Sequences (Ch 2.4)

- Finite or infinite
- Calculus – limits of infinite sequences (proving existence, evaluation…)
- E.g.
  - Arithmetic progression (series)

    1, 4, 7, 10, …
  - Geometric progression (series)

    3, 6, 12, 24, 48 …

Similarity with series

- $S = a_1 + a_2 + a_3 + a_4 + \ldots\ldots$ (n terms)
- Consider the sequence

  $S_1, S_2, S_3, \ldots S_n$, where

  $S_i = a_1 + a_2 + \ldots + a_i$

In general we would like to evaluate sums of series – useful in algorithm analysis.

e.g. what is the total time spent in a nested loop?

Sums of common series

- Arithmetic series

  e.g. $1 + 2 + \ldots + n$ (occurs in the analysis of running time of simple for loops)

  general form $\Sigma_i t_i$, $t_i = a + ib$

- Geometric series

  e.g. $1 + 2 + 2^2 + 2^3 + \ldots + 2^n$

  general form $\Sigma_i t_i$, $t_i = ar^i$

- More general series (not either of the above)

  $1^2 + 2^2 + 3^2 + 4^2 + \ldots + n^2$

Sums of common series - 2

- Technique for summing arithmetic series
- Technique for summing geometric series
- More general series – more difficult

Caveats

- Need to be very careful with infinite series
- In general, tools from calculus are needed to know whether an infinite series sum exists.
- There are instances where the infinite series sum is much easier to compute and manipulate, e.g. geometric series with $r < 1$.