

MATH/EECS 1028: DISCRETE MATH FOR ENGINEERS, WINTER 2017  
Tutorial 7 (Week of Mar 2, 2017)

Notes:

1. Topics: Cardinality, Pigeonhole principle, Proofs.
2. Note to the TA: There will be a quiz this week.

Questions:

1. Prove that if  $m$  is a positive integer and  $m, m+1, m+2$  are the sides of a right triangle then  $m = 3$ .
2. Prove that either  $2 \cdot 10^{500} + 15$  or  $2 \cdot 10^{500} + 16$  is not a perfect square. Is your proof constructive?
3. Show that the product of two of the following numbers is nonnegative:  $65^{1000} - 8^{2001} + 3^{177}$ ,  $79^{1212} - 9^{2399} + 2^{2001}$  and  $24^{4493} - 5^{8192} + 7^{1777}$ . Is your proof constructive?
4. Consider the set of all fractions of the form  $\frac{n}{n+\sqrt{n}}$ , where  $n \in \mathbb{Z}, n > 0$ . Is the set countable? Prove your answer.
5. Prove that among any given  $n+1$  positive integers, there are always two whose difference is divisible by  $n$ .  
Hint: Use the Pigeonhole Principle.
6. Show that if 7 integers are selected from the first 10 positive integers (i.e., the numbers 1 through 10), there must be at least 2 pairs of these integers with sum 11.
7. The digital sum of a number is defined as the sum of its decimal digits. For example, the digital sum of 386 is  $3 + 8 + 6 = 17$ . Suppose 35 two-digit numbers are selected. Prove that there are three of them with the same digital sum.
8. Given any  $n$  natural numbers, the sum of some non-empty subset of them is divisible by  $n$ .