This problem set will not be graded, but will help you consolidate the material from the second half of the course and prepare for the final exam. You are free to work together on these if you prefer. I will post the solutions by Dec 6.

1. Please show the hash table that results from adding the keys below to an initially empty map using hash function \( h(k) = k \mod 11 \) and linear probing.
   Keys to add (in the order given): 98, 31, 39, 21, 33, 91, 90, 34

2. Binary Search Trees
   Insert, into an empty binary search tree, entries with keys 30, 40, 24, 58, 48, 26, 11, 13 (in this order).
   Draw the tree after each insertion.

3. AVL Trees
   Insert, into an empty binary search tree, entries with keys 62, 44, 78, 17, 50, 88, 48, 54 (in this order).
   Now draw the AVL tree resulting from the removal of the entry with key 62.

4. Splay Trees
   Perform the following sequence of operations in an initially empty splay tree and draw the tree after each set of operations.
   (a) Insert keys 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, in this order.
   (b) Search for keys 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, in this order.
   (c) Delete keys 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, in this order.

5. Comparison Sorts
   Of the \( n! \) possible inputs to a given comparison-based sorting algorithm, what is the absolute maximum number of inputs that could be sorted with just \( n \) comparisons?

6. Comparison Sorts
   Give an example input list for which merge-sort and heap-sort take \( O(n \log n) \) time, but for which insertion sort takes \( O(n) \) time. What if the list is reversed?

7. Stack-Based Quicksort
   Describe in pseudocode a non-recursive version of the quick-sort algorithm that explicitly uses a stack.

8. Linear Sorts
   Given an array of \( n \) integers, each in the range \([0, n^2 - 1]\), describe a simple method for sorting the array in \( O(n) \) time.

9. Topological Sorts
   Suppose that you wish to take a sequence of language courses with the following prerequisites:
Graphs

Let’s assume that vertices and edges are processed in the order indicated by the table above.

Then the first call to Topological_Visit will be from LA15, and will produce the list {LA15, LA31, LA16, LA141, LA127, LA32, LA169, LA126}.

The second and last call to Topological_Visit will be from LA22, which will simply prepend LA22 onto the list.

Thus the final linear ordering will be {LA22, LA15, LA31, LA16, LA141, LA127, LA32, LA169, LA126}.

(a) Draw a directed graph that represents these dependencies.

(b) Use the topological sorting algorithm to compute a feasible sequence.

10. DFS and BFS

Let G be an undirected graph whose vertices are labelled by the integers 1 through 8, and having the following edges:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td>2</td>
<td>1, 3, 4</td>
</tr>
<tr>
<td>3</td>
<td>1, 2, 4</td>
</tr>
<tr>
<td>4</td>
<td>1, 2, 3, 6</td>
</tr>
<tr>
<td>5</td>
<td>6, 7, 8</td>
</tr>
<tr>
<td>6</td>
<td>4, 5, 7</td>
</tr>
<tr>
<td>7</td>
<td>5, 6, 8</td>
</tr>
<tr>
<td>8</td>
<td>5, 7</td>
</tr>
</tbody>
</table>

Assume that, in a traversal of G, the adjacent vertices of a given vertex are returned in the order above.

(a) Draw G.

(b) Give the sequence of vertices of G visited using a DFS traversal starting at vertex 1.

(c) Give the sequence of vertices visited using a BFS traversal starting at vertex 1.