EECS-1019C Test #1

Sur / Last Name:
Given / First Name:
Student ID:

- **Instructor:** Parke Godfrey
- **Exam Duration:** 75 minutes
- **Term:** Fall 2015

Answer the following questions to the best of your knowledge. Your answers may be brief, but be precise and be careful. The exam is closed-book and closed-notes. No aids such as calculators, etc., are allowed. Write any assumptions you need to make along with your answers, *if necessary*.

There are five major questions, each with parts. Points for each question and sub-question are as indicated. In total, the test is out of 50 points.

If you need additional space for an answer, just indicate clearly where you are continuing.

Marking Box	
1.	/10
2.	/10
3.	/10
4.	/10
5.	/10
Total	/50

1. (10pts) **Propositional Logic.**

a. (2pts) *Propositional Equivalence.* Prove or disprove by truth table that

$$(p \lor q) \to r \quad \equiv \quad (p \to r) \land (q \to r)$$

p	q	r	$p \lor q$	$(p \lor q) \to r$	$p \to r$	$q \to r$	$(p \to r) \land (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Truth table establishes they are equivalent.
1pt: Constructing TT in right way. 1pt: Correctly filling it, right conclusion

b. (3pts) *English to Compound Proposition.* For Questions 1bi–1biii, write compound propositions (*propositional formula*) using a, s, t, e, and p, the logical connectives, and negation.

a: *You are abducted by aliens.*
s: *You study really hard for EECS-1019.*
t: *You are abducted by UofT students.*
e: *You do every problem in the EECS-1019 textbook.*
p: *You pass EECS-1019.*

i. (1pt) *You will pass EECS-1019 if you study really hard for it and you are not abducted by UofT students.*

$(s \land \neg t) \to p$ *(Or, as some read it,* $(s \to p) \land \neg t$*)*

ii. (1pt) *If you did not pass EECS-1019, then you were not abducted by aliens.*

$\neg p \to \neg a$

iii. (1pt) *You will pass EECS-1019 only if you study it really hard or you do every problem in the textbook*

$p \to (s \lor e)$

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c. (3pts) *Truth Value.* Answer *true* or *false* for the following.

i. (1pt) $(1 + 2 = 7) \rightarrow (2 * 5 = 10)$

True. $((3 = 7) \rightarrow (10 = 10),\ F \rightarrow T,\ T)$

ii. (1pt) $(1 + 2 = 7) \leftrightarrow (2 * 5 = 10)$

False. $((3 = 7) \leftrightarrow (10 = 10),\ F \leftrightarrow T,\ F)$

iii. (1pt) $\neg\,((2 * 7 = 14) \vee (2 * 7 = 15)) \vee (11 * 13 = 145)$

False. $(\neg(T \vee F) \vee F,\ \neg T \vee F,\ F \vee F,\ F)$

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d. (2pts) *Propositional Inference Rules.*

i. (1pt) Show the inference rule of *resolution.*

$$\begin{array}{c} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

ii. (1pt) Show one of *De Morgan's Laws* for propositions.

$$\neg(p \vee q) \quad \equiv \quad \neg p \vee \neg q$$

2. (10pts) **Predicate Logic.**

a. (3pts) *Predicates.* For Questions 2ai–2aiii, determine the truth value of the following statements. Consider the domain to be $\mathbb{Z}$, the set of all integers. Justify briefly each answer.

i. (1pt) $\forall x(x^2 \geq 0)$

True: The square of any non-zero integer is positive. And $0^2 = 0$.

ii. (1pt) $\forall x(x^2 - 1 * x + 5 > x^2 - 2 * x + 3)$

False: $x = -3$ *is a counterexample.* $17 \not> 18$

iii. (1pt) $\exists x \exists y((x > y) \wedge (x < y + 1))$

False: Same as $y < x < y + 1$. *If* y *is an integer, there is no such integer* x.

Mogwai(x): x is a *mogwai*.
Gremlin(x): x is a *gremlin*.
Human(x): x is a *human*.
Sun(x): x is exposed to sunlight.
Wet(x): x is gotten wet.
Fed(x): x is fed after midnight.
Dies(x): x dies.
Likes(x, y): x likes y.
InRoom(x, y): x is in room y.

Figure 1: Predicates for Questions 2b & 2c.

b. (3pts) *Predicate Statements.* For Questions 2bi–2biii, write the statement in predicate logic using the predicates in Figure 2, connectives, negation, and any needed quantifiers.

i. (1pt) *Any mogwai fed after midnight turns into a gremlin.*

$\forall x(\mathit{Mogwai}(x) \wedge \mathit{Fed}(x) \rightarrow \mathit{Gremlin}(x))$

ii. (1pt) *A mogwai or gremlin who is exposed to sunlight dies.*

$\forall x((\mathit{Mogwai}(x) \vee \mathit{Gremlin}) \wedge \mathit{Sun}(x) \rightarrow \mathit{Dies}(x))$

iii. (1pt) *If a human and a gremlin are in the same room, the human dies.*

$\forall x \forall y \exists z(\mathit{Human}(x) \wedge \mathit{Gremlin}(y) \wedge \mathit{InRoom}(x, z) \wedge \mathit{InRoom}(y, z) \rightarrow \mathit{Dies}(x))$

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c. (2pts) *Predicate Statements.* For Questions 2ci & 2cii, write in proper, concise English what the predicate statement says.

i. (1pt) $\exists x \forall y(\textsf{\textbf{Mogwai}}(x) \wedge (\textsf{\textbf{Human}}(y) \rightarrow \textsf{\textbf{Likes}}(y, x)))$

There is a mogwai liked by all people (humans).

ii. (1pt) $\forall x \forall y(\textsf{\textbf{Human}}(x) \wedge (\textsf{\textbf{Gremlin}}(y) \rightarrow \neg\textsf{\textbf{Likes}}(x, y)))$ [1]

No person (human) likes any gremlin.

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d. (2pts) *Predicate Equivalence.* Rewrite the predicate statement

$$\forall x \forall y(\textsf{\textbf{Human}}(x) \wedge (\textsf{\textbf{Gremlin}}(y) \rightarrow \neg\textsf{\textbf{Likes}}(x, y)))$$

(this is the same statement as in Question 2cii) into an equivalent predicate statement *without* any negation ("¬") inside the scope of the quantifiers.

This is for $\forall x \forall y(\textsf{\textbf{Human}}(x) \wedge \textsf{\textbf{Gremlin}}(y) \rightarrow \neg\textsf{\textbf{Likes}}(x, y)$*:*

$\forall x \forall y(\textsf{\textbf{Human}}(x) \wedge \textsf{\textbf{Gremlin}}(y) \rightarrow \neg\textsf{\textbf{Likes}}(x, y)$

$\equiv \quad \forall x \forall y(\neg(\textsf{\textbf{Human}}(x) \wedge \textsf{\textbf{Gremlin}}(y)) \vee \neg\textsf{\textbf{Likes}}(x, y))$

$\equiv \quad \forall x \forall y(\neg\textsf{\textbf{Human}}(x) \vee \neg\textsf{\textbf{Gremlin}}(y) \vee \neg\textsf{\textbf{Likes}}(x, y))$

$\equiv \quad \forall x \forall y(\neg(\textsf{\textbf{Human}}(x) \wedge \textsf{\textbf{Gremlin}}(y) \wedge \textsf{\textbf{Likes}}(x, y)))$

$\equiv \quad \neg\exists x \exists y(\textsf{\textbf{Human}}(x) \wedge \textsf{\textbf{Gremlin}}(y) \wedge \textsf{\textbf{Likes}}(x, y)$

And for the way it written, $\forall x \forall y(\textsf{\textbf{Human}}(x) \wedge (\textsf{\textbf{Gremlin}}(y) \rightarrow \neg\textsf{\textbf{Likes}}(x, y)))$ *(which was not really my intention):*

$\forall x \forall y(\textsf{\textbf{Human}}(x) \wedge (\textsf{\textbf{Gremlin}}(y) \rightarrow \neg\textsf{\textbf{Likes}}(x, y)))$

$\equiv \quad \forall x \forall y(\textsf{\textbf{Human}}(x) \wedge (\neg\textsf{\textbf{Gremlin}}(y) \vee \neg\textsf{\textbf{Likes}}(x, y)))$

$\equiv \quad \forall x \forall y((\textsf{\textbf{Human}}(x) \wedge \neg\textsf{\textbf{Gremlin}}(y)) \vee (\textsf{\textbf{Human}}(x) \wedge \neg\textsf{\textbf{likes}}(x, y)))$

$\equiv \quad \forall x \forall y(\neg\neg((\textsf{\textbf{Human}}(x) \wedge \neg\textsf{\textbf{Gremlin}}(y)) \vee (\textsf{\textbf{Human}}(x) \wedge \neg\textsf{\textbf{likes}}(x, y))))$

$\equiv \quad \forall x \forall y(\neg(\neg(\textsf{\textbf{Human}}(x) \wedge \neg\textsf{\textbf{Gremlin}}(y)) \wedge \neg(\textsf{\textbf{Human}}(x) \wedge \neg\textsf{\textbf{likes}}(x, y))))$

$\equiv \quad \forall x \forall y(\neg((\neg\textsf{\textbf{Human}}(x) \vee \textsf{\textbf{Gremlin}}(y)) \wedge (\neg\textsf{\textbf{Human}}(x) \vee \textsf{\textbf{likes}}(x, y))))$

$\equiv \quad \forall x \forall y(\neg((\textsf{\textbf{Human}}(x) \rightarrow \textsf{\textbf{Gremlin}}(y)) \wedge (\textsf{\textbf{Human}}(x) \rightarrow \textsf{\textbf{likes}}(x, y))))$

$\equiv \quad \neg\exists x \exists y((\textsf{\textbf{Human}}(x) \rightarrow \textsf{\textbf{Gremlin}}(y)) \wedge (\textsf{\textbf{Human}}(x) \rightarrow \textsf{\textbf{likes}}(x, y)))$

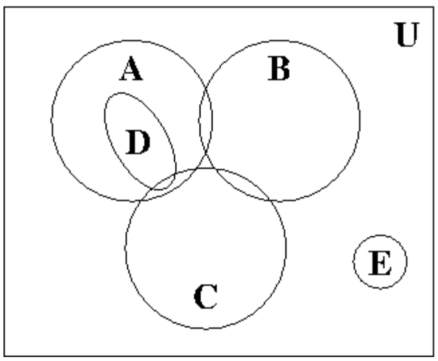
Credit for either solution. Lesson learned: Be careful with parentheses! They can easily change the meaning.

[1] See 2d for clarification.

3. (10pts) **Sets.**

a. (4pts) *Venn Diagram.* Draw the Venn diagram for sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, and $\mathcal{E}$ that obeys the following statements.

1. $\mathcal{A} \cap \mathcal{B} \neq \emptyset$
2. $\mathcal{A} \cap \mathcal{C} \neq \emptyset$
3. $\mathcal{B} \cap \mathcal{C} \neq \emptyset$
4. $\mathcal{A} \cap \mathcal{B} \cap \mathcal{C} = \emptyset$
5. $\mathcal{D} \subsetneq \mathcal{A}$
6. $\mathcal{B} \cap \mathcal{D} = \emptyset$
7. $\mathcal{C} \cap \mathcal{D} \neq \emptyset$



$\mathcal{E}$ is unconstrained. Note that $\mathcal{D} \subsetneq \mathcal{A}$ is notation that means $\mathcal{D}$ is a proper *subset of $\mathcal{A}$ (subset but not equal).*

1pt: (1)–(3)
1pt: (4)
1pt: (5)
1pt: (6) & (7)

b. (6pts) *Membership & Cardinality.*
For Questions 3bi–3biii, consider

$$\mathcal{A} = \{x \in \mathbb{Z}^+ | (x > 1) \wedge \exists y \in \mathbb{Z}^+ (x * y = 12)\}$$

$$\mathcal{B} = \{x \in \mathbb{Z}^+ | (x > 1) \wedge \exists y \in \mathbb{Z}^+ (x * y = 21)\}$$

i. (1pt) Prove that $\mathcal{A} \cap \mathcal{B} \neq \emptyset$ or that $\mathcal{A} \cap \mathcal{B} = \emptyset$.

$\mathcal{A} = \{2, 3, 4, 6, 12\}$ *and* $\mathcal{B} = \{3, 7, 21\}$. *Thus,* $\mathcal{A} \cap \mathcal{B} = \{3\}$, *so is non-empty.*

ii. (1pt) What is the cardinality of $\mathcal{A}$? That is, $|\mathcal{A}|$?

$|\{2, 3, 4, 6, 12\}| = 5$

iii. (1pt) Show $\mathcal{P}(\mathcal{B})$, the powerset of $\mathcal{B}$.

$\mathcal{P}(\{3, 7, 21\}) = \{\emptyset, \{3\}, \{7\}, \{21\}, \{3, 7\}, \{3, 21\}, \{7, 21\}, \{3, 7, 21\}\}$

For Questions 3biv–3bvi, consider

$$\mathcal{C} = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}\}$$

iv. (1pt) What is the cardinality of $\mathcal{C}$?

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v. (1pt) $\{\emptyset, \{\emptyset\}\} \in \mathcal{C}$?

True.

vi. (1pt) $\{\emptyset, \{\emptyset, \{\emptyset\}\}\} \in \mathcal{C}$?

False.

4. (10pts) **Functions.**

a. (2pts) *General.*

i. (1pt) A *function* is said to map from its *domain* to its *codomain*.

ii. (1pt) Can f where $f(1) = 17$, $f(2) = 19$, $f(2) = 23$, $f(3) = 29$, $f(4) = 31$, $f(4) = 37$, and $f(5) = 41$ be a *function*?
Why or why not?

No. 2 *is mapped to* 19 and 23. *This violates the definition of function.*

b. (2pts) *Injective I.* Show that $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $f(x, y) = x^3 + y^5$ is not injective.[2]

$f(-1, 1) = 0$
$f(1, -1) = 0$

2pt: show counterexample
1pt: for no, but not supported

c. (3pts) *Surjective.* Show whether or not $f : \mathbb{N} \to \{0, 1, 2\}$ where $f(x) = x^2 \bmod 3$ is surjective.[3]

Let x *mod* $3 = 0$. *Then* x^2 *mod* $3 = 0$.
Let x *mod* $3 = 1$. *Then* x^2 *mod* $3 = 1$.
Let x *mod* $3 = 2$. *Then* x^2 *mod* $3 = 2^2$ *mod* $3 = 1$.
Thus, there is no $x \in \mathbb{N}$ *such that* x^2 *mod* $3 = 2$.
$\therefore$ *f is not surjective on* $\{0, 1, 2\}$.

3pt: full explanation
2pt: show by expanding
1pt: right answer, but not justified

d. (3pts) *Injective II.* Show whether or not $f : \mathbb{Z}^+ \to (0, 1]$ for

$$(0, 1] = \{x \in \mathbb{Q} \mid (0 < x) \wedge (x \leq 1)\}$$

where $f(x) = 1/x$ is injective.

Consider $x \neq y$, *where* $x, y \in \mathbb{Z}^+$.
Then $1/x \neq 1/y$. $1/x, 1/y \in (0, 1]$.
$\therefore$ *f is injective on* $(0, 1]$.

3pt: for yes, and proper argument
2pt: for yes, but incomplete argument
1pt: for yes, but not supported

[2] Recall that $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, $\mathbb{Z}^+ = \{1, 2, \ldots\}$, $\mathbb{N} = \{0, 1, 2, \ldots\}$, and that $\mathbb{Q}$ is the set of all rationals.

[3] Recall that the modulo operator "mod" returns the *remainder*. E.g., $7 \div 2$ returns a *quotient* of 3 and a *remainder* of 1. Thus, 7 mod 2 returns 1.

5. (10pts) **Proofs.**

a. (5pts) *Analytical Proof I.* Show that

$$\neg p \wedge ((p \vee q) \rightarrow r)$$

and

$$\neg p \wedge (q \rightarrow r)$$

are logically equivalent by an *analytical proof*, *valid-argument style*; that is, show step-by-step equivalent ("≡") logical statements, and label with the *law* used (e.g., *distributive*) to obtain that statement.

$\neg p \wedge ((p \vee q) \rightarrow r) \quad \equiv$

$\equiv$	$\neg p \wedge (\neg(p \vee q) \vee r)$	*[Implication Equivalence]*
$\equiv$	$\neg p \wedge ((\neg p \wedge \neg q) \vee r)$	*[De Morgan]*
$\equiv$	$(\neg p \wedge \neg p \wedge \neg q) \vee (\neg p \wedge r)$	*[Distribution]*
$\equiv$	$(\neg p \wedge \neg q) \vee (\neg p \wedge r)$	*[Idempotent]*
$\equiv$	$(\neg p \vee (\neg p \wedge r)) \wedge (\neg q \vee (\neg p \wedge r))$	*[Distribution]*
$\equiv$	$\neg p \wedge (\neg q \vee (\neg p \wedge r))$	*[Absorbtion]*
$\equiv$	$\neg p \wedge (\neg q \vee \neg p) \wedge (\neg q \vee r)$	*[Distribution]*
$\equiv$	$\neg p \wedge (\neg q \vee r)$	*[Absorbtion]*
$\equiv$	$\neg p \wedge (q \rightarrow r)$	*[Implication Equivalence]*

3pt: for proof

- +1pt: a start
- +2pt: nearly correct
- +3pt: correct

1pt: proper labelled steps

1pt: proper valid-argument style (not skipping steps, etc.)

$\equiv \quad \neg p \wedge (q \rightarrow r)$

b. (5pts) *Analytical Proof II.* Given the sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, and $\mathcal{E}$ and the axioms

1. $\mathcal{A} \cap \mathcal{B} \neq \emptyset$
2. $\mathcal{A} \cap \mathcal{C} \neq \emptyset$
3. $\mathcal{B} \cap \mathcal{C} \neq \emptyset$
4. $\mathcal{A} \cap \mathcal{B} \cap \mathcal{C} = \emptyset$
5. $\mathcal{D} \subsetneq \mathcal{A}$
6. $\mathcal{B} \cap \mathcal{D} = \emptyset$
7. $\mathcal{C} \cap \mathcal{D} \neq \emptyset$

about them (these are the very same as in Question 3a), prove by an *analytical proof*, *valid-argument style*, that $\mathcal{A} \not\subseteq \mathcal{C}$ (that is, $\mathcal{A}$ is not a subset of $\mathcal{C}$).

8. $\mathcal{A} \subseteq \mathcal{C}$	*[Hypothesis]*
9. $\exists x (x \in \mathcal{A} \cap \mathcal{B})$	*[from (1)]*
10. $e \in \mathcal{A} \cap \mathcal{B}$	*[Existential Instantiation over (9)]*
11. $e \in \mathcal{A}$	*[by Def. of Intersection and (10)]*
12. $e \in \mathcal{C}$	*[by (8) and Def. of Subset]*
13. $e \in \mathcal{A} \cap \mathcal{B} \cap \mathcal{C}$	*[by Def. of Intersection over (10) and (12)]*
14. Contradiction	*[(4) and (13)]*
15. $\mathcal{A} \not\subseteq \mathcal{C}$	$\square$

4pt: for proof

- 0pt: nothing
- 1pt: a start
- 2pt: general idea
- 3pt: nearly correct
- 4pt: correct

1pt: proper valid-argument style

Extra space.

Relax. Turn in your test. You have reached the end.