MATH/EECS 1028: Discrete Math for Engineers, Winter 2017
Tutorial 9 (Week of Mar 17, 2017)

Notes:

1. Topics: Induction, Counting, Pigeonhole principle,

2. Note to the TA: There will be a quiz this week.

3. I have used the notations ^nC_k and $\binom{n}{k}$ interchangeably.

Questions:

1. From a group of 12 people – 7 of which are men and 5 women – in how many ways may choose a committee of 4 with 1 man and 3 women?

2. What is the number of 5 digit numbers that do not have a 9 in their decimal representation?

3. What is the number of 5 digit numbers that have exactly two 9's in their decimal representation?

4. How many three-digit numbers (100, 101, . . . , 999) have three different digits in strictly increasing order or in strictly decreasing order?

5. How many different functions $f : \{0, 1, \ldots, n\} \to \{0, 1, \ldots, n, n + 1\}$ are there? How many different one-to-one functions $f : \{0, 1, \ldots, n\} \to \{0, 1, \ldots, n, n + 1\}$ are there?

6. . Repeat the previous question, but require that $f(x) < f(x + 1)$ for all $0 \leq x < n$.

7. How many bit strings of length 8 contain an equal number of 0s and 1s?

8. Let $f_1 = 1, f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ when $n > 1$ (the Fibonacci sequence). Prove using induction that $f_n > 2n$ when $n \geq 8$ (note that $f_8 = 21$).

9. There are 8 different courses available, and each student must choose 5 courses to put in his/her plan of studies. What is the minimum number of students such that, no matter what they choose, there will be at least 10 students with the same plan?