Sampling (cont.)

Example [sampling of bandpass signal]

A complex <u>bandpass</u> signal has a bandwidth of 200 kHz.
What is the minimum sampling rate for this signal?

We cannot find the minimum sampling rate in this case, as we do not know where the bandwidth starts (or ends).

Example [sampling of low-pass signal]

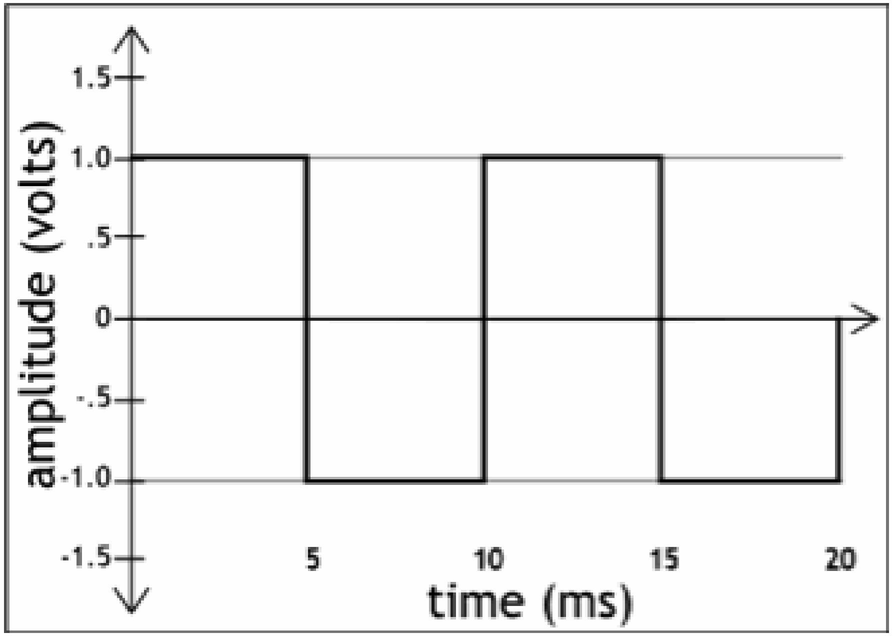
A complex <u>low-pass</u> signal has a bandwidth of 200 kHz.
What is the minimum sampling rate for this signal?

Since this is a low-pass signal, it frequency characteristics starts at 0 Hz.
Hence, the highest frequency in the spectrum is 200 kHz.
Thus, the minimum sampling rate is 400 kHz.

Sampling (cont.)

Example **[sampling of square-wave]**

What is the minimum sampling rate for the below signal?



Frequency spectrum of a square-wave is infinite – signal cannot be sampled !!!

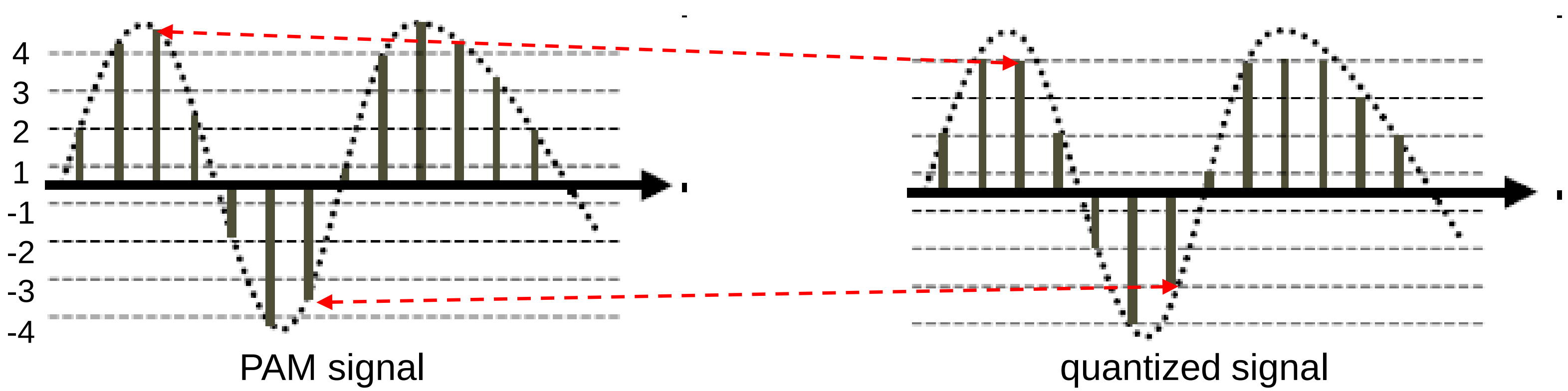
Quantization

Quantization

- **PAM signal samples have amplitudes of '∞ precision" – direct encoding of such amplitudes would require ∞ number of bits (digital pulses) per sample**
- **to convert PAM signal to digital signal (that is practical for transmission), each sample has to be 'rounded up' to the nearest of M possible quantization levels**

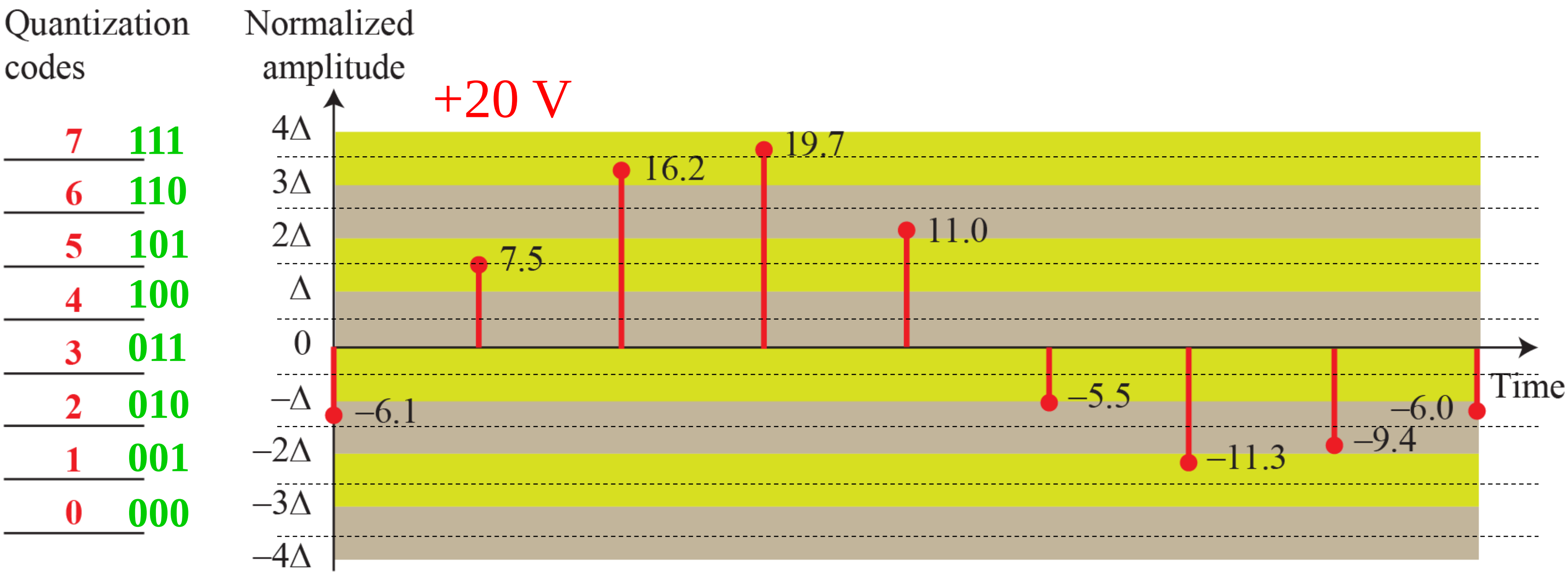
M quantization levels ⇔ $m=\log_2(M)$ bits per level

- **M↑ ⇒ better precision ☺, more bits per sample ☹**
- **M↓ ⇒ poor precision ☹, fewer bits per sample ☺**



Quantization (cont.)

Example [Quantization of PAM Signal using 3 bits per sample]

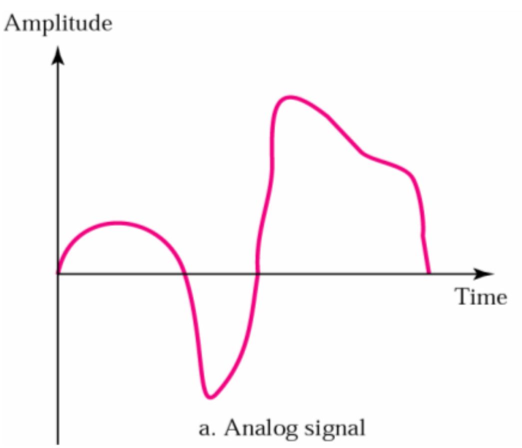


Normalized PAM values	–1.22	1.50	3.24	3.94	2.20	–1.10	–2.26	–1.88	–1.20
Normalized quantized values	–1.50	1.50	3.50	3.50	2.50	–1.50	–2.50	–1.50	–1.50
Normalized error	–0.28	0	+0.26	–0.44	+0.30	–0.40	–0.24	+0.38	–0.30
Quantization code	2	5	7	7	6	2	1	2	2
Encoded words	010	101	111	111	110	010	001	010	010

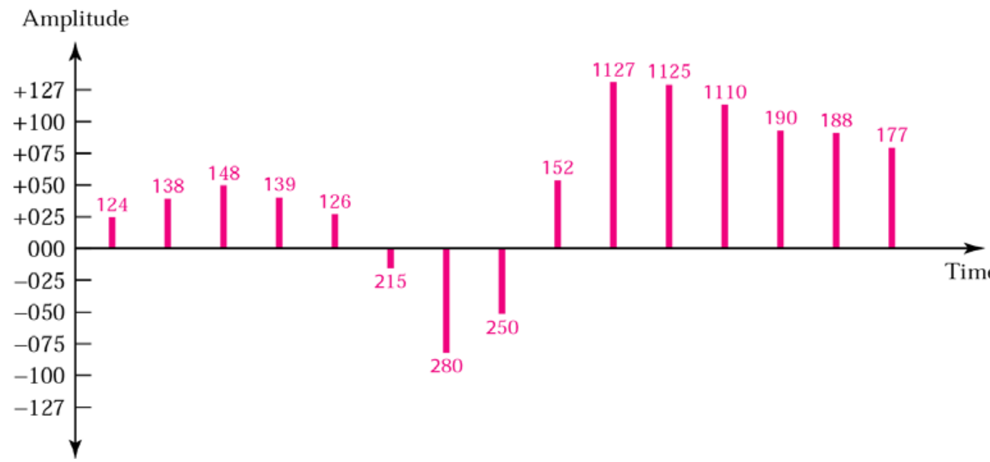
Quantization (cont.)

Example [Quantization of PAM Signal]

Assume an analog signal, as shown below, has to be quantized using at most 8-bits per sample.
How many different quantization levels are allowed / should be used?



a. Analog signal

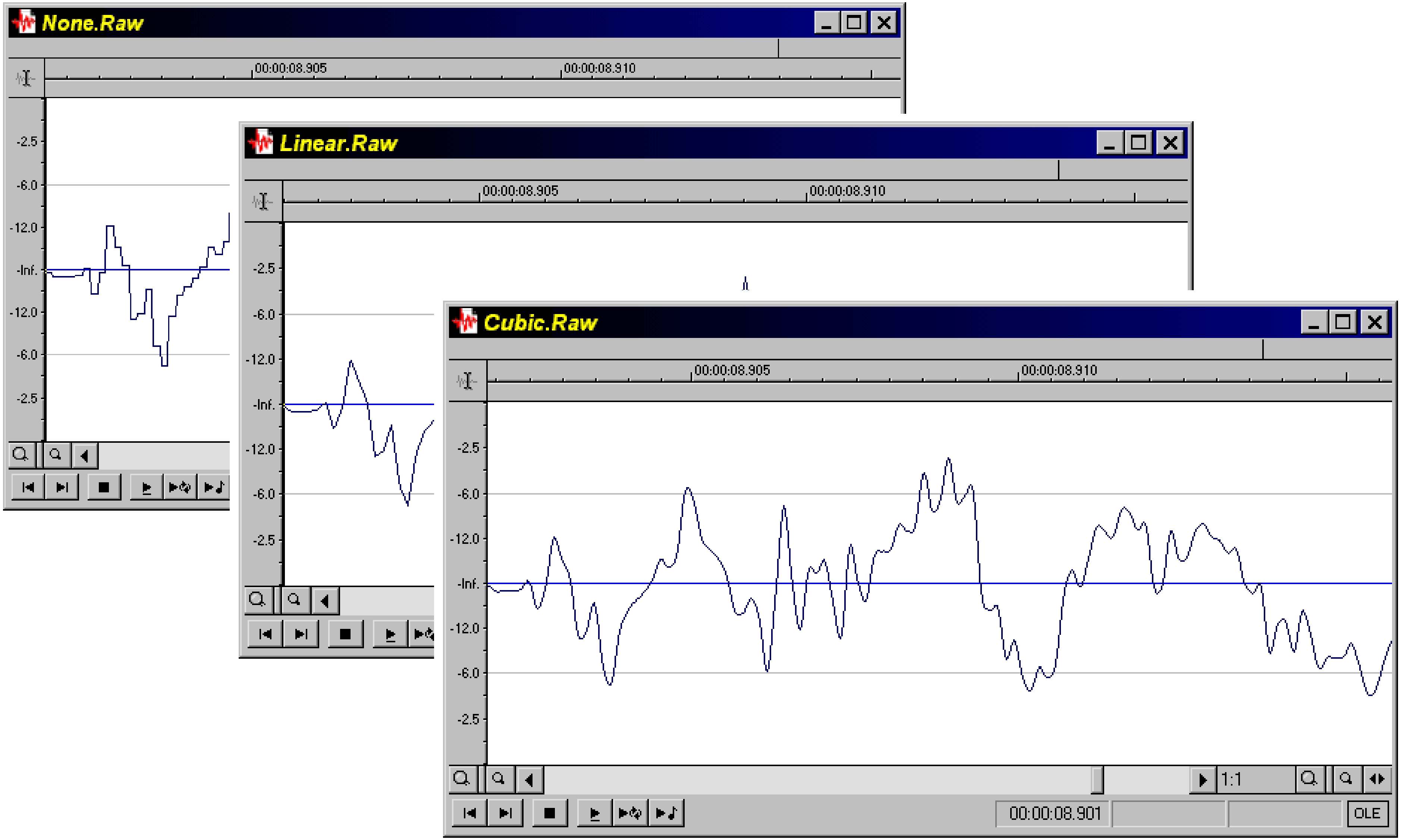


+024	00011000	–015	10001111	+125	01111101
+038	00100110	–080	11010000	+110	01101110
+048	00110000	–050	10110010	+090	01011010
+039	00100111	+052	00110110	+088	01011000
+026	00011010	+127	01111111	+077	01001101

Sign bit
+ is 0 – is 1

Quantization (cont.)

Example [Reconstruction from PAM signal]



Quantization (cont.)

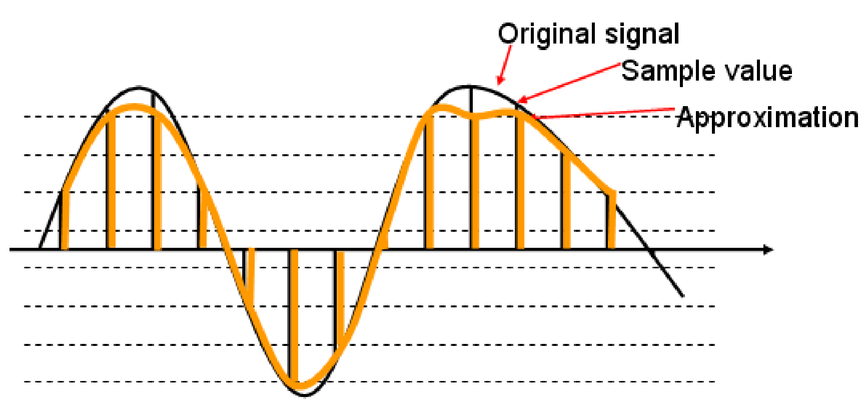
Quantization Error – **by quantizing the PAM signal, the original signal is now only approximated and cannot be recovered exactly**

- **this effect is known as quantizing error or quantizing noise**
- **signal-to-noise ratio due to quantizing noise can be expressed as**

$$\text{SNR [dB]} \approx 6m + 1.76 \text{ [dB]}$$

bits per sample

- **every additional bit used in quantizer will increase the SNR by 6 [dB]**
  - **# of quantization levels ↑ ⇒ higher SNR ⇒ better (received) signal quality**



signal interpolation

Quantization (cont.)

Example [voice signal in telephone system]

Natural human voice occupies the range of 80 – 3,400 [Hz].
Human ear can tolerate SNR of 40 [dB].

Assume we want to transmit human voice in digitized form.
What bit rate [bps] should be supported by the channel to enable such transmission?

---

(1) Sampling rate?!

Based on Nyquist Sampling Theorem:

max frequency = 4 [kHz] ⇒ sampling rate = 2*4 [kHz] = 8000 [samples/sec]

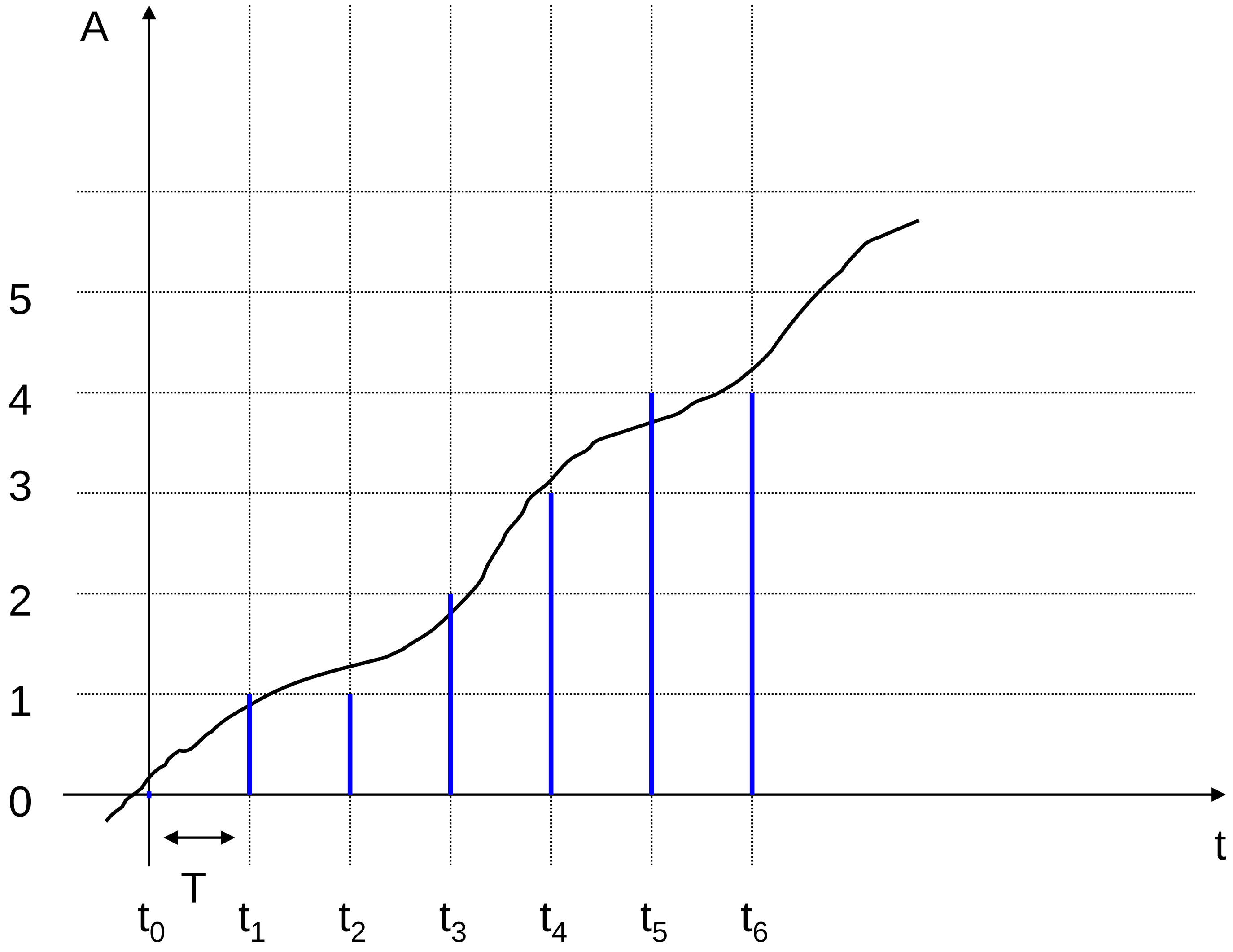
(2) # of bits per sample?!

Based on SNR formula:

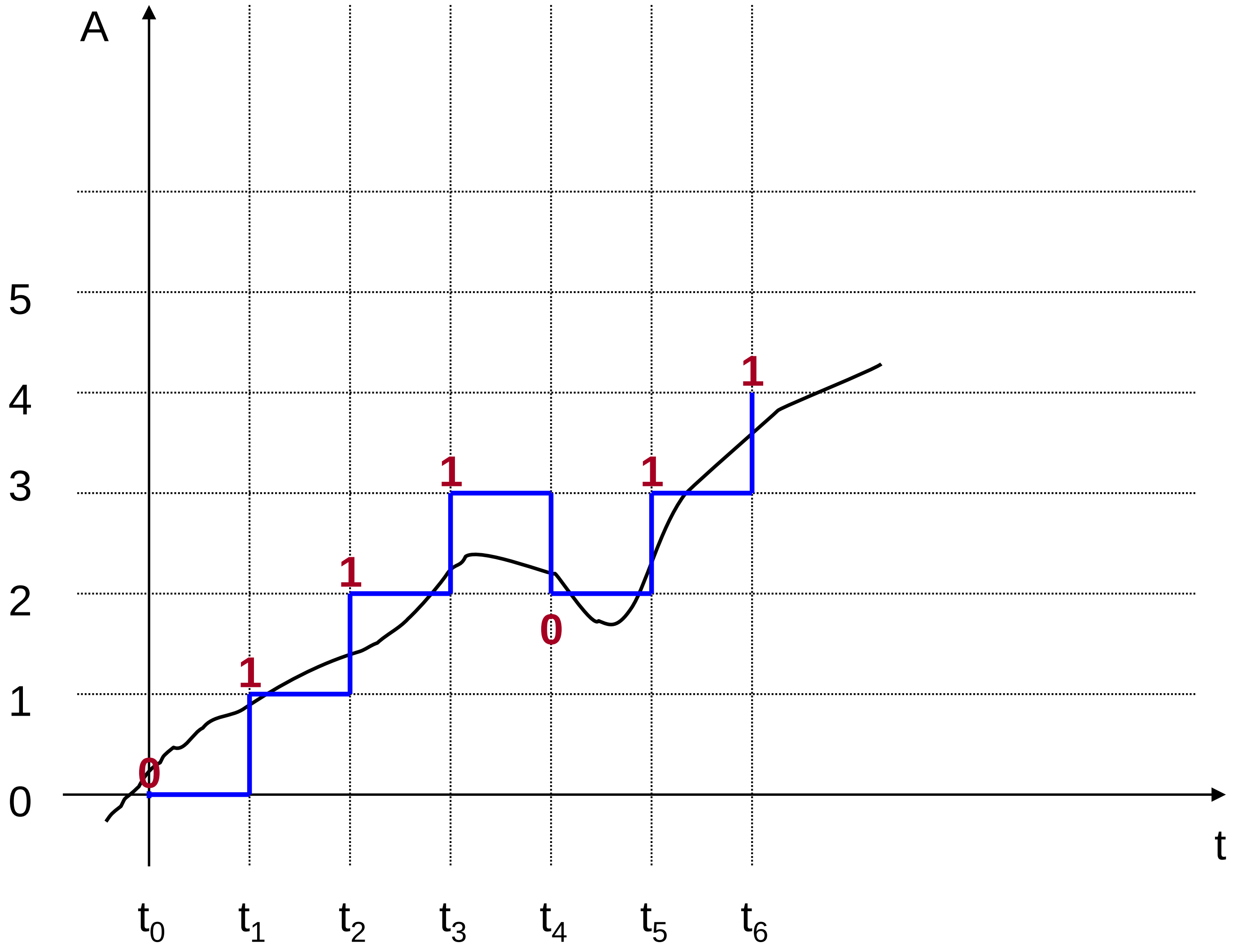
40 [dB] = 6*m + 1.76 ⇒ # bits per sample = 7 ⇒ # of levels = 2^7 = 127

data rate = # samples per second * # bits per sample = 56 kbps

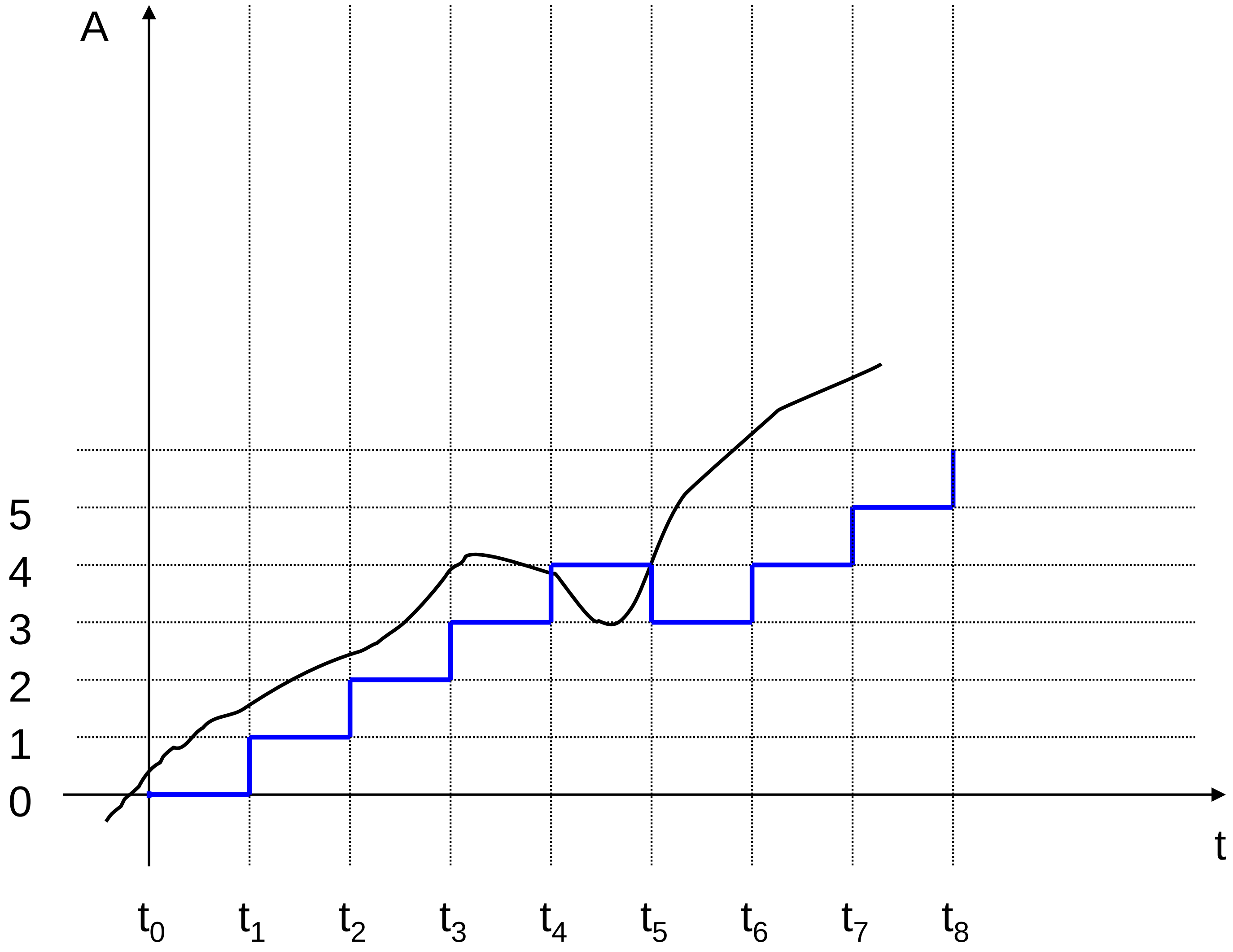
Example [PCM]



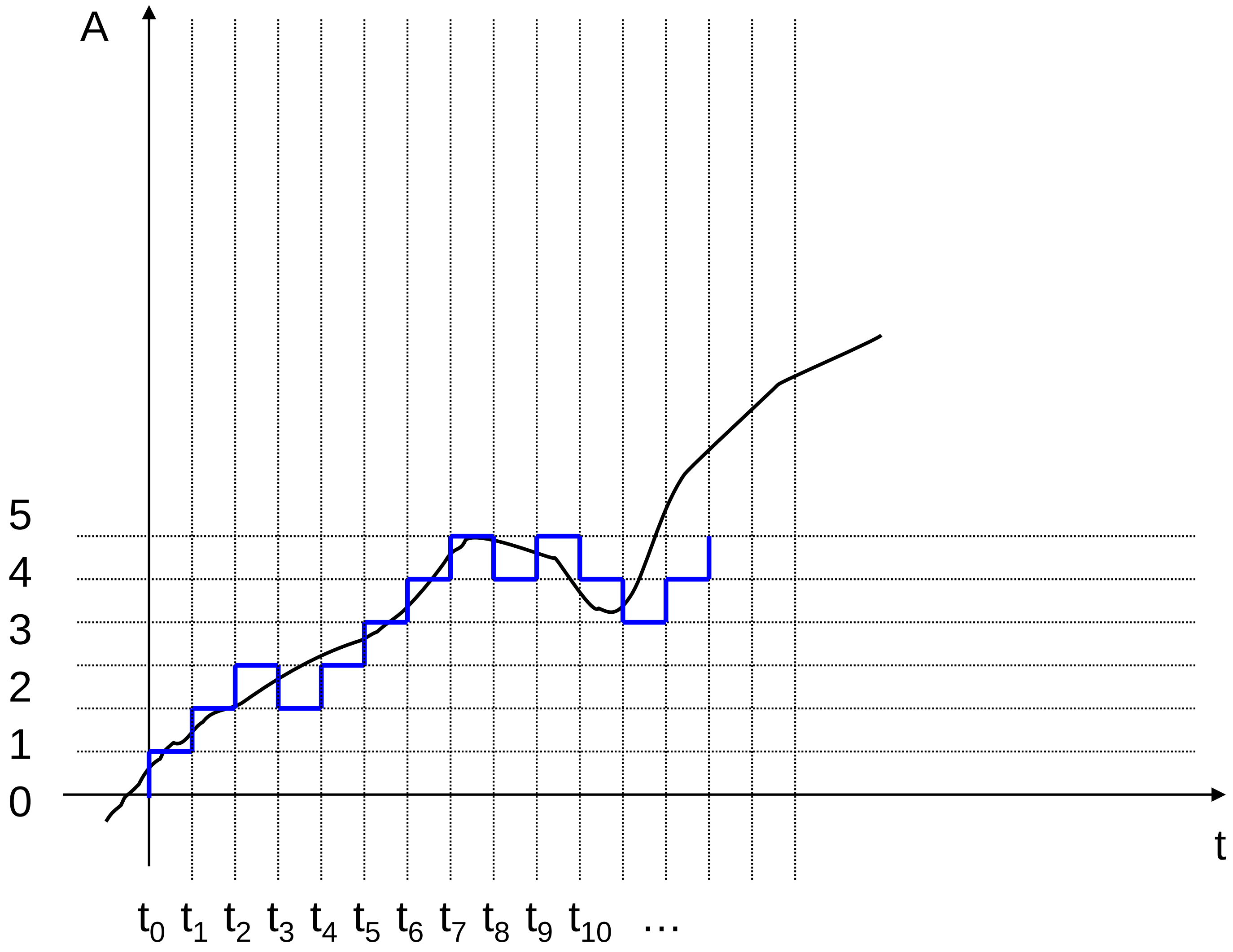
Example [Delta Modulation]

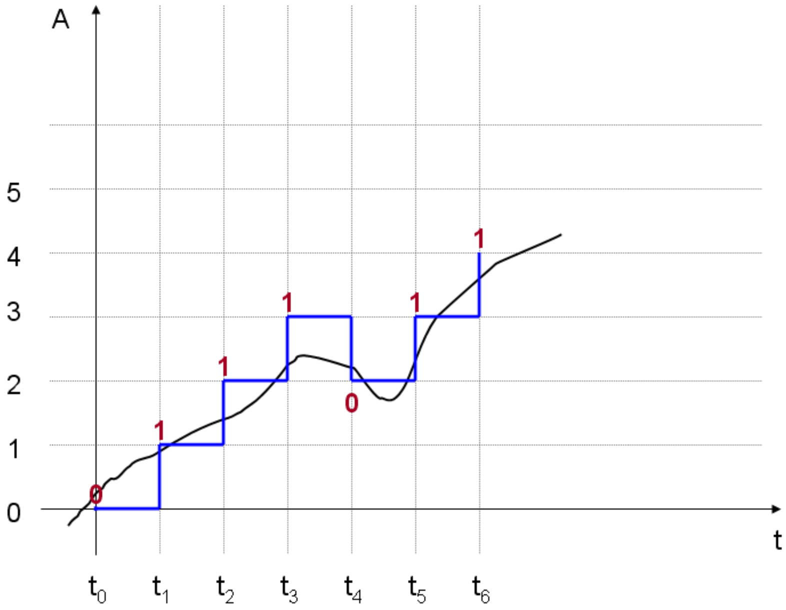


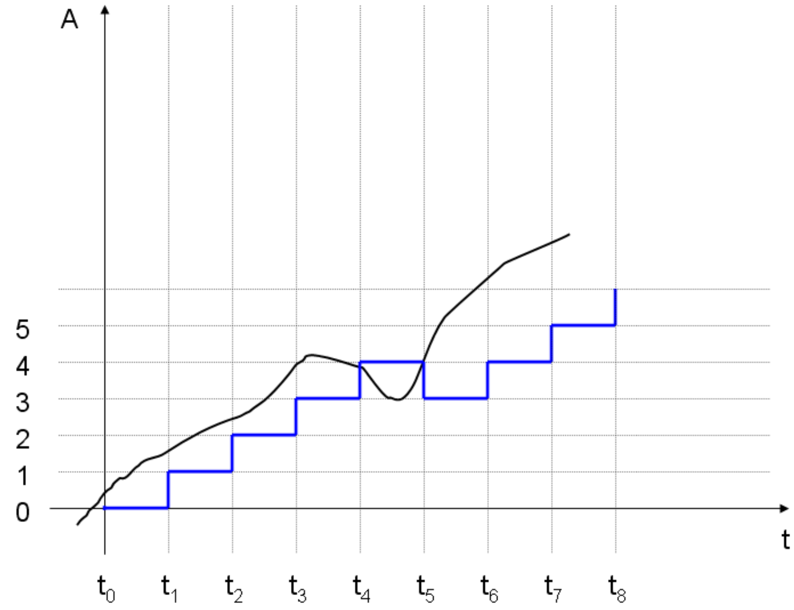
Example [Delta Modulation: δ step reduced 50%, T remains the same]

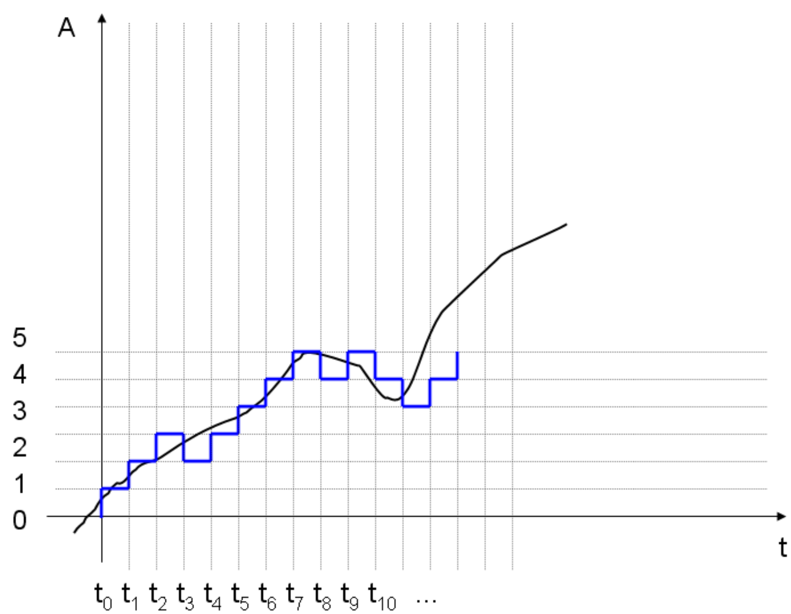


Example [Delta Modulation: both δ-step and T reduced 50%]




A
5
4
3
2
1
0
0
1
1
1
0
1
1
t
t0 t1 t2 t3 t4 t5 t6

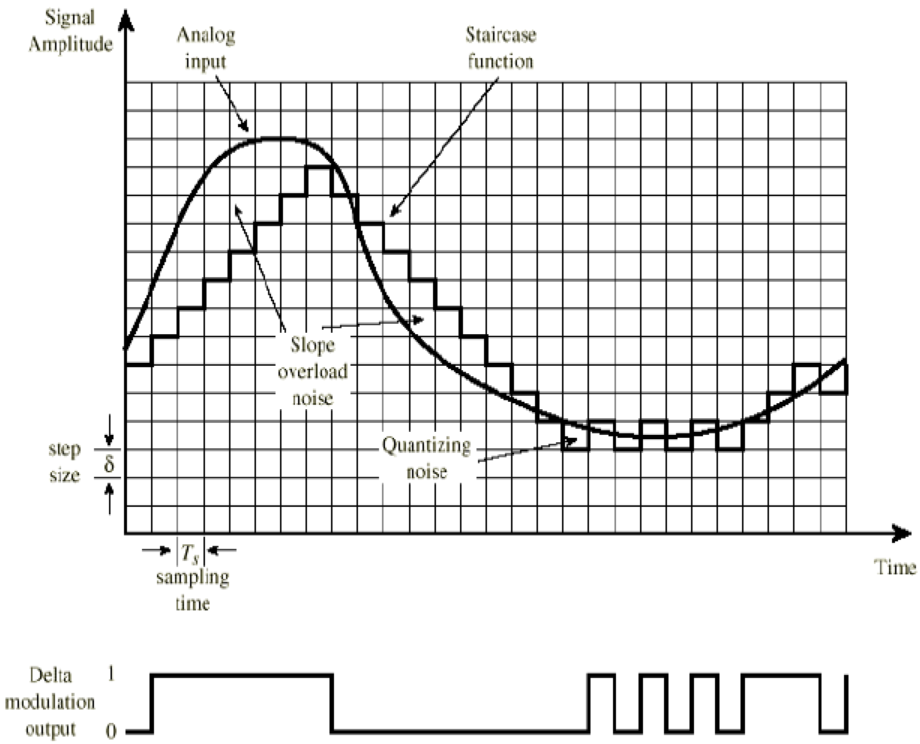

A
5
4
3
2
1
0
t
t0 t1 t2 t3 t4 t5 t6 t7 t8


A
5
4
3
2
1
0
t
t0 t1 t2 t3 t4 t5 t6 t7 t8 t9 t10 …

Delta Modulation

Delta-Modulation – **most popular alternative to PCM**

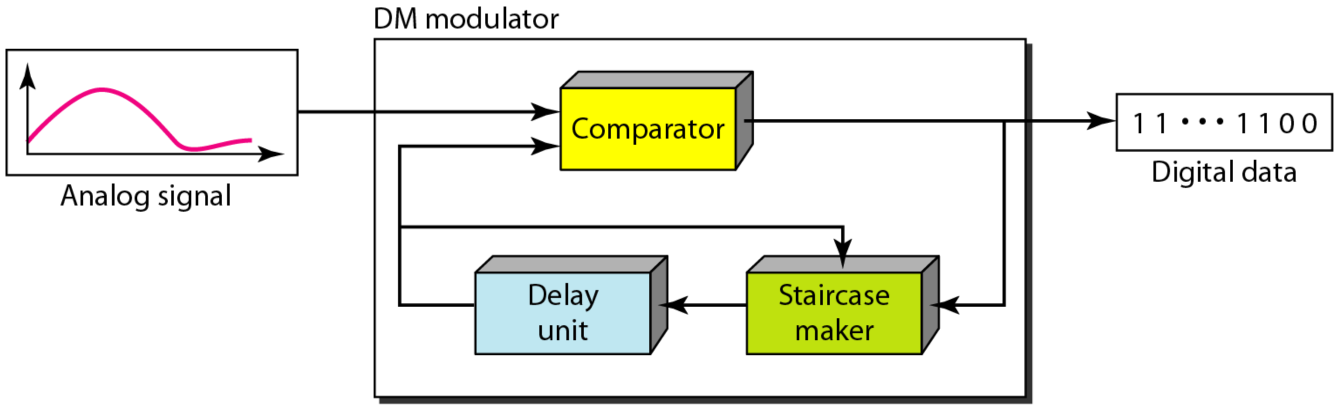
- **analog signal is approximated by staircase function**
- **only a single binary digit is required for each sample !!!**

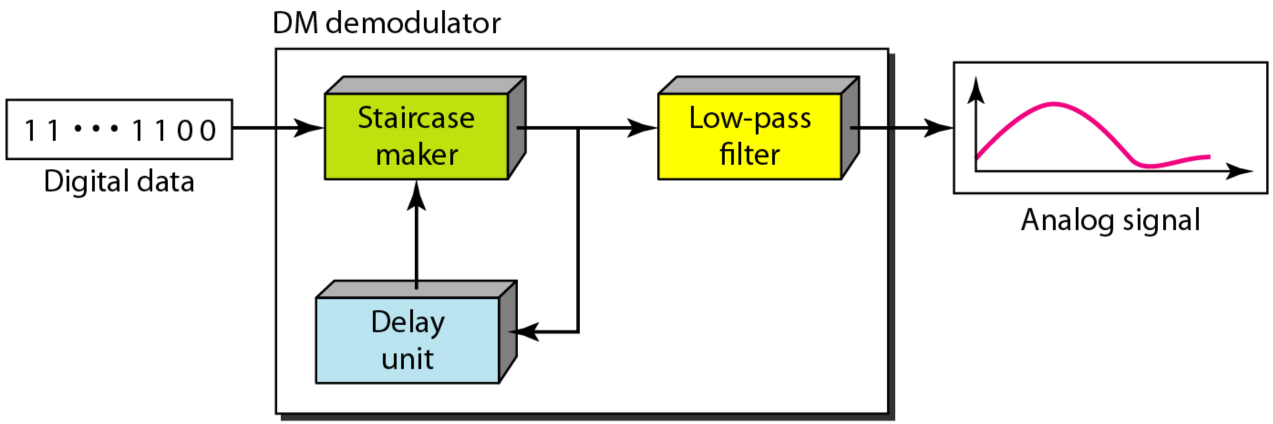


- **at each sampling time (kT), the function moves up or down a constant amount δ (step size) – the staircase function attempts to track the original waveform as closely as possible**
- **at each sampling time, the analog input is compared to the most recent value of the approximating staircase function**
- **binary-1 is generated if the function goes up, binary-0 otherwise**

Delta Modulation (cont.)

Example [Delta modulation / demodulation]





Delta Modulation (cont.)

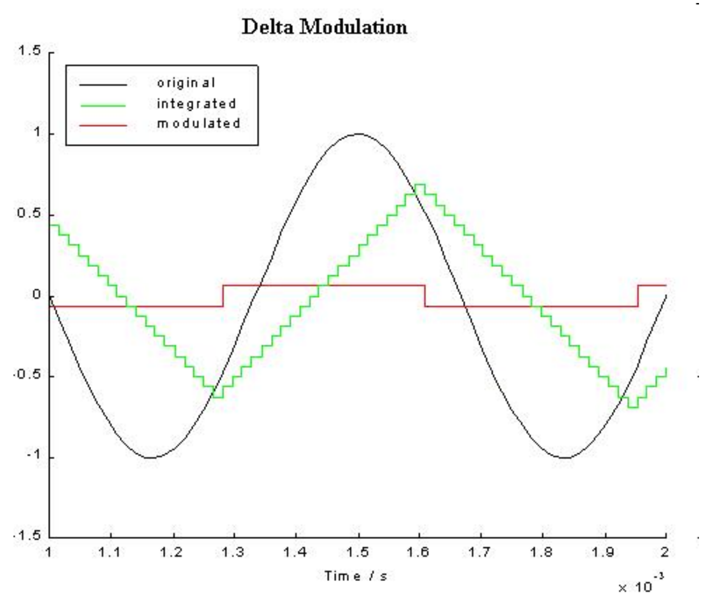
Delta-Modulation Parameters

(1) step size (δ) – should not be too small, nor too large

- **small δ + signal changes rapidly ⇒ underestimation**
- **large δ + signal changes slowly ⇒ overestimation**

(2) sampling time (T)

- **smaller T increase overall accuracy**
- **but, small T increases output data rate, i.e. # of bps**



Delta-modulation rule: smaller δ ⇒ smaller T, larger δ ⇒ larger T .