MATH/EECS 1028: Discrete Math for Engineers
Winter 2017
Tutorial 6 (Feb 17 and Feb 27, 2017)

Notes:

1. Topics: Logic, proofs.
2. Note to the TA: Attendance will be taken as usual. No quiz this week.

Questions:

1. Express using logical operators, quantifiers and predicates: "The negation of a contradiction is a tautology".
2. Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \rightarrow \neg S(x))$ and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.
3. Proof by cases.

   Prove that $n, n+7$ cannot both be perfect cubes where n is an integer greater than 1.
4. Let $p < q$ be two consecutive odd primes. Prove that $p + q$ is a composite number, having at least three, not necessarily distinct, prime factors.
5. A function $f(x)$ is said to be strictly increasing if $f(b) > f(a)$ for all $b > a$. Prove that a strictly increasing function from $\mathbb{R}$ to itself is one-to-one.
6. Suppose A, B, C are sets. Prove or disprove: $(A - B) - C = A - (B - C)$.
7. Prove that there is no positive integer n such that $n^2 + n^3 = 99$.
8. Prove that there is no positive integer n such that $n^2 + n^4 = 200$.
9. Prove that $p, p+2, p+4$ cannot all be primes except when $p = 3$.
   Hint: Consider the different cases for p mod 3 i.e. the remainder obtained after dividing p by 3.
10. Prove that if n is a nonnegative integer then $n^3 - n$ is divisible by 6.