Name (LAST, FIRST): $\qquad$
Student number:

## Instructions:

1. If you have not done so, put away all books, papers, cell phones and pagers. Write your name and student number NOW!
2. Check that this examination has 5 pages. There should be 6 questions together worth 30 points.
3. You have 75 minutes to complete the exam. Use your time judiciously.
4. Show all your work. Partial credit is possible for an answer, but only if you show the intermediate steps in obtaining the answer.
5. If you need to make an assumption to answer a question, please state the assumption clearly.
6. Points will be deducted for vague and ambiguous answers.
7. Your answers MUST be LEGIBLE.
8. Feel free not to use the hints supplied.

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

1. (6 points) Proofs I: prove that $\sqrt{3}$ is irrational.
2. (6 points) Proofs II: show that if $n$ is an odd integer, there is a unique integer $k$ such that $n$ is the sum of $k-2$ and $k+3$.
3. (6 points) Proofs III: Prove that there are infinitely many prime numbers.

Hint: Suppose not. Then there is a finite set of primes $p_{1}, \ldots, p_{n}$ for some positive integer $n$. Define $p=1+p_{1} p_{2} \ldots p_{n}$. What can you say about $p$ ?
4. (6 points) Sets: let $A, B, C$ be sets. Prove (without using Venn diagrams) that
(a) $(A-B)-C \subseteq A-C$
(b) $(B-A) \cup(C-A)=(B \cup C)-A$
5. (6 points) Functions: define a function $f$ on the real numbers as $f(0)=0$; for $x \neq 0, f(x)=\frac{3 x-1}{x}$. What is the range of $f$ ? Is $f$ onto? Justify your answer.
6. (6 points) Series sums: define the $i^{t h}$ term of a series as $t_{i}=2^{i}+3 i$; evaluate the sum

$$
\sum_{i=1}^{n} t_{i}
$$

> Use this page if you need extra space

