MATH/EECS 1019: Discrete Math for Computer Science
Fall 2014
Assignment 1 - Solutions (Released September 23 , 2014)

Question 1

[2 points] Form the contrapositive of these statements:

1. If you drop the course, you will not get a grade for the course at the end of the term.

   Solution: If you get a grade for the course at the end of the term, you did not drop the course.

2. If a triangle is equilateral, it has 3 equal angles.

   Solution:

   If a triangle does not have three equal angles, it is not equilateral.

Question 2

[6 points] Decide whether the inferences are valid in each case. Give the reason behind each step. Do not use truth tables in this question.

1. $(p \vee q) \rightarrow r$
   p
   $\therefore r$

   Solution:

1.	p	Premise
2.	$p \vee q$	Addition
3.	$(p \vee q) \rightarrow r$	Premise
4.	r	Modus Ponens

2. $p \rightarrow r$
   $p \vee q$
   $\neg q$
   $\therefore r$

   Solution: Many different solutions are possible.
   Solution 1:

1.	$p \vee q$	Premise
2.	$\neg q$	Premise
3.	p	Disjunctive Syllogism
4.	$p \rightarrow r$	Premise
5.	r	Modus Ponens

   Solution 2: Rewrite $p \rightarrow r$ as $\neg p \vee r$.Then

1.	$p \vee q$	Premise
2.	$\neg p \vee r$	Premise
3.	$q \vee r$	Resolution
4.	$\neg q$	Premise
5.	r	Disjunctive Syllogism

3. $\forall x \in \mathbb{R}, p(x) \vee q(x)$
   $a \in \mathbb{R}$
   $q(a) \to r(a)$
   $\therefore p(a) \vee r(a)$

   Solution: Rewrite $q(a) \to r(a)$ as $\neg q(a) \vee r(a)$. Then,

1.	$\forall x \in \mathbb{R}, p(x) \vee q(x)$	Premise
2.	$p(a) \vee q(a)$	Universal instantiation
3.	$q(a) \vee p(a)$	Commutativity
4.	$\neg q(a) \vee r(a)$	Premise
5.	$p(a) \vee r(a)$	Resolution

Question 3

[4 points] Write down the following sentences in predicate logic.

1. The equation $x^2 + 2x + 1 = 0$ has no solutions over the natural numbers.

   Solution: Assume the domain is the set of natural numbers $\mathbb{N}$. Define a predicate $Sol(x), x \in \mathbb{N}$ that is true if x is a solution of the given equation and false otherwise. Then the sentence is

   $\forall x \neg Sol(x)$.

2. Every positive real number has a **unique** positive real square root.

   Solution: There are different correct answers. One is as follows.

   Suppose the domain is the set of positive real numbers. Define a predicate $Sq(x, y)$ to be true if $x = y^2$ and false otherwise.

$$(\forall x \exists y Sq(x, y)) \wedge (\forall x (\exists y \exists z Sq(x, y) \wedge Sq(x, z)) \to (y = z))$$

Question 4

[4 points] Write down the negations of the following expressions, so that the $\neg$ symbol does not arise to the left of any quantifier. Indicate whether the negated statement is true.

1. $\forall x \in \mathbb{N}, x^2 \in \mathbb{N}$ and $\frac{1}{2x} \notin \mathbb{N}$.

   Solution:

   $\exists x \in \mathbb{N}, x^2 \notin \mathbb{N}$ or $\frac{1}{2x} \in \mathbb{N}$.

   The given statement is true and hence the negated statement is false.

2. $\forall x \in \mathbb{R}, \exists n \in \mathbb{Z}, x^n > 0$.

   Solution: $\exists x \in \mathbb{R}, \forall n \in \mathbb{Z}, x^n \leq 0$.

   The given statement is true, and so the the negated statement is false.

   [Note: To see that the negated statement is false, we can use the counterexample of $n = 0$. For any x, $x^0 = 1 \not\leq 0$.

Question 5

[4 points] Negate the following sentences

1. Truth is not always popular, but it is always right.

   Solution: We can do this directly but let us do it by using the propositions p : "Truth is always popular" and q : "Truth is always right". Then the given statement is $\neg p \wedge q$. According to De Morgan's laws, the negation of this expression is $p \vee \neg q$, which is "Truth is always popular or truth is not always right".

2. Some young people are not athletic.

   Solution:

   All young people are athletic.