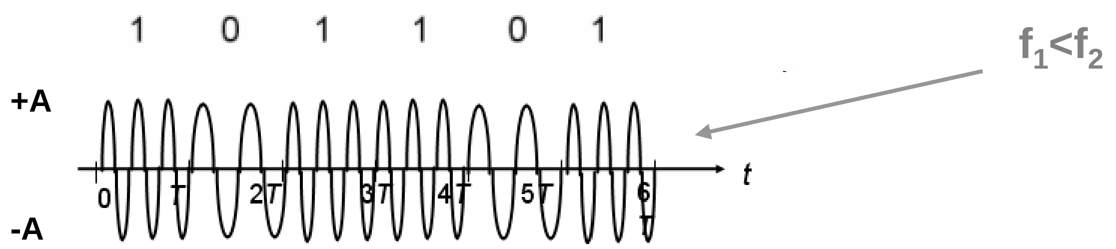
Modulation of Digital Data: FSK

FSK – **frequency of the carrier signal is varied to represent binary 1 or 0**

- **both peak amplitude and phase remain constant during each bit interval**

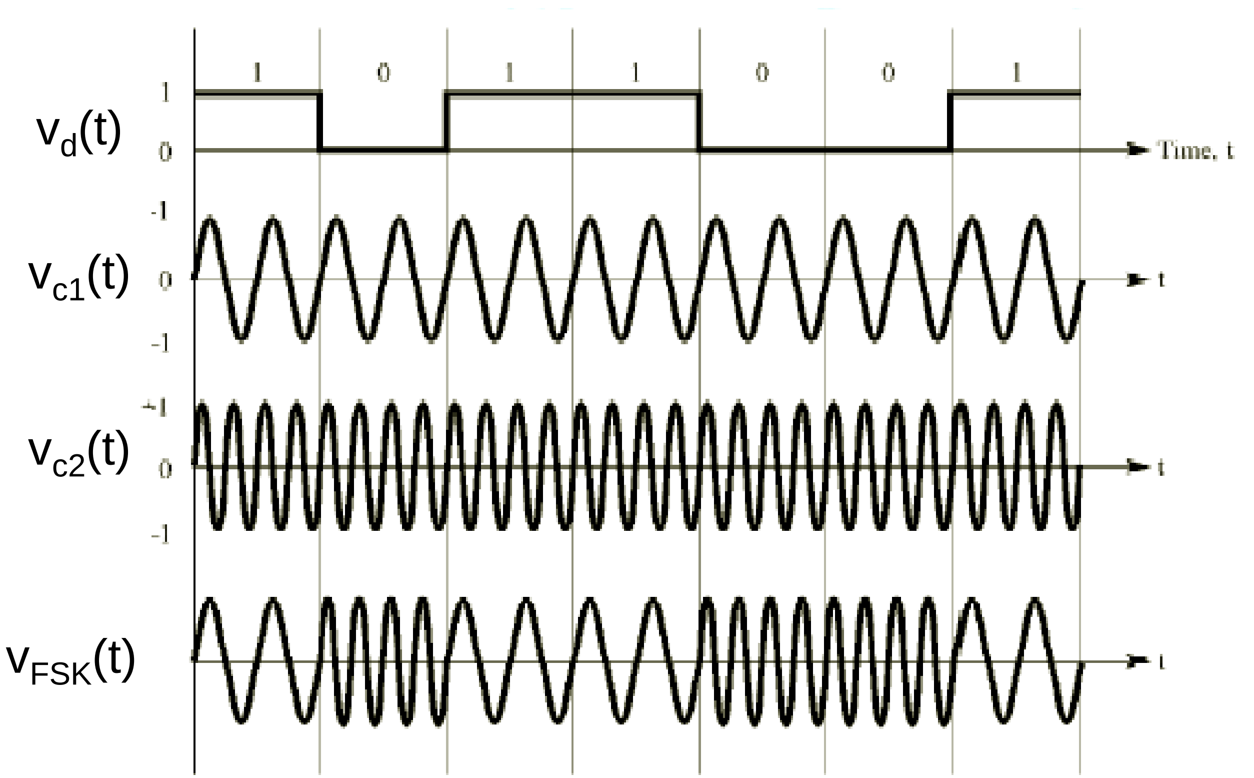
$$s(t) = \begin{cases} A\cos(2\pi f_1 t), & \text{binary } 0 \\ A\cos(2\pi f_2 t), & \text{binary } 1 \end{cases}$$

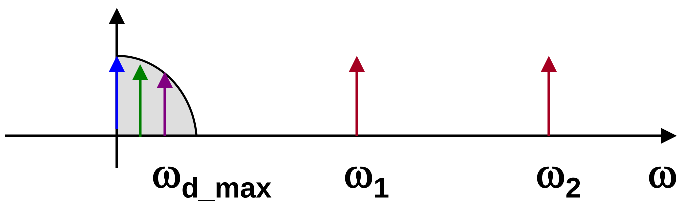


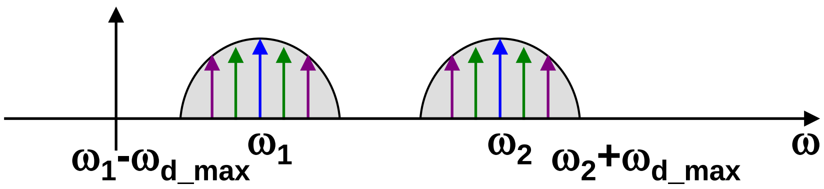
- **demodulation**: **demodulator must be able to determine which of two possible frequencies is present at a given time**
- **advantage**: **FSK is less susceptible to errors than ASK – receiver is looking for specific frequency changes over a number of intervals, so voltage (noise) spikes can be ignored**
- **disadvantage**: **FSK spectrum is 2 x ASK spectrum**
- **application**: **over voice lines, in high-frequency radio transmission, etc.**

Modulation of Digital Data: FSK (cont.)

Example [FSK]







Modulation of Digital Data: FSK (cont.)

FSK-Modulated Signal: Frequency Spectrum

Digital signal: $v_d(t)$ **- modulated with ω_1, and**

$v_d'(t) = 1 - v_d(t)$ **- modulated with ω_2**

Modulated signal:

$$v_{FSK}(t) = \cos\omega_1 t \cdot v_d(t) + \cos\omega_2 t \cdot (1 - v_d(t)) =$$

$$= \cos\omega_1 t \cdot \left[\frac{1}{2} + \frac{2}{\pi}\cos\omega_0 t - \frac{2}{3\pi}\cos 3\omega_0 t + \frac{2}{5\pi}\cos 5\omega_0 t - \ldots\right] +$$

$$+ \cos\omega_2 t \cdot \left[\frac{1}{2} - \frac{2}{\pi}\cos\omega_0 t + \frac{2}{3\pi}\cos 3\omega_0 t - \frac{2}{5\pi}\cos 5\omega_0 t - \ldots\right] =$$

$$= \ldots$$

$$= \frac{1}{2}\cos\omega_1 t + \frac{1}{\pi}\left[\cos(\omega_1 - \omega_0)t + \cos(\omega_1 + \omega_0)t\right] -$$

$$- \frac{1}{3\pi}\left[\cos(\omega_1 - 3\omega_0)t + \cos(\omega_1 + 3\omega_0)t\right] + \ldots +$$

$$\frac{1}{2}\cos\omega_2 t - \frac{1}{\pi}\left[\cos(\omega_2 - \omega_0)t + \cos(\omega_2 + \omega_0)t\right] -$$

$$+ \frac{1}{3\pi}\left[\cos(\omega_2 - 3\omega_0)t + \cos(\omega_2 + 3\omega_0)t\right] + \ldots +$$

Modulation of Digital Data: PSK

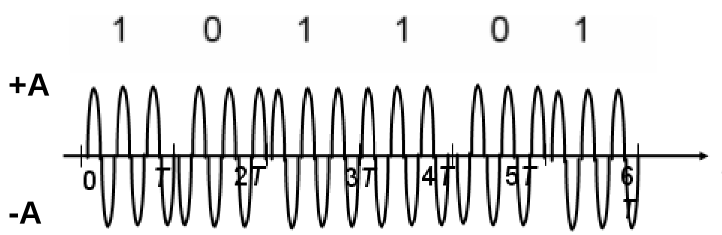
PSK – **phase of the carrier signal is varied to represent binary 1 or 0**

- **peak amplitude and frequency remain constant during each bit interval**
- **example: binary 1 is represented with a phase of 0º, while binary 0 is represented with a phase of 180º=πrad ⇒ PSK is equivalent to multiplying the carrier signal by +1 when the information is 1, and by -1 when the information is 0**

2-PSK, or Binary PSK, since only 2 different phases are used.

$$s(t) = \begin{cases} A\cos(2\pi f_c t), & \text{binary } 1 \\ A\cos(2\pi f_c t + \pi), & \text{binary } 0 \end{cases}$$

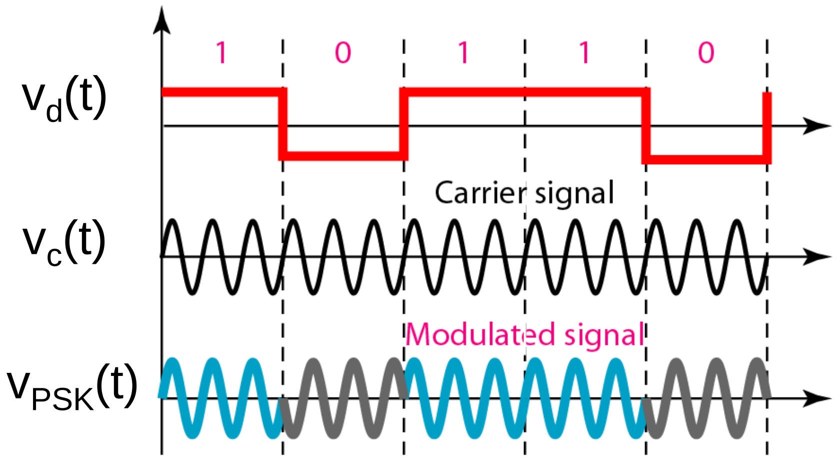
$$s(t) = \begin{cases} A\cos(2\pi f_c t), & \text{binary } 1 \\ -A\cos(2\pi f_c t), & \text{binary } 0 \end{cases}$$

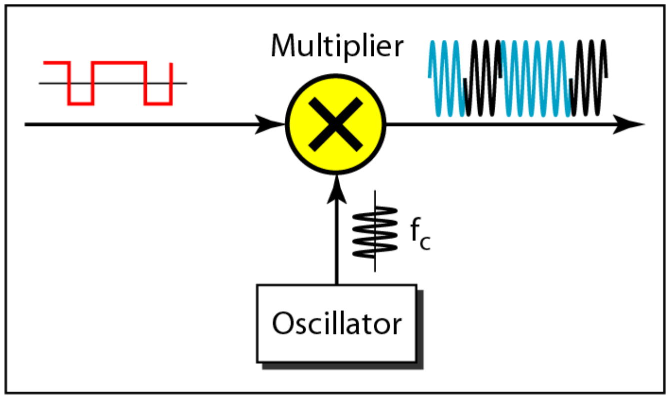


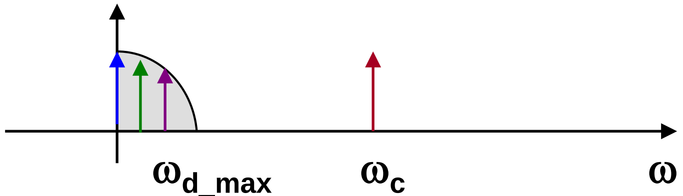
- **demodulation: demodulator must be able to determine the phase of received sinusoid with respect to some reference phase**
- **advantage:**
  - **PSK is less susceptible to errors than ASK, while it requires/occupies the same bandwidth as ASK**
  - **more efficient use of bandwidth (higher data-rate) are possible, compared to FSK !!!**
- **disadvantage: more complex signal detection / recovery process, than in ASK and FSK**

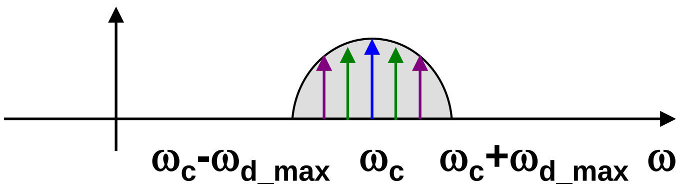
Modulation of Digital Data: PSK (cont.)

Example [PSK]









Modulation of Digital Data: PSK (cont.)

PSK Detection / Recovery – **multiply the received / modulated signal** $\pm A\cos(2\pi f_c t)$ **by 2*cos(2πf$_c$t)**

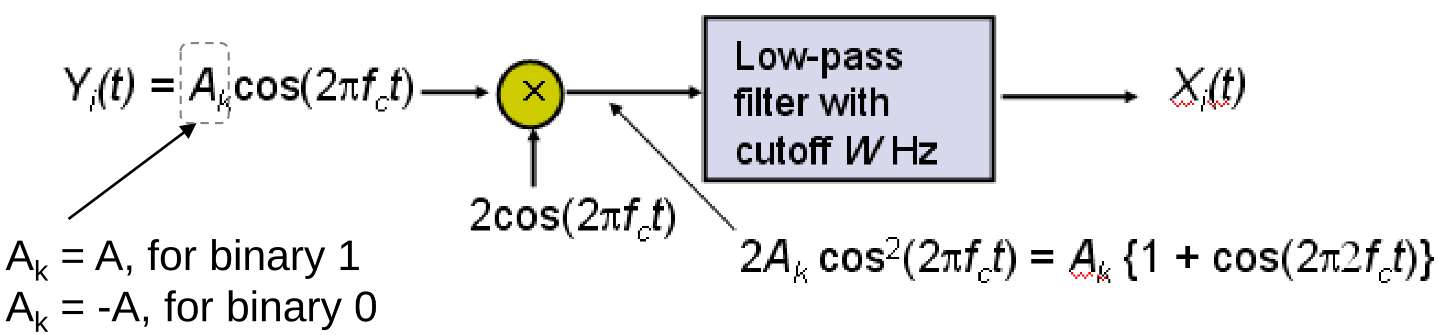
$$\cos^2 A = \frac{1}{2}(1+\cos 2A)$$

- **resulting signal**

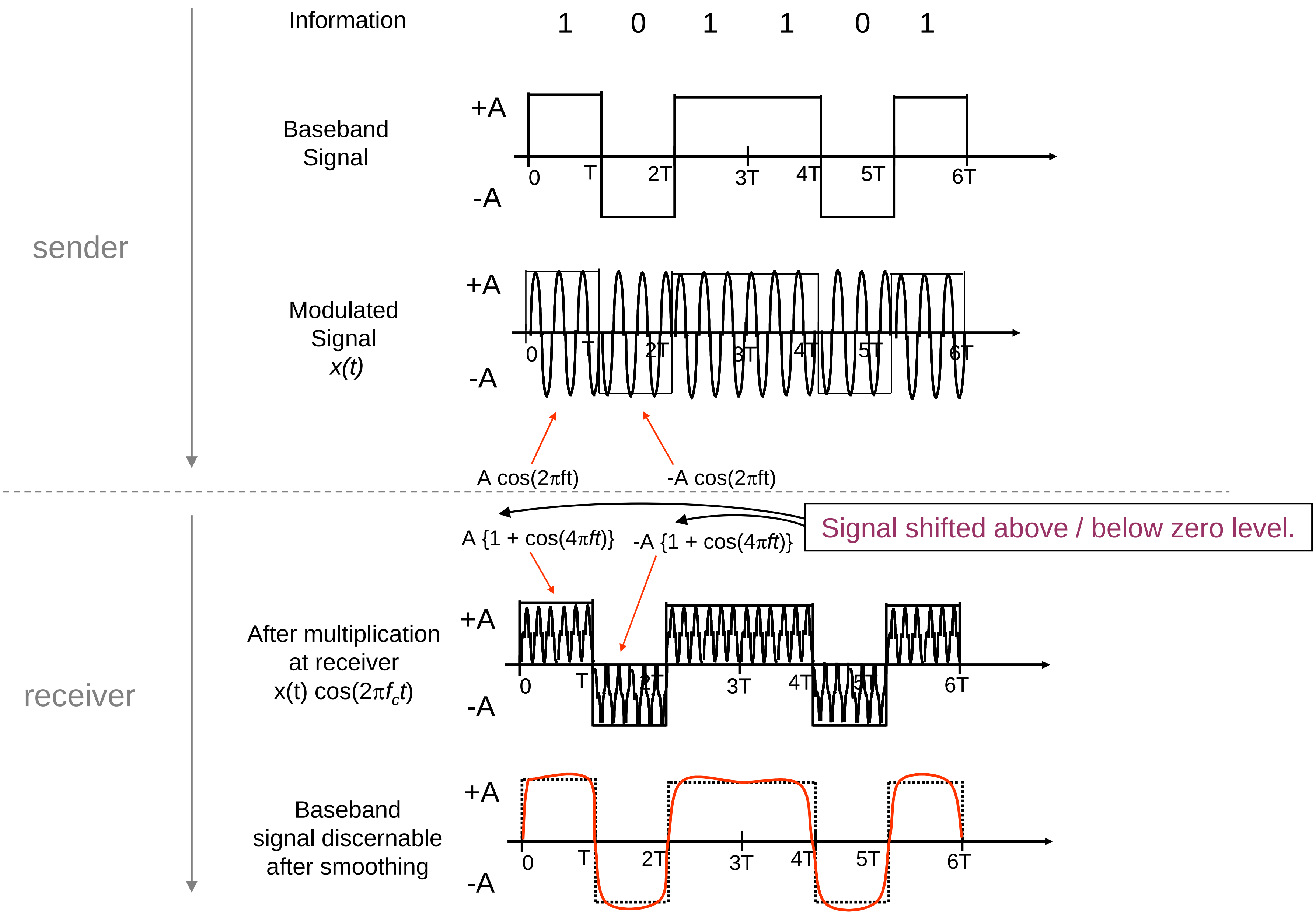
$$2A\cos^2(2\pi f_c t) = A[1+\cos(4\pi f_c t)], \quad \text{binary } 1$$

$$-2A\cos^2(2\pi f_c t) = -A[1+\cos(4\pi f_c t)], \quad \text{binary } 0$$

- **by removing the oscillatory part with a low-pass filter, the original baseband signal (i.e. the original binary sequence) can be easily determined**



Modulation of Digital Data: PSK (cont.)



Modulation of Digital Data: PSK (cont.)

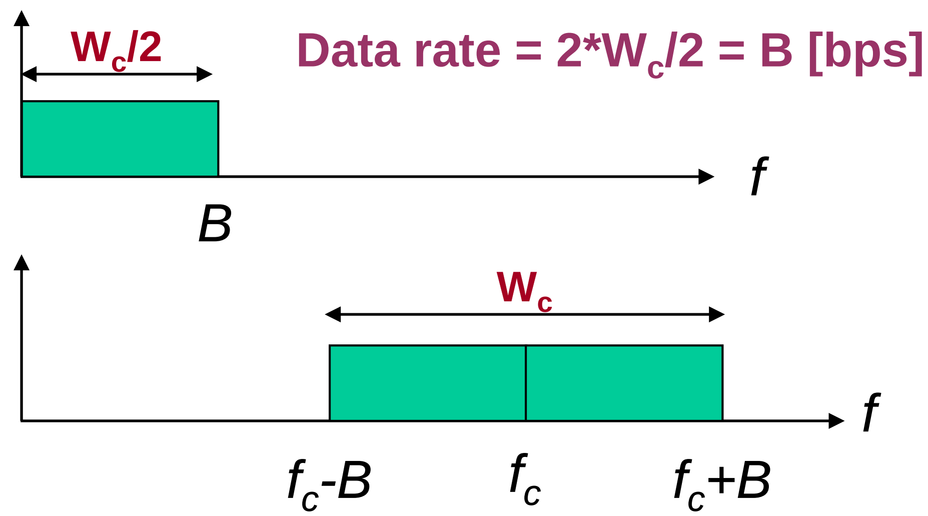
Facts from Modulation Theory

If

Baseband signal *x(t)* occupies bandwidth $W_c/2$

then

Modulated signal $x(t)\cos(2\pi f_c t)$ occupies bandwidth W_c Hz



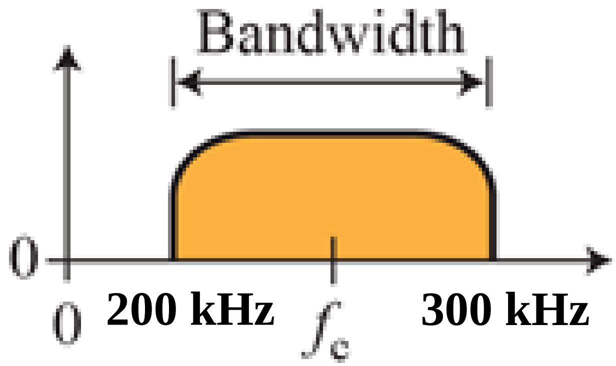
- **If bandpass channel has bandwidth W_c [Hz],**
  - **then baseband channel has $W_c/2$ [Hz] available, so**
  - **modulation system supports C = 2*($W_c/2$) = W_c [pulses/second]**
  - **recall Nyqyist Law: baseband transmission system of bandwidth W_c [Hz] can theoretically support 2 W_c pulses/sec**

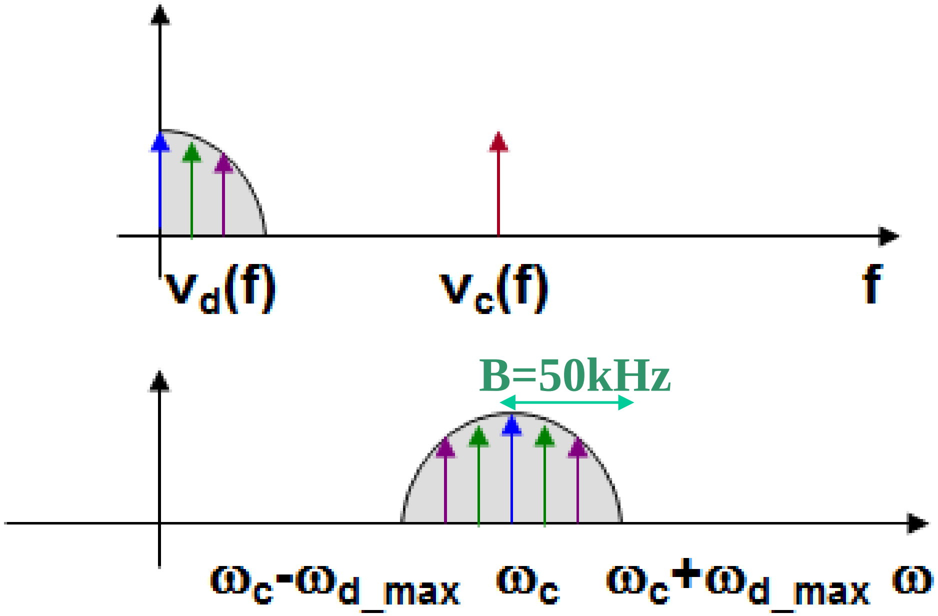
How can we recover the factor 2 in supported data-rate !?

Modulation of Digital Data: PSK (cont.)

Example **[data rate of ASK signal]**

We have an available bandwidth of 100 kHz which spans from 200 – 300 kHz. What are the carrier frequency and the bit rate if we modulated our data by using ASK?





carrier signal = middle of the bandwidth = 250 kHz

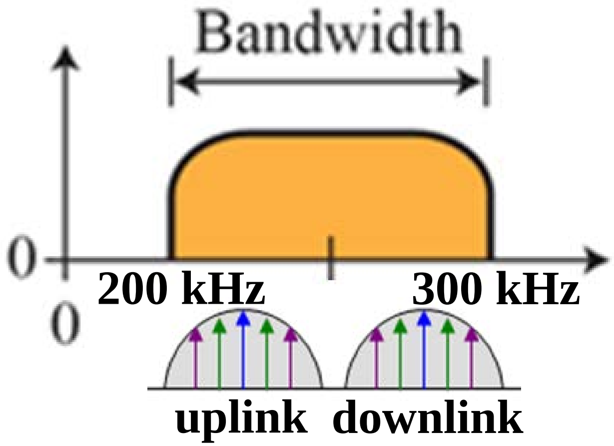
bandwidth of digital signal B = 50 kHz

digital data rate = 2* B = 100 kbps

Modulation of Digital Data: PSK (cont.)

Example [data rate of ASK signal]

Assumptions are the same as in the previous example (200-300 kHz Available, for ASK modulated signal), but the line is full-duplex.



bandwidth for uplink = bandwidth for downlink = 50 kHz

carrier for uplink = 225 kHz carrier for downlink = 275 kHz

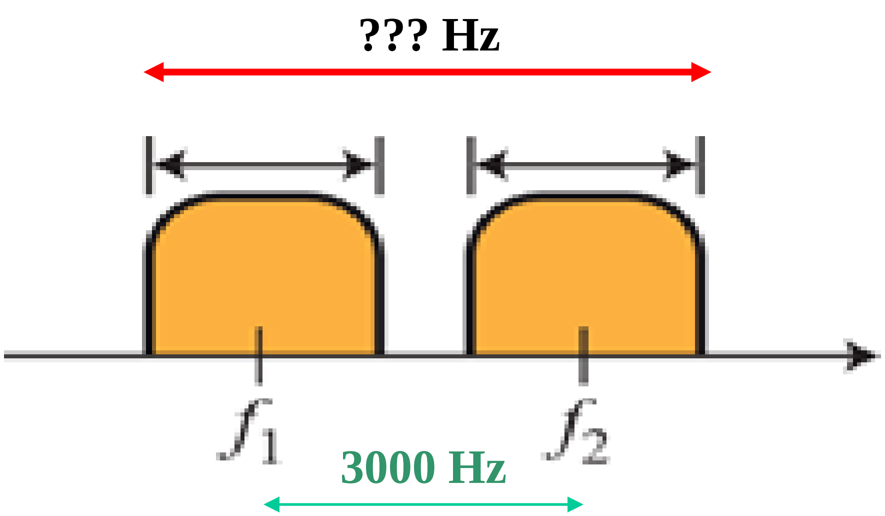
bandwidth of digital signal = 25 kHz

digital data rate = 2* B = 50 kbps

Modulation of Digital Data: PSK (cont.)

Example [data rate of FSK signal]

Find the minimum bandwidth for FSK signal transmitting at 2000 bps using half-duplex mode, and the carriers are separated by 3000 Hz?



overall bandwidth = band. dig. sig. + 3000 + band dig. sig.

band. dig. sig. = C / 2 = 1000 Hz

overall bandwidth = 5000 Hz

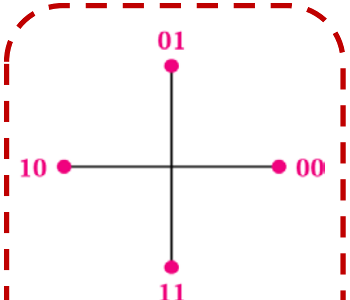
Modulation of Digital Data: PSK (cont.)

QPSK = 4-PSK – **PSK that uses phase shifts of 90º=π/2 rad ⇒ 4 different signals are generated, each representing 2 bits**

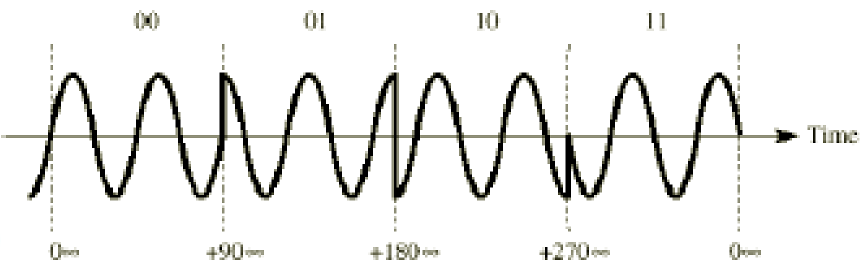
$$s(t)=\begin{cases} A\cos(2\pi f_c t), & \text{binary } 00 \\ A\cos(2\pi f_c t+\frac{\pi}{2}), & \text{binary } 01 \\ A\cos(2\pi f_c t+\pi), & \text{binary } 10 \\ A\cos(2\pi f_c t+\frac{3\pi}{2}), & \text{binary } 11 \end{cases}$$

Dibit	Phase
00	0
01	90
10	180
11	270

Dibit (2 bits)



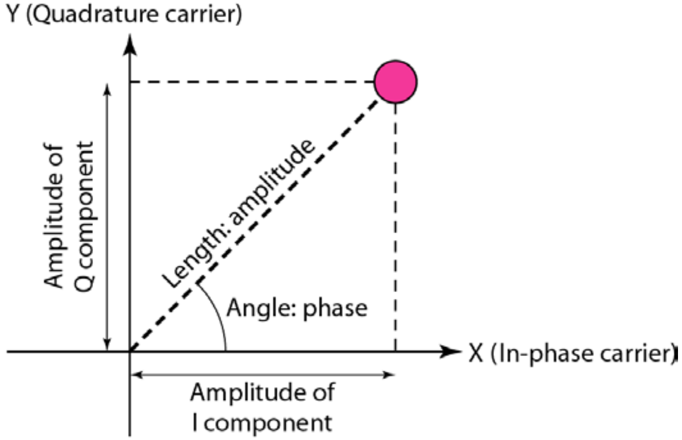
Constellation diagram



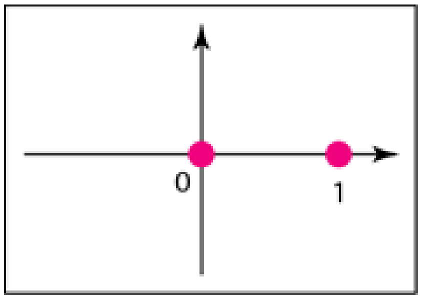
- **advantage**: **higher data rate than in PSK (2 bits per bit interval), while bandwidth occupancy remains the same**
- **4-PSK can easily be extended to 8-PSK, i.e. n-PSK**
- **however, higher rate PSK schemes are limited by the ability of equipment to distinguish small differences in phase**

Signal Constellation

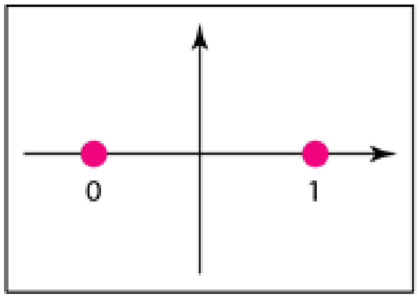
Constellation Diagram – used to represents possible symbols that may be selected by a given modulation scheme as <u>points</u> in 2-D plane



- **X-axis is related to in-phase carrier:** $\cos(\omega_c t)$
  - **the projection of the point on the X-axis defines the peak amplitude of the in-phase component**
- **Y-axis is related to the quadrature carrier:** $\sin(\omega_c t)$
  - **the projection of the point on the Y-axis defines the peak amplitude of the quadrature component**
- **the length of the line that connects the point to the origin is the peak amplitude of the signal element (combination of X and Y components)**
- **the angle the line makes with the X-axis is the phase of the signal element**



a. ASK (OOK)



b. BPSK

Signal Constellation (cont.)

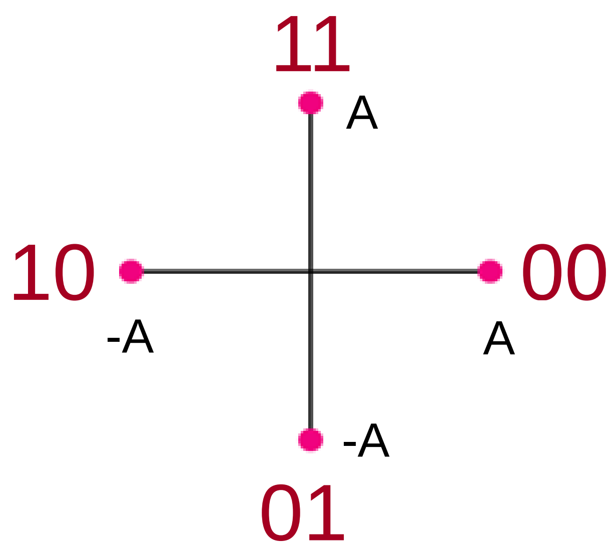
Example **[signal constellation of QPSK]**

$$
s(t)=\begin{cases}
A\cos(2\pi f_c t), & \text{binary } 00 \\
A\cos(2\pi f_c t+\frac{\pi}{2}), & \text{binary } 01 \\
A\cos(2\pi f_c t+\pi), & \text{binary } 10 \\
A\cos(2\pi f_c t+\frac{3\pi}{2}), & \text{binary } 11
\end{cases}
$$

$A\cos(2\pi f_c t+\frac{\pi}{2}) \rightarrow -A\sin(2\pi f_c t)$

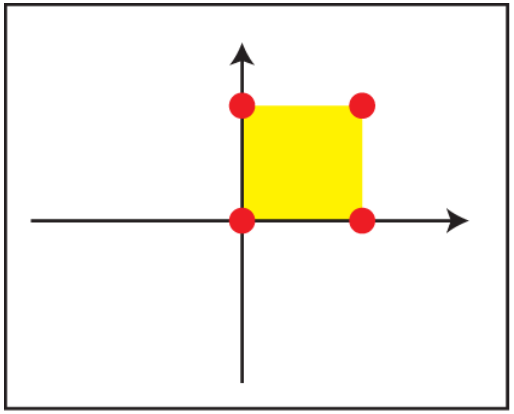
$A\cos(2\pi f_c t+\pi) \rightarrow -A\cos(2\pi f_c t)$

$A\cos(2\pi f_c t+\frac{3\pi}{2}) \rightarrow A\sin(2\pi f_c t)$

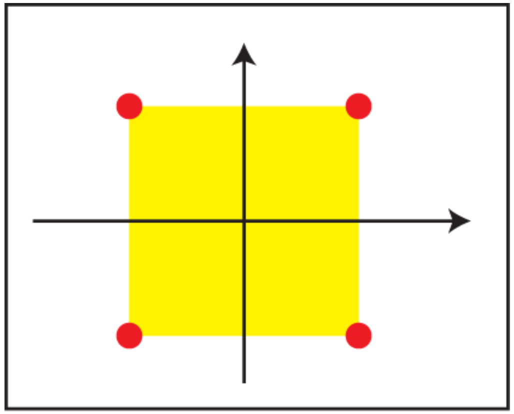


Signal Constellation (cont.)

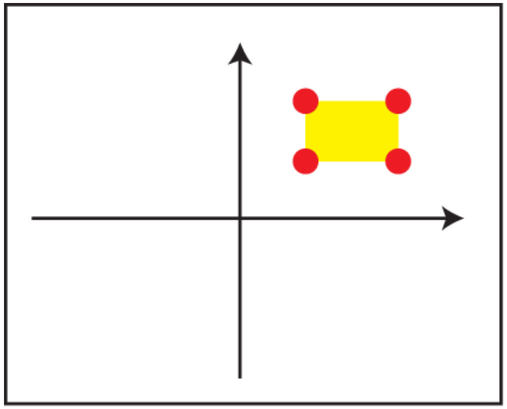
Example [other types of modulations]



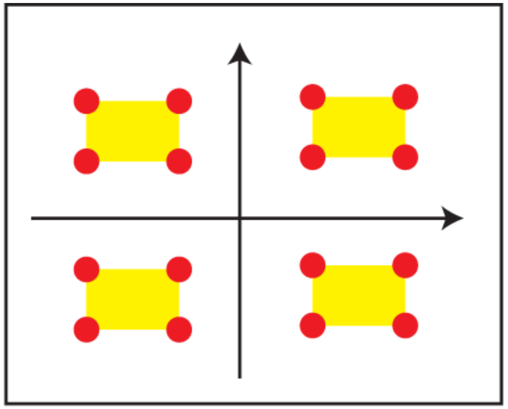
a. 4-QAM



b. 4-QAM



c. 4-QAM



d. 16-QAM