Assertions

How to write correct programs and know it

– Harlan Mills

Assertions

- Boolean expressions or predicates that evaluate to **true** or **false** in every state
- In a program they express **constraints** on the state **that must be true** at that point
- Associate with
  - » **Individual program statements**
  - » **functions**
  - » **classes**

Assertions & Correct Programs

- Specify clearly, precisely and succinctly
  - » **What is expected and guaranteed by each component – class, function and statement**
- The essence of documentation
- Essential for debugging
- Aids in fault tolerance

Assertion Language Symbols

- Arithmetic operators

  + – * / ^ (exponent)

  // div (integer division)
  \\ mod (modulus / remainder)

- Relational operators

  $= \quad \neq \quad \leq \quad \geq \quad < \quad >$

- Boolean operators & logic

  ∧ **and** ∨ **or** ⊕ **xor** ¬ **~ not**

  → **implies** ↔ **iff**

Assertion Language Symbols – 2

- Semi-strict **and** and **or** – Eiffel only for practical and efficiency reasons
  - » **Also called lazy evaluation in other programming languages**

and then

A and then B **Evaluate B only if A is true**

or else

A or else B **Evaluate B only if A is false**

Assertion Language Symbols – 3

- Predicate logic

  ∀ **forall**   ∃ **exists (there exists)**

  | **such that**

  • **it is the case that (it holds that)**

- Set operators

  ∈ **member_of**   ∉ **not_member_of**

  ⊃ ⊇ ⊂ ⊆ **contains**

  ⊄ **does_not_contain**

  ∩ **intersection**   ∪ **union**   \ **set difference**

  #S number of members of the set S

Assertion Language Special Symbols

- Special variables related to program semantics

Result	**– result of a function**
Current @	**– current object**
Void	**– not attached**

Variable before and after values

- Mathematical notation

  Unlimited context

  name

  value of the variable name before its value is changed

  name'

  value of the variable name after a its value has changed

- Eiffel notation

  Limited context

  name

  value of the variable name after a routine terminates

  old name

  value of the variable name before a routine starts

Quantified Expression

- Used to express properties about sets of objects

such that — it holds / it is the case that

Quantifier Range_Expr [| Restriction] • Property

Quantifier ∀ **forall** ∃ **exists (there exists)**

Range_Expr **var_name : set_of_values**

Restriction **Boolean expression or, recursively, a quantified expression**

Property **Boolean expression or, recursively, a quantified expression**

Range Expression examples

- Type range – each value is of a given type

  v : VEHICLE

- Sequence range – each value is in a sequence

  k : low .. high

- Member range – each value is a member in a set

  c ∈ **children**

Feature name of a collection of items

Mathematical Notation example

```
class CITIZEN feature
    name, sex, age : VALUE
    spouse : CITIZEN
    children, parents : SET[CITIZEN]
    single : BOOLEAN ensure Result ↔ ( spouse = Void )
    divorce
        require ~ single
        ensure single ∧ ( old spouse ) . single
     end
invariant
        single ∨ spouse . spouse = @
        parents . count = 2
      ∀ c ∈ children • ( ∃ p ∈ c . parents • p = @ )
end
```

Textual Notation example

```
class CITIZEN feature
    name, sex, age : VALUE
    spouse : CITIZEN
    children, parents : SET[CITIZEN]

    single : BOOLEAN ensure Result iff ( spouse = Void )

    divorce
        require  not single
        ensure  single and ( old spouse ) . single
    end

invariant
        single or spouse.spouse = Current
        parents.count = 2
        for_all  c member_of children  it_holds
            ( exists  p  member_of  c . parents  it_holds  p = Current)

end
```

Specifying Members of a Set

- Set enumeration – list the members

  S = { a, e, i, o, u }

  The set of vowels in the English alphabet

- Set comprehension – logically specify members
  Notice that the forall is implicit not explicit

  $$\{ x , y : \text{Integer} \mid (0 < x < 10) \land (1 \leq y \leq 9) \cdot x^3 + y^3 \}$$

  The set of the sums of pairs of the cubes of single digit integers greater than zero

Pre-Conditions

- Statement syntax
  - » **require boolean expression**
- Where within function/procedure
  - » **write immediately after the routine header**

```
add( new_item : T)
-- add the new_item to the collection
require  not is_full
         not has(new_item)
do
...
end
```

Post-Conditions

- Statement syntax
  - » **ensure boolean expression**
- Where within function/procedure
  - » **write just before the end for the routine body**

```
add( new_item : T)
-- add the new_item to the collection
do
...
ensure  has(new_item)
end
```

State changes

- Show relationship between initial and final values
- At the end of the body the final values are in effect
- Refer to initial values using the keyword **old**

```
addElement ( element : TYPE )
require  size < Capacity
do

...

ensure size = old size + 1
end
```

Assertions are tagged

- Tag names are used to identify assertions

```
addElement ( element : TYPE )
require enough_space: size < Capacity
do

...

ensure one_larger: size = old size + 1
end
```

Non-executable assertions

- Use **comments** if you cannot write an executable assertion
- Use already defined functions or custom written functions

```
insert_in_row(matElem : MATRIX_ELEMENT)
    -- Insert the matrix element in the current row
  require …
  local …
  do …
ensure
  -- contains(MatrixElement(data, row, column)) at < row, column >
end
```

Loop Invariants & Loop Syntax

```
from
    init statements
invariant
    assertions for invariant
until
    exit condition
loop
    body statements
variant
    integer expression
end
```

Assertions for invariant:

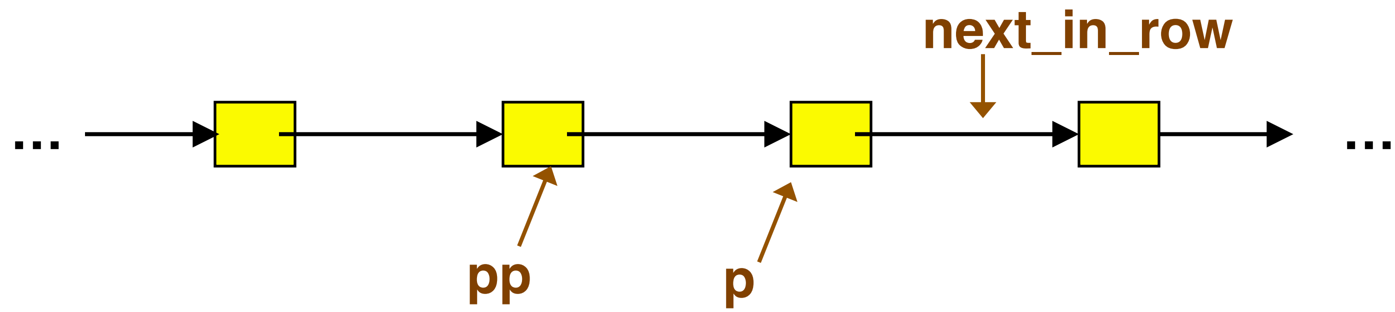
- Can invoke Boolean functions
- Use agents to implement predicate calculus expressions

Integer expression (variant):

- Always non negative
- Body decreases value on every iteration
- Ideally 0 on loop exit

Loop Invariant Example

- Insert an element into a sorted by column singly linked list




```
row := matElem.row ; column := matElem.column
from   p := rowList @ row
invariant  ???
until
  p = void or p.column >= column
loop
  pp := p   ;   p := p.next_in_row
end
```

Loop Invariant Example – 2

- Using mathematical notation

invariant

predecessor_relation: $(pp = void \wedge p = head)$
$\vee (pp \neq void \wedge pp\ .\ next = p)$

predecessor_before : $pp = void \vee pp\ .\ column < column$

column_less_than: $\forall\ k : head\ ..\ pp \bullet k\ .\ column < column$

Loop Invariant Example – 3

- Eiffel executable assertion.
- Column_less_than uses an agent to implement the invariant
  - > **Agents and loop invariant details are in other slide sets**

```
from   p := rowList @ row
invariant
  predecessor_relation :      (pp = void and p = rowList @ row)
                           or  (pp /= void and pp.next_in_row = p)

  predecessor_before : pp = void or pp.column < column

 -- forall k : rowList @ row .. pp :: k.column < column
  data_less_than : column_limit( rowList @ row, pp,
                                         agent less_than(?, column) )
end
```

Check Assertion

- Within the body of a routine you can insert a **check** clause
- The **check** clause is executed and if an assertion is false then an exception occurs
- Used to remind the reader of a non obvious fact

```
If full then  error := overflow
else
   check
      representation_exists : representation /= Void
   end
   representation.put(x) ; error := none
end
```

Class Invariants

- Appear in the **invariant** clause just before the end of the class definition

```
class RING_BUFFER
...
invariant
    -- Abstract properties
  enough_capacity: Capacity >= 3
  unique_items: for_all(agent unique_item(?))

-- Implementation properties
  proper_buffer_size: Buffer_size = Capacity + 1
  count_and_ptrs_related:
     count = (Buffer_size + last - first + 1) \\ Buffer_size
end
```

Class Invariants – 2

- Class invariants define which states of the ADT are valid
- True at stable times
  - » **After make (object creation)**
  - » **Before and after every exported feature call**
    - > **Could be false during a feature call as various sub-states change**
- Invariant is implicitly a part of every pre and post condition

Class Invariants – examples

- See slides 11 & 12 in this set of slides
  - » **Relationship between parents and children**
  - » **Relationship between spouses**
- See Abstract data type documentation slides 18 .. 23
  - » **Relationship between first and last pointers in a circular queue and the length of the queue**
- Case studies
  - » **Sparse matrix**
  - » **Dictionary**
- Report 1 system

General Guideline

- Assertions may be written in many ways
  - » **Select the representation to be as clear and easy to understand as possible**
    - > **Point is to convey information, not provide a puzzle to be solved**
  - » **Use notation that is close to the meaning of the relationships involved**
    - > **Set notation { … }**
    - > **Bag notation [[…]]**
    - > **Sequence notation < … >**

Assertion Monitoring

- Eiffel provides multiples levels of assertion monitoring
- Always should be on during debugging
- Turn off as little as possible only if time is critical and the system can be trusted