\omega_x \cos\omega_z & -\sin\omega_x \cos\omega_y \\ -\cos\omega_x \sin\omega_y \cos\omega_z + \sin\omega_x \sin\omega_z & \cos\omega_x \sin\omega_y \sin\omega_z + \sin\omega_x \cos\omega_z & \cos\omega_x \cos\omega_z \end{pmatrix}$$

Remarks

- The order of rotations about the coordinate axes matters (rotations do not commute).
- We have $\mathbf{R}^T\mathbf{R} = \mathbf{R}\mathbf{R}^T = \mathbf{I}$, with $\mathbf{I}$ the 3 x 3 identity matrix.
- There are several alternative ways to represent 3D rotations.

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Geometry of image formation: Extrinsic parameters

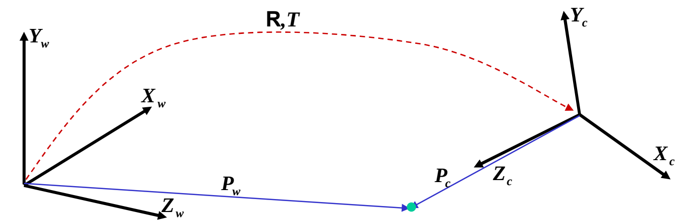
Camera to world transformation

- Typically given via two sets of parameters

1. A 3D translation vector, $\boldsymbol{T}$, describing the relative positions of the two reference frames.
2. A 3x3 rotation matrix, **R**, that brings the corresponding axes of the two frames into alignment.

- Letting $\boldsymbol{P}_c$ and $\boldsymbol{P}_w$ be the camera and world coordinates of the same point, we write.

$$\boldsymbol{P}_c = \mathbf{R}(\boldsymbol{P}_w - \boldsymbol{T})$$



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$$\boldsymbol{P}_c = \mathbf{R}(\boldsymbol{P}_w - \boldsymbol{T})$$

Remark

- By definition, **R**, is completely specified be 3 parameters (e.g., rotation about each of the coordinate axes); so, there are 6 extrinsic parameters in total (3 for $\boldsymbol{T}$; 3 for **R**).

Geometry of image formation: Intrinsic parameters

Camera to pixel transformation

- Typically given via two sets of parameters

1. The focal length, f, serving to capture the (perspective) projection.
2. The pixel coordinates of the principle point (image center), (o_x, o_y), and the effective pixel horizontal and vertical dimensions, (s_x, s_y), serving to capture the transformation between camera frame coordinates and pixel coordinates.

- We already have considered how to incorporate the focal length.
- Letting (x_i, y_i) be the pixel coordinates, we incorporate the second set of parameters via

$$x = -(x_i - o_x)s_x$$

$$y = -(y_i - o_y)s_y$$

Remarks

- For this simple analysis, there are 5 intrinsic parameters in total (f, o_x, o_y, s_x, s_y)
- More generally, additional parameters might come into play, e.g., lens distortion parameters.

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Geometry of image formation: World-image transformation

Component formulation

- Recall the camera frame expression of perspective projection

$$x = f\frac{X}{Z}$$

$$y = f\frac{Y}{Z}$$

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Geometry of image formation: World-image transformation

Component formulation

- Recall the camera frame expression of perspective projection

$$x = f\frac{X}{Z}$$

$$y = f\frac{Y}{Z}$$

- We substitute the intrinsic parameterization on the lhs

$$-(x_i - o_x)s_x =$$

$$-(y_i - o_y)s_y =$$

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Geometry of image formation: World-image transformation

Component formulation

- Recall the camera frame expression of perspective projection

$$x = f\frac{X}{Z}$$

$$y = f\frac{Y}{Z}$$

- We substitute the intrinsic parameterization on the lhs and the extrinsic parameterization on the rhs to find that (with $\boldsymbol{R}_i$ the i-th row of $\mathbf{R}$)

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = \boldsymbol{P}_c = \mathbf{R}(\boldsymbol{P}_w - \boldsymbol{T})$$

$$-(x_i - o_x)s_x = f\frac{\boldsymbol{R}_1^T(\boldsymbol{P}_w - \boldsymbol{T})}{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})}$$

$$-(y_i - o_y)s_y = f\frac{\boldsymbol{R}_2^T(\boldsymbol{P}_w - \boldsymbol{T})}{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})}$$

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Geometry of image formation: World-image transformation

Component formulation

- Recall the camera frame expression of perspective projection

$$x = f\frac{X}{Z}$$

$$y = f\frac{Y}{Z}$$

- We substitute the intrinsic parameterization on the lhs and the extrinsic parameterization on the rhs to find that (with $\boldsymbol{R}_i$ the i-th row of **R**)

$$-(x_i - o_x)s_x = f\frac{\boldsymbol{R}_1^T(\boldsymbol{P}_w - \boldsymbol{T})}{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})}$$

$$-(y_i - o_y)s_y = f\frac{\boldsymbol{R}_2^T(\boldsymbol{P}_w - \boldsymbol{T})}{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})}$$

- Which relates the 3D coordinates of a point in the world frame to its corresponding image coordinates.

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Geometry of image formation: World-image transformation

Matrix formulation

- Starting with the component formulation

$$-(x_i - o_x)s_x = f\frac{\boldsymbol{R}_1^T(\boldsymbol{P}_w - \boldsymbol{T})}{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})}$$

$$-(y_i - o_y)s_y = f\frac{\boldsymbol{R}_2^T(\boldsymbol{P}_w - \boldsymbol{T})}{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})}$$

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Geometry of image formation: World-image transformation

Matrix formulation

- Starting with the component formulation

$$-(x_i - o_x)s_x = f\,\frac{\boldsymbol{R}_1^T(\boldsymbol{P}_w - \boldsymbol{T})}{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})}$$

$$-(y_i - o_y)s_y = f\,\frac{\boldsymbol{R}_2^T(\boldsymbol{P}_w - \boldsymbol{T})}{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})}$$

Neglecting world to camera transformation

$$x_i = -\frac{f}{s_x}\frac{X_w}{Z_w} + o_x$$

$$y_i = -\frac{f}{s_y}\frac{Y_w}{Z_w} + o_y$$

- Define matrices that encapsulate intrinsic, $\mathbf{M}_{\text{int}}$

$$\mathbf{M}_{\text{int}} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix},$$

$$\begin{pmatrix} x_i \\ y_i \\ w_i \end{pmatrix} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_w/Z_w \\ Y_w/Z_w \\ 1 \end{pmatrix}$$

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Geometry of image formation: World-image transformation

Matrix formulation

- Starting with the component formulation

$$-(x_i - o_x)s_x = f\,\frac{\boldsymbol{R}_1^T(\boldsymbol{P}_w - \boldsymbol{T})}{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})}$$

$$-(y_i - o_y)s_y = f\,\frac{\boldsymbol{R}_2^T(\boldsymbol{P}_w - \boldsymbol{T})}{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})}$$

Neglecting camera to image transformation

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = \boldsymbol{P}_c = \mathbf{R}(\boldsymbol{P}_w - \boldsymbol{T})$$

- Define matices that encapsulate intrinsic, $\mathbf{M}_{\text{int}}$, and extrinsic, $\mathbf{M}_{ext}$, parameters

$$\mathbf{M}_{\text{int}} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix}, \mathbf{M}_{ext} = \begin{pmatrix} \bullet & \bullet & \bullet & -\boldsymbol{R}_1^T T \\ \bullet & \mathbf{R} & \bullet & -\boldsymbol{R}_2^T T \\ \bullet & \bullet & \bullet & -\boldsymbol{R}_3^T T \end{pmatrix}$$

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = \begin{pmatrix} \bullet & \bullet & \bullet & -\boldsymbol{R}_1^T T \\ \bullet & \mathbf{R} & \bullet & -\boldsymbol{R}_2^T T \\ \bullet & \bullet & \bullet & -\boldsymbol{R}_3^T T \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

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Geometry of image formation: World-image transformation

Matrix formulation

- Starting with the component formulation

$$-(x_i - o_x)s_x = f\,\frac{\boldsymbol{R}_1^T(\boldsymbol{P}_w - \boldsymbol{T})}{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})}$$

$$-(y_i - o_y)s_y = f\,\frac{\boldsymbol{R}_2^T(\boldsymbol{P}_w - \boldsymbol{T})}{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})}$$

- Define matices that encapsulate intrinsic, $\mathbf{M}_{\text{int}}$, and extrinsic, $\mathbf{M}_{ext}$, parameters

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- And concatenate them to write

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \mathbf{M}_{\text{int}}\mathbf{M}_{ext} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

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Geometry of image formation: World-image transformation

Matrix formulation

- Starting with the component formulation

$$-(x_i - o_x)s_x = f\,\frac{\boldsymbol{R}_1^T(\boldsymbol{P}_w - \boldsymbol{T})}{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})}$$

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- And concatenate them to write

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \mathbf{M}_{\text{int}}\mathbf{M}_{ext} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

- Remark: $(x_i, y_i) = (x_1/x_3, x_2/x_3)$

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Geometry of image formation: Perspective camera

Model

- The perspective camera projection matrix, **M**, is formed by explicitly concatentating the interior and exterior parameter matrices.

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Geometry of image formation: Perspective camera

Model

- The perspective camera projection matrix, **M**, is formed by explicitly concatentating the interior and exterior parameter matrices.
- As an example, letting $(o_x, o_y) = (0,0)$ and $(s_x, s_y) = (1,1)$

$$\mathbf{M} = \mathbf{M}_{\text{int}}\mathbf{M}_{ext} = \begin{pmatrix} -f & 0 & 0 \\ 0 & -f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} & & & -\boldsymbol{R}_1^{\mathrm{T}}\boldsymbol{T} \\ & \mathbf{R} & & -\boldsymbol{R}_2^{\mathrm{T}}\boldsymbol{T} \\ & & & -\boldsymbol{R}_3^{\mathrm{T}}\boldsymbol{T} \end{pmatrix} = \begin{pmatrix} -fr_{11} & -fr_{12} & -fr_{13} & f\boldsymbol{R}_1^T\boldsymbol{T} \\ -fr_{21} & -fr_{22} & -fr_{23} & f\boldsymbol{R}_2^T\boldsymbol{T} \\ r_{31} & r_{32} & r_{33} & -\boldsymbol{R}_3^T\boldsymbol{T} \end{pmatrix}$$

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Geometry of image formation: Perspective camera

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Geometry of image formation: Perspective camera

Model

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- Following through on the transformation, we find

$$\mathbf{M}\begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} = \begin{pmatrix} f\boldsymbol{R}_1^T(\boldsymbol{T}-\boldsymbol{P}_w) \\ f\boldsymbol{R}_2^T(\boldsymbol{T}-\boldsymbol{P}_w) \\ \boldsymbol{R}_3^T(\boldsymbol{P}_w-\boldsymbol{T}) \end{pmatrix}$$

Geometry of image formation: Weak perspective camera

Model

- We note that the third component of the perspective projection, $\boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T})$, gives the distance of the point $\boldsymbol{P}_w$ from the center of projection along the optical axis.
- The weak perspective condition is (with $\overline{\boldsymbol{P}}$ the centrode of the points under consideration)

$$\left\| \frac{\boldsymbol{R}_3^T(\boldsymbol{P}_w - \overline{\boldsymbol{P}})}{\boldsymbol{R}_3^T(\overline{\boldsymbol{P}} - \boldsymbol{T})} \right\| << 1$$

- Which suggests that

$$\mathbf{M}\begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} = \begin{pmatrix} f\boldsymbol{R}_1^T(\boldsymbol{T} - \boldsymbol{P}_w) \\ f\boldsymbol{R}_2^T(\boldsymbol{T} - \boldsymbol{P}_w) \\ \boldsymbol{R}_3^T(\boldsymbol{P}_w - \boldsymbol{T}) \end{pmatrix} \approx \begin{pmatrix} f\boldsymbol{R}_1^T(\boldsymbol{T} - \boldsymbol{P}_w) \\ f\boldsymbol{R}_2^T(\boldsymbol{T} - \boldsymbol{P}_w) \\ \boldsymbol{R}_3^T(\overline{\boldsymbol{P}} - \boldsymbol{T}) \end{pmatrix}$$

- So that the corresponding projection matrix has the form

$$\mathbf{M}_{wp} = \begin{pmatrix} -fr_{11} & -fr_{12} & -fr_{13} & f\boldsymbol{R}_1^T\boldsymbol{T} \\ -fr_{21} & -fr_{22} & -fr_{23} & f\boldsymbol{R}_2^T\boldsymbol{T} \\ 0 & 0 & 0 & \boldsymbol{R}_3^T(\overline{\boldsymbol{P}} - \boldsymbol{T}) \end{pmatrix}$$

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Geometry of image formation: Affine camera

Model

- A generalization of the weak perspective camera model.
- Defined from **M** by
  - setting the first three entries of its last row to 0
  - leaving all remaining entries be unconstrained.
- Does not appear to correspond to any standard physical camera
- But often used in the computer vision research for its simplicity

$$\mathbf{M}_{aff} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & a_{34} \end{pmatrix}$$

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Optics (focus & lens)

Radiometry

Geometry

Image acquisition

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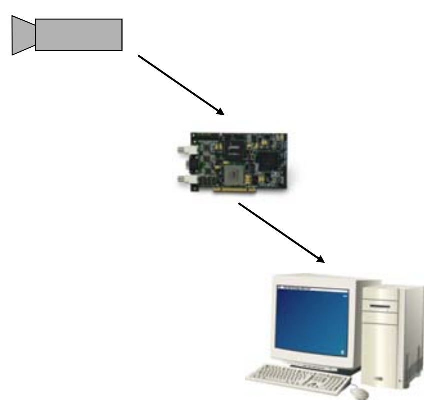
Digital image acquisition: Basics

Typical image acquisition system

- Three major hardware components
1. A viewing camera
2. A frame grabber
3. A host computer



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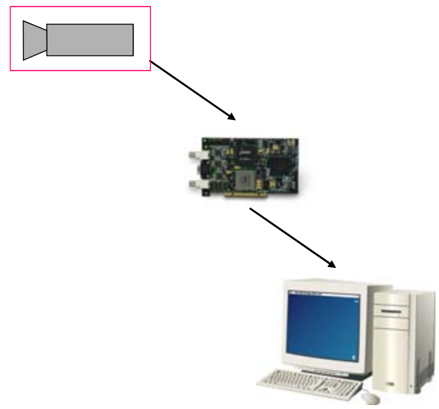
Digital image acquisition: Basics

Typical image acquisition system

- Three major hardware components

1. A viewing camera
2. A frame grabber
3. A host computer



Digital camera

- Optics, radiometry, geometry as before.
- Sensor typically CCD (Charge Coupled Device) technology
  - An *nxm* rectangular grid of photosensors
  - Each photosensor converts light energy to a voltage
- Output is a continuous electrical signal, the video signal,
  - generated by scanning the CCD array (e.g., line by line)
  - and reading the voltages.

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Digital image acquisition: Basics

Typical image acquisition system

- Three major hardware components

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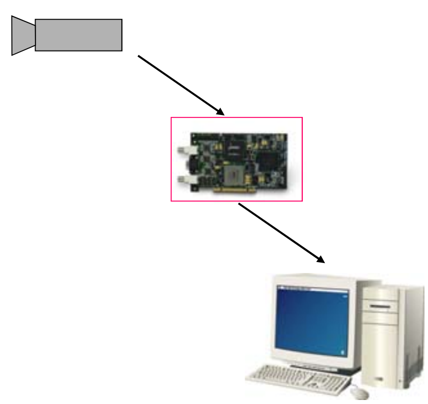
2. A frame grabber
3. A host computer



Frame grabber

- Digitizes the video signal into a 2D rectangular array of NxM integer values, typically [0,255].
- Stores the digitized result into a memory buffer.
- Refer to the array as the digitized image $E(i,j)$,
- with (i,j) indexing individual picture elements, called pixels

Remark:

- The number of elements in the CCD (nxm) may be different from those of the frame grabber (NxM)
- The pixel (x_i, y_i) and CCD coordinates (x_c, y_c) are related by

$$x_i = \frac{N}{n} x_c$$

$$y_i = \frac{M}{m} y_c$$

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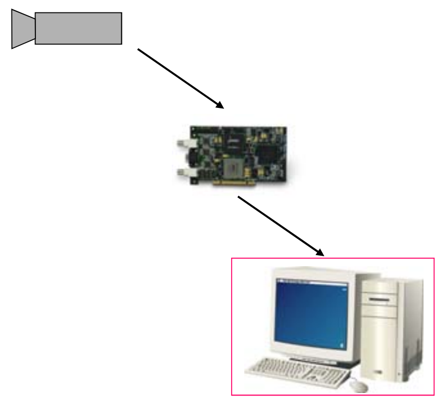
Digital image acquisition: Basics

Typical image acquisition system

- Three major hardware components
1. A viewing camera
2. A framegrabber
3. A host computer

Host computer

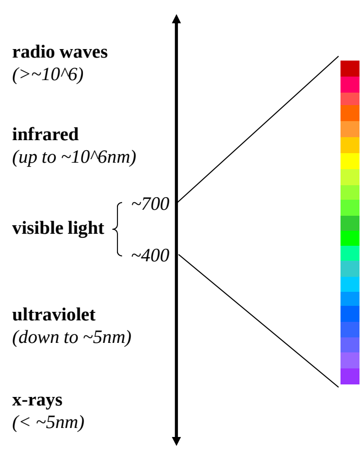
- Recipient of the digitzed image.
- Computational platfom on which processing takes place.



Digital image acquisition: Colour

Wavelength dependence of sensing

- The sensitivity of a sensing device varies with the wavelength of the incident light.
- Photons with very little energy tend to go right through the material.
- Very energetic photons may be stopped before they reach the sensitive layer.
- Quantum efficiency is the ratio of electron flux generated by the sensor for an incident photon flux, denoted as $q(\lambda)$, with λ wavelength.
- Each sensing material has its characteristic variation in quantum efficiency with wavelength.



Electromagnetic spectrum (Wavelength in nanometers; not to scale)

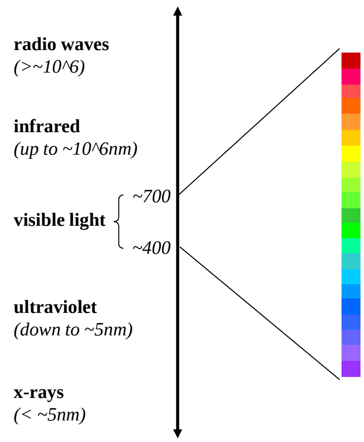
Digital image acquisition: Colour

Wavelength dependence of image

- Consider a small wavelength interval $\delta\lambda$
- Let the flux of photons with energy >= λ but < $\lambda + \delta\lambda$ be $b(\lambda)\delta\lambda$
- The number of electrons liberated during sensing is then

$$\int_{-\infty}^{\infty} b(\lambda)q(\lambda)d\lambda$$

- If different photosensitive materials are used in sensors, then the obtained images differ because their spectral sensitivities differ.
- Another way to achieve the same effect
  - Use the same sensing material, but
  - Place differentially absorbing filters in front of the camera
  - If the transmission of the i-th filter is $f_i(\lambda)$, then the effective quantum efficiency of the combined filter and sensor is $f_i(\lambda)q(\lambda)$.



Electromagnetic spectrum (Wavelength in nanometers; not to scale)

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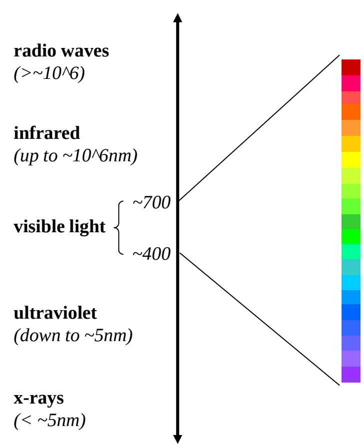
Digital image acquisition: Colour

How many filters should we use?

- The ability to distinguish among materials grows as more images are taken through more filters.
- The measurements are correlated, as most surfaces have a smooth variation of reflectance with wavelength.
- Typically, little is gained by using many filters.
- Remark: Sensing systems that use a small number of sensor types having different spectral sensitivities
  - will provide the same output for many different impinging spectral distributions.
  - The spectral distributions themselves are not being measured,
  - rather integrals of their product with spectral sensitivity of particular sensor types.



Electromagnetic spectrum (Wavelength in nanometers; not to scale)

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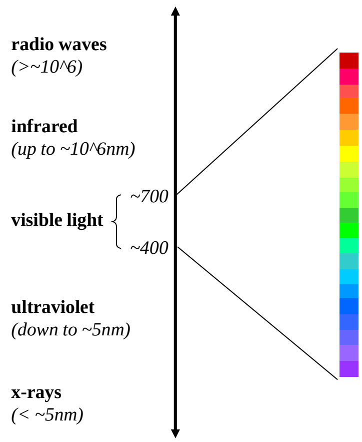
Digital image acquisition: Colour

How many filters should we use?

- As an example, the human visual system
  - Uses three types of photoreceptors called cones.
  - They cover the range of roughly [400,700] nanometers (visible light).
  - There is considerable overlap in their spectral sensitivity.
- Since they often have been designed with human viewing in mind, standard video cameras have sensitivities similar to that of humans.
- However, an emerging area of computer vision research is in “vision beyond the visible spectrum”.



Electromagnetic spectrum (Wavelength in nanometers; not to scale)

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Digital image acquisition: Noise

It is difficult to make accurate measurements of image irradiance.

- Measurements are affected by fluctuations in the signal being measured.
- If the measurements are repeated, somewhat different results might be obtained.
- Typically, measurements will cluster around the correct value.
- It can be useful to consider the probability that a measurement will fall within a certain interval, roughly
  - This is the limit of the ratio of the number of measurements that fall in that interval
  - to the total number of trials,
  - as the total number or trials tends to infinity.

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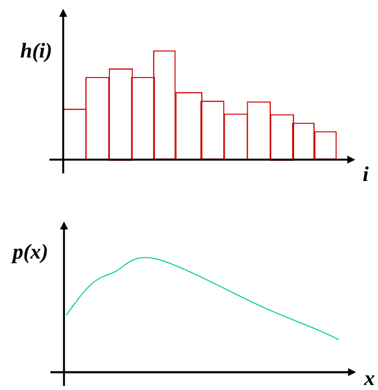
Digital image acquisition: Noise

Probability density distribution

- The probability that a random variable will be equal to or greater that *x*, but less than or equal to $x + \delta x$ tends to $p(x)\delta x$ as δx tends to zero.
- Define *p(x)* as a probability density distribution.
- A probability distribution can be estimated from a histogram, *h(i)*, obtained from a finite number of trials.
- Two important properties of any probability distribution *p(x)*

1. $\forall x, \; p(x) \geq 0$
2. $\int_{-\infty}^{\infty} p(x)dx = 1$



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Digital image acquisition: Noise

Mean

- Often the probability distribution has a strong peak near the correct or expected value.
- Define the mean as the center of area, μ , of this peak, according to

$$\mu \int_{-\infty}^{\infty} p(x)dx = \int_{-\infty}^{\infty} xp(x)dx$$

- Since the integral of *p(x)* from minus to plus infinity is *1*, we have

$$\mu = \int_{-\infty}^{\infty} xp(x)dx$$

- We call the integral on the rhs the first moment of *p(x)*.

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Digital image acquisition: Noise

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Variance

- To estimate the spread of the peak of *p(x),* we compute the second moment about the mean, called the variance

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx$$

- Define the standard deviation as the square root of the variance of a distribution, $\sigma = \sqrt{\sigma^2}$
  - Conventionally, this is a typical characterization of the width of a distribution.

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Digital image acquisition: Noise

Signal to noise ratio (SNR)

- We often want to speak of the relative strength of the signal and noise for a given situation.
- For this purpose, we introduce the signal to noise ratio

$$SNR = \frac{\sigma_s}{\sigma_n}$$

where σ_s and σ_n are the standard deviations of the signal and noise, respectively.

- Signal to noise ratio is often given in decibel units (*dB*)

$$SNR_{dB} = 10\log_{10}\frac{\sigma_s}{\sigma_n}$$

- For example, assuming a signal to noise ratio of 100 we have

$$10\log_{10} 100 = 20dB$$

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Digital image acquisition: Quantization

Spatial quantization as a requirement

- Because we can only transmit a finite number of measurements to a digital computer, spatial quantization of the image is necessary.
- It is common to make measurements at nodes of a rectangular array of integers.
- For example, the photoreceptors of a CCD sensor are organized in a rectangular array of closely packed sensing elements.

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Digital image acquisition: Quantization

Spatial quantization as a requirement

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How does the quantization rate influence the image formation process?

- Let d be the distance between adjacent samples
  - for simplicity assume equal sampling in the vertical and horizontal dimensions
- The sampling theorem tells us that d determines the highest spatial frequency, ν_c, that can be captured by the system.
- We will consider the sampling theorem in detail during the next unit of our course.

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Digital image acquisition: Quantization

How does the highest spatial frequency captured by quantization compare with the spatial frequency content of images?

- The optical component (lens, aperture, etc.) of typical camera systems are capable of imaging spatial frequencies approximately an order of magnitude higher than what could be properly sampled by the sensor array.
  - We are to expect undersampled images with corresponding artifacts
  - In particular, aliasing – the masquerading of high frequencies as spurious low frequencies (jaggies).
- However,
  - The amplitude of such components as derived from common image sources contain little energy in such regions of the spectrum.
  - We do not sample at points; rather each sampling element reports the average irradiance over a finite area and thereby eliminates the highest frequency components before they can be aliased.

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(Digital) image acquisition

- **Basics**
- **Colour**
- **Noise**
- **Quantization**

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Our visual world: Final thoughts on image formation

Why is vision possible?

- At first consideration, it might seems that there is little hope of recovering information about a 3D world from one (or more) 2D images.
- However, as we have started to understand, the relationship between the image and the impinging world is highly constrained.
  - We are immersed in a homogeneous, transparent medium.
  - Light rays are (mostly) not refracted or absorbed in the medium.
  - Surfaces are (mostly) opaque.
  - We can follow a ray from an image point through the lens until we intersect a surface.
  - The irradiance at an image point depends (mostly) on the radiance of the surface patch.
  - Surfaces are 2D manifolds; their shape can be represented, e.g., by giving the distance to the surface as a function of image coordinates.
- A principled approach to computer vision must be based in understanding and exploiting these constraints.

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Would it be possible if?

- Imagine being immersed instead in a world with varying concentrations of pigments dispersed within a gelatinous substance.
- What could be seen then?

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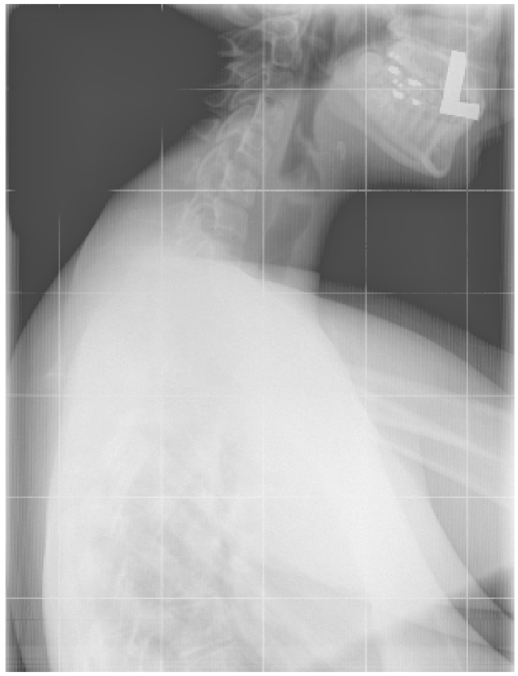

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Summary

- Introduction
- Basic optics
- Basic radiometry
- Geometric image formation
- Image acquisition
- Our visual world

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