EECS-1019C: ASSIGNMENT #4

Out of 50 points.

Section 2.1 [18pt]

6. [4pt] Suppose that $A = \{2, 4, 6\}$, $B = \{2, 6\}$, $C = \{4, 6\}$, and $D = \{4, 6, 8\}$. Determine which of these sets are subsets of which other of these sets.

> $B \subset A$, $C \subset A$, *and* $C \subset D$.

20. [4pt] What is the cardinality of each of these sets?

a. $\emptyset$

> 0.

b. $\{\emptyset\}$

> 1.

c. $\{\emptyset, \{\emptyset\}\}$

> 2.

d. $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

> 3.

32. [4pt] Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

a. $A \times B \times C$

$\{ (a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1), (b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1), (c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1) \}$

b. $C \times A \times B$

$\{ (0, a, x), (0, a, y), (0, b, x), (0, b, y), (0, c, x), (0, c, y), (1, a, x), (1, a, y), (1, b, x), (1, b, y), (1, c, x), (1, c, y) \}$

c. $C \times B \times A$

$\{ (0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b), (0, y, c), (1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c) \}$

d. $B \times B \times B$

$\{ (x, x, x), (x, x, y), (x, y, x), (x, y, y), (y, x, x), (y, x, y), (y, y, x), (y, y, y) \}$

44. [6pt] Find the truth set of each of these predicates where the domain is the set of integers.

a. [2pt] $P(x) : x^3 \geq 1$

False. Consider $x = 0$.

b. [2pt] $Q(x) : x^2 = 2$

False. Consider $x = 1$.

c. [2pt] $R(x) : x < x^2$

False. Consider $x = 1$.

Section 2.2 [18pt]

16. [10pt] Let A and B be sets. Show that

a. [2pt] $A \cap B \subseteq A$.

For any $x \in A \cap B$, $x \in A$ *and* $x \in B$. *Therefore, any such* x *is in* A. $A \cap B \subseteq A$.

b. [2pt] $A \subseteq (A \cup B)$.

If $x \in A$, *then* $x \in (A \cup B$ *by definition of* union.

c. [2pt] $A - B \subseteq A$.

For any $x \in A - B$, $x \in A$ *and* $x \notin B$. *Therefore, any such* x *is in* A. $A - B \subseteq A$.

d. [2pt] $A \cap (B - A) = \emptyset$.

If $x \in A$, *then* $x \notin (B - A)$. *Therefore, there is no* x *in both* A *and in* $(B - A)$. $A \cap (B - A) = \emptyset$.

e. [2pt] $A \cup (B - A) = A \cup B$.

Consider any $x \in A$. *Then* $x \in A \cup (B - A)$, *by definition of* union, *and* $x \in A \cup B$, *by definition of* union.
Consider any $x \in B$ *but* $x \notin A$. *Then* $x \in A \cup (B - A)$, *since* $x \in (B - A)$ *by definition of* set minus, *and then by definition of* union. *And* $x \in A \cup B$, *by definition of* union.
Consider any $x \notin B$ *and* $x \notin A$. *Then* $x \notin A \cup (B - A)$, *as* $x \notin (B - A)$ *as* $x \notin B$ *(by definition of* set minus*) and then by definition of* union. *And* $x \notin A \cup B$, *by definition of* union.

50. [8pt] Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i

a. [2pt] $A_i = \{i, i+1, i+2, \ldots\}$.

$$\bigcup_{i=1}^{\infty} A_i = \mathbb{Z}^+$$
$$\bigcap_{i=1}^{\infty} A_i = \emptyset$$

b. [2pt] $A_i = \{0, i\}$.

$$\bigcup_{i=1}^{\infty} A_i = \mathbb{N}$$
$$\bigcap_{i=1}^{\infty} A_i = \{0\}$$

c. [2pt] $A_i = (0, i)$, that is, the set of real numbers x with $0 < x < i$.

$$\bigcup_{i=1}^{\infty} A_i = \mathbb{R}^+$$
$$\bigcap_{i=1}^{\infty} A_i = (0, 1)$$

d. [2pt] $A_i = (i, \infty)$, that is, the set of real numbers x with $x > i$.

$$\bigcup_{i=1}^{\infty} A_i = (1, \infty)$$
$$\bigcap_{i=1}^{\infty} A_i = \emptyset$$

Section 2.3 [14pt]

12. [4pt] Determine whether each of these functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one.

a. $f(n) = n1$.

One-to-one since if $n_1 - 1 = n_2 - 1$ *then* $n_1 = n_2$.

b. $f(n) = n^2 + 1$.

Not one-to-one. Consider that $f(3) = f(-3) = 10$.

c. $f(n) = n^3$.

One-to-one since if $n_1^3 = n_2^3$ *then* $n_1 = n_2$ *(the cube-root of each side).*

d. $f(n) = \lceil n/2 \rceil$.

Not one-to-one. Consider that $f(1) = f(2) = 1$.

34. [5pt] If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Justify your answer.

It does. Consider if g *were not one-to-one. Then there exist* x *and* y *such that* $x \neq y$, *but* $g(x) = g(y)$. *Clearly then,* $f(g(x)) = f(g(y))$. *Thus,* $f \circ g$ *is not one-to-one. But this contradicts our assumption.*

36. [5pt] Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from $\mathbb{R}$ to $\mathbb{R}$.

$(f \circ g)(x) = (x + 2)^2 + 1 = x^2 + 2x + 5$.
$(g \circ f)(x) = (x^2 + 1) + 2 = x^2 + 3$.