Test 2

First Name: _____

Last Name: _____

Student Number: _____

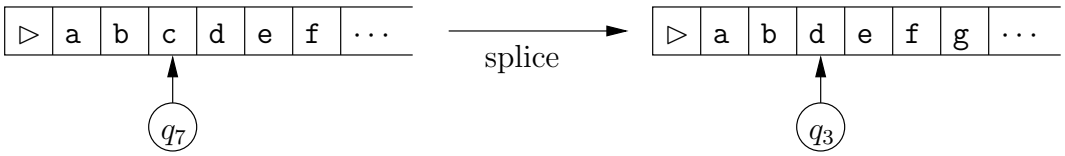
This test lasts 80 minutes. *No aids allowed.*
You may use any result that was proved in class or in the textbook without reproving it.
You may also use the Church-Turing Thesis.
Make sure your test has 5 pages, including this cover page.
Answer in the space provided. (If you need more space, use the reverse side of the page and indicate ***clearly*** *which part of your work should be marked.)*
Write legibly.

Question 1	/2
Question 2	/3
Question 3	/4
Question 4	/4
Question 5	/4
Total	/17

1. [2 marks] Let L be a language. Explain the difference between a Turing machine that recognizes L and a Turing machine that decides L.

2. [3 marks] If L is a language, let $ROT(L)$ be the set of rotations of strings in L.
More precisely, $ROT(L) = \{xy : x \text{ and } y \text{ are strings such that } yx \in L\}$.
E.g., if $L = \{\texttt{dog}, \texttt{horse}\}$, then $ROT(L) = \{\texttt{dog}, \texttt{ogd}, \texttt{gdo}, \texttt{horse}, \texttt{orseh}, \texttt{rseho}, \texttt{sehor}, \texttt{ehors}\}$.
Prove (for all languages L) that if L is decidable, then $ROT(L)$ is also decidable.

3. [4 marks] A splicing Turing machine (STM) is similar to an ordinary Turing machine. The only difference is that it has the additional ability to perform a splice, which cuts the square at the location of the STM's head out of the tape and then glues the two remaining pieces of the tape together. The head of the STM then moves to the square either to the left or right of the removed square, depending on the transition specified. An example of a splice step is shown below, if the transition function of the STM specifies that $\delta(q_7, \texttt{c}) = (q_3, \text{splice}, R)$.



Let M_s be any STM. Explain (at a high level) how you could construct an ordinary Turing machine M that simulates the actions of M_s. Your explanation should be in clear, precise English.

4. [4 marks] We call a Turing machine M *spacious* if there is a string w such that M writes to at least $|w|^2$ different tape squares when it runs on the input string w. Give pseudocode for a *deterministic* algorithm that *recognizes* $SPC_{TM} = \{\langle M \rangle : M \text{ is a spacious Turing machine}\}$. Explain why your algorithm is correct.

5. [4 marks] Let $R_{TM} = \{\langle M, w\rangle : M \text{ is a Turing machine that rejects the input string } w\}$.

(a) Explain why $R_{TM} \neq \overline{A_{TM}}$.

(b) Prove that R_{TM} is undecidable.