Data Rate Limits in Digital Transmission

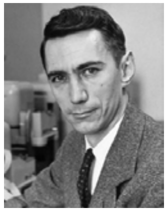
Max Data Rate [bps] Over a Channel? – depends on three factors:

- bandwidth available
- # of levels in digital signal
- quality of channel – level of noise

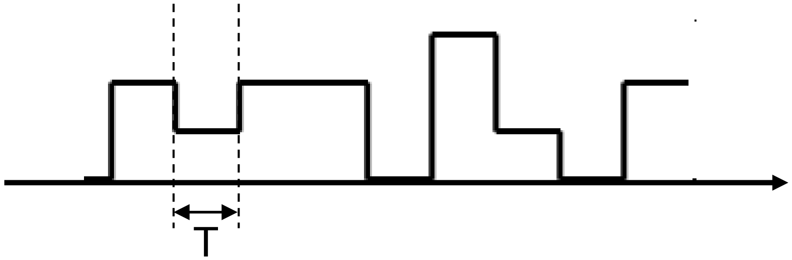
Nyquist Theorem – defines theoretical max bit rate in noiseless channel [1924]

- even perfect (noiseless) channels have limited capacity

Shannon Theorem – Nyquist Theorem extended - defines theoretical max bit rate in noisy channel [1949]



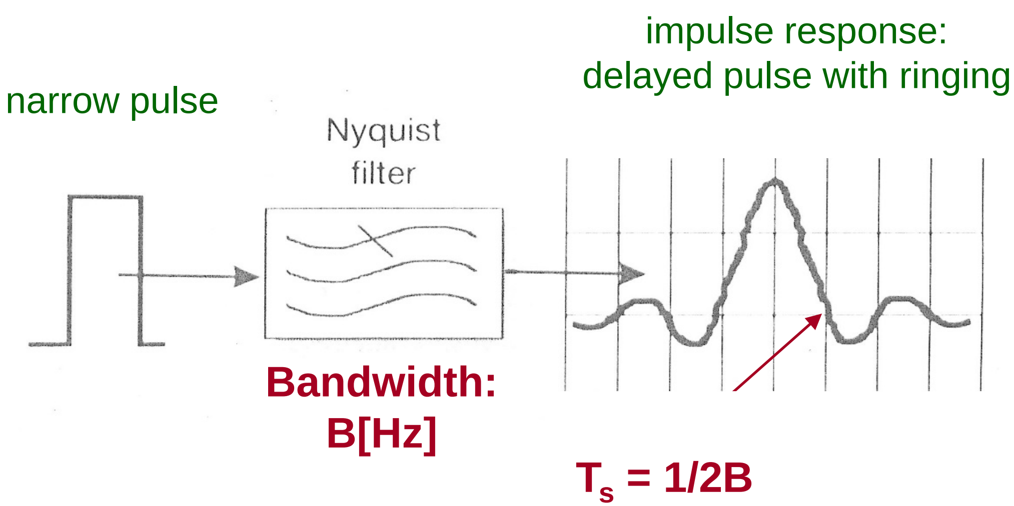
- if random noise is present, situation deteriorates rapidly!



Data Rate Limits: Nyquist Theorem

Intersymbol Interference – **the inevitable filtering effect of any practical channel will cause spreading of individual data symbols that pass through the channel**

- **this spreading causes part of symbol energy to overlap with neighbouring symbols causing intersymbol interference (ISI)**
- **ISI can significantly degrade the ability of the data detector to differentiate a current symbol from the diffused energy of the adjacent symbols**



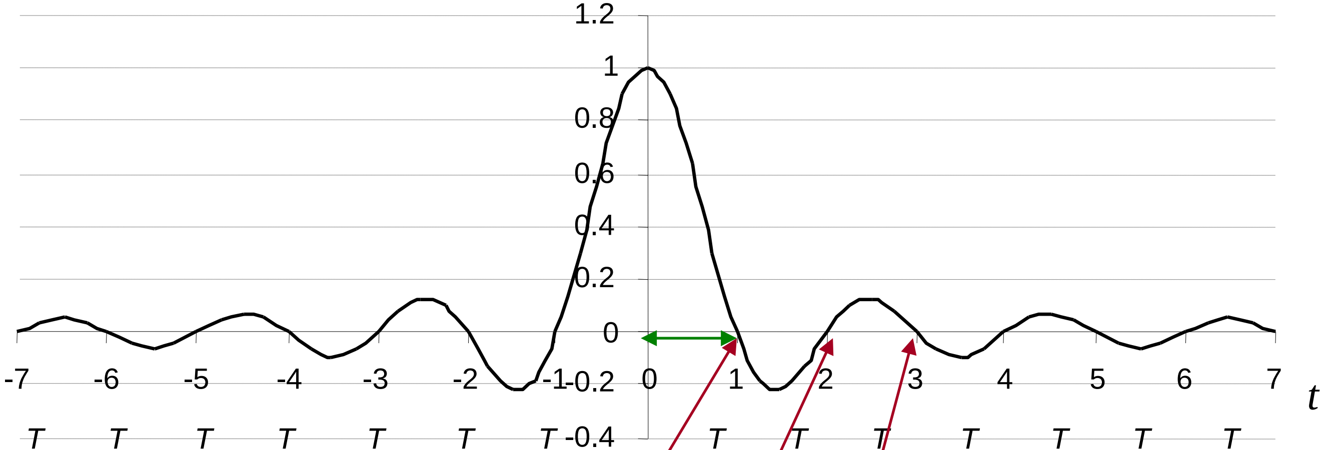
As the channel bandwidth B increases, the width of the impulse response decreases
⇒ pulses can be input in the system more closely spaced, i.e. at a higher rate.

Data Rate Limits: Nyquist Theorem

Impulse Response – response of a low-pass channel (of bandwidth B) to a narrow pulse h(t), aka Nyquist pulse:

$$s(t) = \frac{\sin(2\pi Bt)}{2\pi Bt}$$

- **zeros: where sin(2πBt)=0** $\Rightarrow \; t = k \cdot \frac{1}{2B}, \quad k = 1,2,3,\ldots$

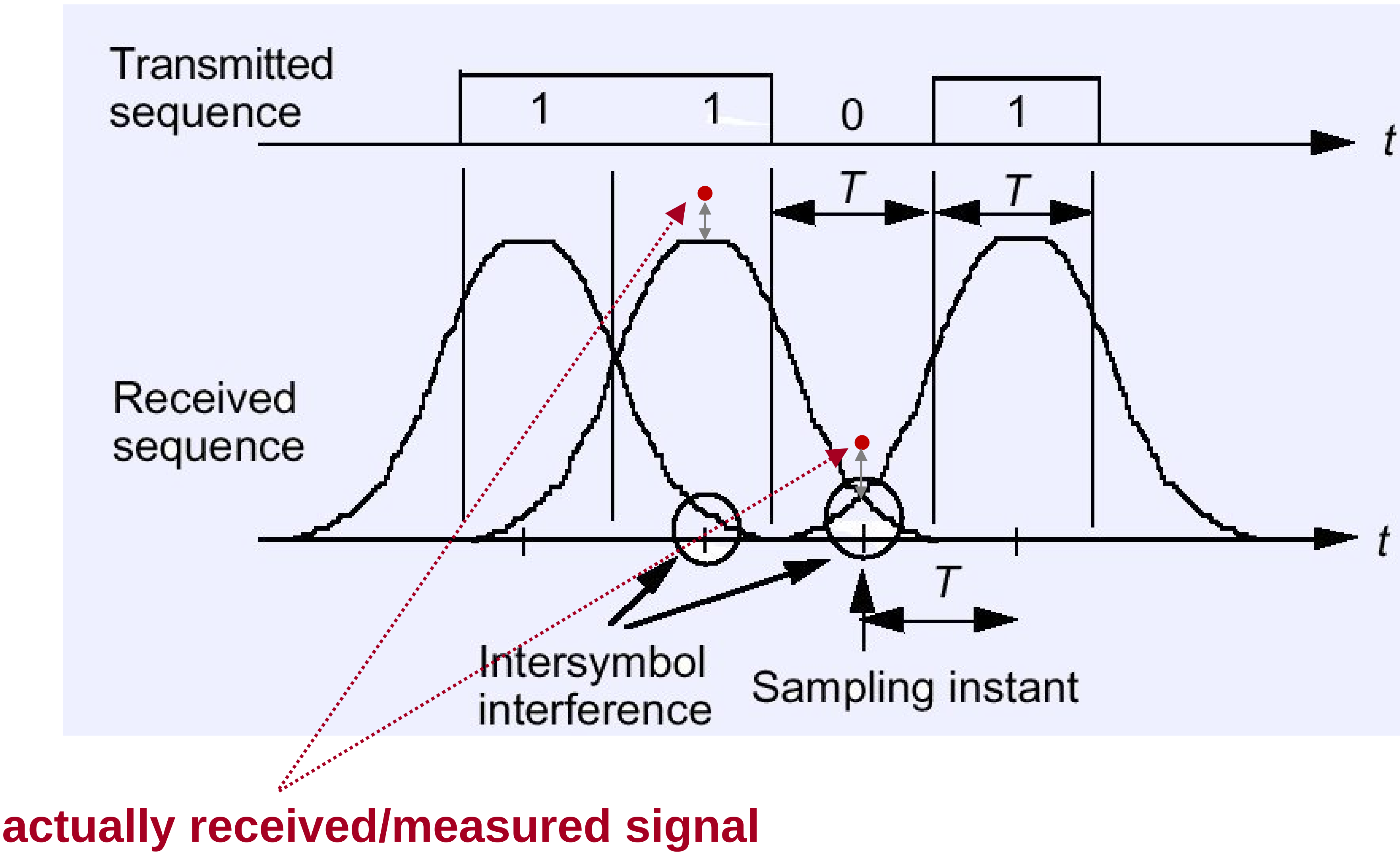


$$T_S = \frac{1}{2B} \qquad \frac{2}{2B} \qquad \frac{3}{2B}$$

What is the minimum pulse/bit duration time to avoid significant ISI?!

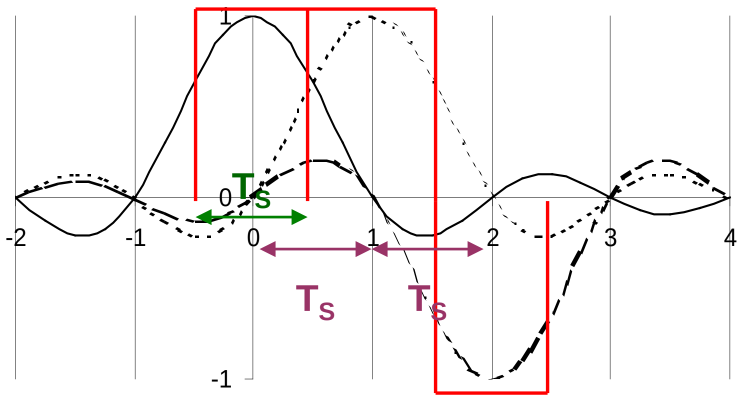
Data Rate Limits: Nyquist Theorem

Example **[problems associated with intersymbol interference]**

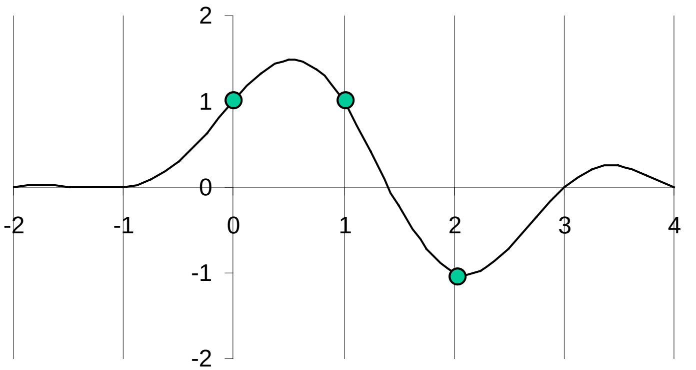


Data Rate Limits: Nyquist Theorem

Example [system response to binary input 110]



three separate pulses



combined signal

Assume: channel bandwidth = max analog frequency passed = B [Hz].

New pulse is sent every T_S sec $\Rightarrow$ data rate = $1/T_S$ [bps] = 2B [bps]

The combined signal has the correct values at t = 0, 1, 2.

$$r_{max} = \frac{1 \text{ pulse}}{T_S \text{ second}} = 2W = 2B\left[\frac{\text{pulses}}{\text{second}}\right]$$

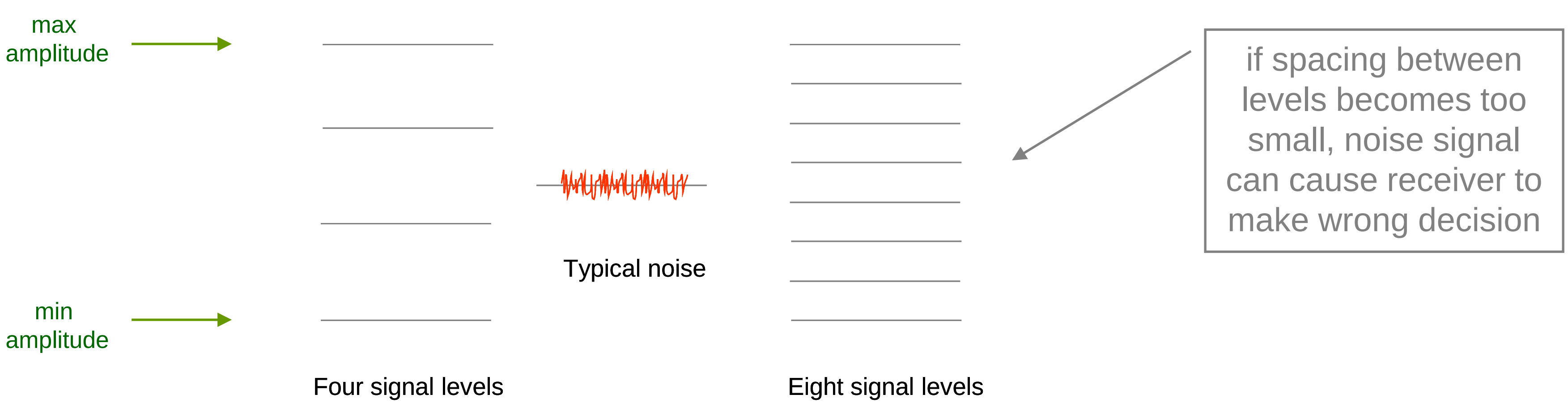
Maximum <u>signaling rate</u> that is achievable through an ideal low-pass channel.

Data Rate Limits: Nyquist Theorem

Nyquist Law – **max rate at which digital data can be transmitted over a communication channel of bandwidth B [Hz] is**

$$C_{noiseless} = 2 \cdot B \cdot \log_2 M \ \ [bps]$$

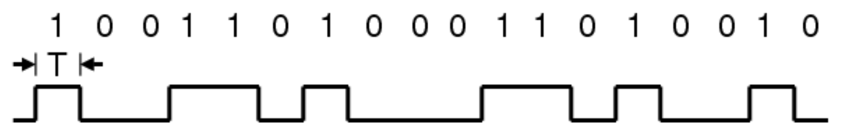
- **M – number of discrete levels in digital signal**
- **$M\uparrow \Rightarrow C\uparrow$, however this places increased burden on receiver – instead of distinguishing one of two possible signals, now it must distinguish between M possible signals**
  - **especially complex in the presence of noise**



Data Rate Limits: Nyquist Theorem

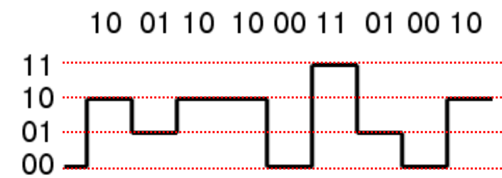
Example [multilevel digital transmission]

2-level encoding: C=2B [bps]
one pulse – one bit



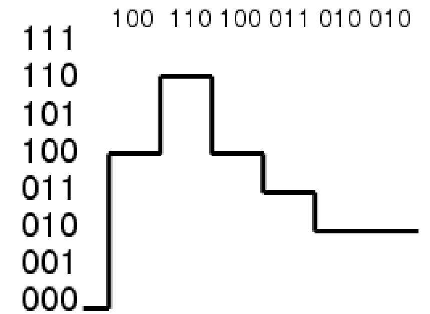
4-level encoding: C=2*2=4B [bps]
one pulse – two bits

1001101000110100 10 ⇒



8-level encoding: C=2*3=6B [bps]
one pulse – three bits

1001101000110100 10 ⇒



Data Rate Limits: Shannon Theorem

Shannon Law – **maximum transmission rate over a channel with bandwidth B, with Gaussian distributed noise, and with signal-to-noise ratio SNR=S/N, is**

$$C_{noisy} = B \cdot \log_2(1 + SNR) \ \ [bps]$$

- **theoretical limit – there are numerous impairments in every real channel besides those taken into account in Shannon's Law (e.g. attenuation, delay distortion, or impulse noise)**
- **no indication of levels – no matter how many levels we use, we cannot achieve a data rate higher than the capacity of the channel**
- **in practice we need to use both methods (Nyquist & Shannon) to find what data rate and signal levels are appropriate for each particular channel:**

The Shannon capacity gives us the upper limit!
The Nyquist formula tells us how many levels we need!

Data Rate Limits

Example [data rate over telephone line]

What is the theoretical highest bit rate of a regular telephone line?
A telephone line normally has a bandwidth of 3000 Hz (300 Hz to 3300 Hz). The signal-to-noise ratio is usually 35 dB (3162) on up-link channel (user-to-network).

Solution:

We can calculate the theoretical highest bit rate of a regular telephone line as

$$C = B \log_2 (1 + SNR) =$$
$$= 3000 \log_2 (1 + 3162) =$$
$$= 3000 \log_2 (3163)$$

$$C = 3000 \times 11.62 = 34{,}860 \text{ bps}$$

Data Rate Limits

Example [data rate / number of levels]

We have a channel with a 1 MHz bandwidth. The SNR for this channel is 63; what is the appropriate bit rate and number of signal level?

<u>Solution:</u>

First use Shannon formula to find the upper limit on the channel's data-rate

$$C = B \log_2 (1 + SNR) = 10^6 \log_2 (1 + 63) = 10^6 \log_2 (64) = 6 \text{ Mbps}$$

Although the Shannon formula gives us 6 Mbps, this is the upper limit. For better performance choose something lower, e.g. 4 Mbps.

Then use the Nyquist formula to find the number of signal levels.

$$C = 2 \cdot B \cdot \log_2 M \ \ [\text{bps}]$$

$$4 \text{ Mbps} = 2 \times 1 \text{ MHz} \times \log_2 L \ \Rightarrow \ L = 4$$