MATH/EECS 1028: Discrete Math for Engineers
Winter 2017
Tutorial 10 (Week of Mar 24, 2017)

Notes:

1. Topics: Counting, Pigeonhole principle, Strong Induction.
2. Note: Attendance will be taken this week. No quiz this week.

Questions:

1. (Q48, p 398.)

   How many bit strings of length 7 either begin with two 0's or end with three 1's?

2. (Q24, p 396.)

   How many positive integers between 1000 and 9999 inclusive

   (a) are divisible by 5 or 7?

   (b) are divisible by 5 but not divisible by 7?

   (c) have distinct digits?

3. Let us represent the result of three tosses of a standard six-sided die (faces 1 through 6) as an ordered list of length 3. How many different results are possible? In how many of these are all three tosses different?

4. How many arrangements are there of the letters of the word MATCH? Of these how many of them have the letters MA together? How many arrangements have the letters M,A together but not necessarily in that order?

5. (Q10, p 342 of the text.)

   Assume that a chocolate bar consists of n squares arranged in a rectangular pattern. The entire bar, or any smaller rectangular piece of the bar, can be broken along a vertical or a horizontal line separating the squares. Assuming that only one piece can be broken at a time, determine how many breaks you must successfully make to break the bar into n separate squares. Use strong induction to prove your answer.

6. (Q14, p 342 of the text.)

   Suppose you begin with a pile of n stones and split the pile into n piles of 1 stone each by successively splitting a pile of stones into two smaller piles. Each time you split a pile, you multiply the number of stones in each of the two smaller piles you form and add them to a running sum. So if these piles have r and s stones in them, respectively, you compute rs and add it to the sum. Show that no matter how you split the piles, the sum of the products computed at each step equals $n(n-1)/2$.

7. (Q47, p 398)

   In how many ways can a photographer at a wedding arrange 6 people in a row, including the bride and the groom, if

   (a) The bride must be next to the groom?

   (b) The bride is not next to the groom?

   (c) The bride is positioned somewhere to the left of the groom?

8. Given any 7 integers there will be four integers such that the sum of the squares of those integers is divisible by 4.

9. How many positive integral solutions are there to $a + b + c = 10$?

10. Expand and simplify: $\left(\sqrt{1-x^2}+1\right)^7 - \left(\sqrt{1-x^2}-1\right)^7$.