

MATH/EECS 1028: DISCRETE MATH FOR ENGINEERS  
WINTER 2017  
Assignment 2 (Released March 8, 2017)  
Submission deadline: 1:25 pm, Mar 20, 2017

Notes:

1. The assignment can be handwritten or typed. It MUST be legible.
2. You must do this assignment individually.
3. Submit this assignment only if you have read and understood the policy on academic honesty on the course web page. If you have questions or concerns, please contact the instructor.
4. Use the dropbox near the EECS main office to submit your assignments. No late submissions will be accepted. Please do not send files by email.
5. Your answers should be precise and concise. Points may be deducted for long, rambling arguments.

## Question 1

There are 12 chairs in a row, and 9 people sitting (so that 3 chairs are free). Prove that there are 3 consecutive chairs occupied.

## Question 2

Prove using induction that 12 divides  $2^{2n+2} - 3n^4 + 3n^2 - 4$ , for all  $n \in \mathbb{N}$ .

## Question 3

Prove using induction that

$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \cdots + \frac{1}{(3n-2) \cdot (3n+1)} = \frac{n}{3n+1}$$

for all  $n \in \mathbb{N}$ .

## Question 4

Recall the definition of the Fibonacci sequence:  $f_0 = 1, f_1 = 1$ , and  $f_n = f_{n-1} + f_{n-2}$  for  $n > 1$ . Prove that  $f_n \leq (1 + \sqrt{5})^n$ .

## Question 5

Define a sequence recursively as follows.  $x_1 = 1$  and for  $n \in \mathbb{N}$ ,  $x_{n+1} = \sqrt{x_n^2 + \frac{1}{x_n^2}}$ . Prove using mathematical induction that for all  $n \in \mathbb{N}$ ,  $1 \leq x_n \leq \sqrt{n}$ .