MATH/CSE 1019 Third test
Fall 2011
Nov 21, 2011
Instructor: S. Datta

Name (LAST, FIRST): _____

Student number: _____

Instructions:

1. If you have not done so, put away all books, papers, cell phones and pagers. Write your name and student number NOW!
2. Check that this examination has 5 pages. There should be 2 questions together worth 30 points.
3. You have 75 minutes to complete the exam. Use your time judiciously.
4. Show all your work. Partial credit is possible for an answer, but only if you show the intermediate steps in obtaining the answer.
5. If you need to make an assumption to answer a question, please state the assumption clearly.
6. Points will be deducted for **vague and ambiguous** answers.
7. Your answers MUST be LEGIBLE.

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

1. (15 points) Cardinality: Determine which of these sets is countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

   (a) (5 points) The odd negative integers

   (b) (5 points) The integers that are multiples of 7.

(c) (5 points) The integers less than 100.

2. (15 points) Induction:

(a) (5 points) Prove using induction that $2^n > n^2$ if n is an integer greater than 4.

(b) (5 points) Prove using induction that 3 divides $n^3 + 2n$ whenever n is a positive integer.

(c) (5 points) Use strong induction to show that every positive integer can be written as a sum of distinct powers of 2, that is, as a subset of the integers $2^0 = 1, 2^1 = 2, 2^2 = 4$ and so on.
Hint: For the inductive step, separately consider the case where $k + 1$ is even and where it is odd. Where it is even, note that $\frac{k+1}{2}$ is an integer.

Use this page if you need extra space