MATH/EECS 1028: DISCRETE MATH FOR ENGINEERS
WINTER 2017
Tutorial 4 (Week of Feb 3, 2017)

Notes:

1. Assume $\mathbb{R}$ to denote the real numbers, $\mathbb{Z}$ to denote the set of integers $(\ldots, -2, -1, 0, 1, 2, \ldots)$ and $\mathbb{N}$ to denote the natural numbers $(1, 2, 3, \ldots)$.

2. Topics: Sequences, Logic.

3. Note to the TA: Attendance will be taken this week.

Questions:

1. Evaluate
$$\sum_{i=1}^{n} \sum_{k=1}^{n} ik.$$

2. Evaluate
$$\sum_{i=1}^{n} \sum_{k=1}^{i} ik.$$

3. Show that
$$\sum_{k=2}^{n} \frac{1}{(k-1)k} = \frac{n-1}{n}.$$

4. Write down the truth table for the following proposition. Then indicate whether is a tautology, contradiction or neither.
$$(p \wedge q) \to (p \to q)$$

5. Let p, q, r be the propositions:
   p: You have the flu
   q: You miss the final examination
   r: You pass the course.

   Write the following proposition as an English sentence.
   $$(p \to \neg r) \vee (q \to \neg r)$$

6. Form the contrapositive of these statements:

   (a) If you don't take the final examination, you will get an F for the course.

   (b) If a quadrilateral is a rectangle, it has 4 equal angles.

   (c) If a triangle has either two equal sides or two equal angles, then it is an isosceles triangle.

7. Decide whether the following statements are tautologies or contradictions or neither. Prove your answer in each case.

   (a) $(p \to q) \vee (q \to p)$.

   (b) $(p \wedge q) \vee (q \to \neg p)$.

   (c) $(p \vee \neg q) \to (q \wedge \neg p)$.

8. Each argument below is either correct or it has a fallacy (but not both!). Write the argument in symbols and then determine whether the argument is valid. If it is valid, write whether it uses *modus ponens* or *modus tollens*.

   (a) If both numbers are even, then the sum is even. They are not both even. Therefore the sum is not even.

   (b) If this University is large, then it has large departments. This University has large departments. Therefore, it is large.

9. Let p be the proposition "Grizzly bears have been seen in the area", q be the proposition "Hiking is safe on the trail" and r be the proposition "Berries are ripe on the trail". Express the following using propositions p, q, r and logical connectives (including negation).

   (a) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

   (b) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

10. Show that $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ are logically equivalent.