MATH/EECS 1019: Discrete Math for Computer Science
Fall 2014
Assignment 1 (Released September 30 , 2014)
Submission deadline: 5 pm, Oct 14, 2014

Notes:

1. The assignment can be handwritten or typed. It MUST be legible.
2. You must do this assignment individually.
3. Submit this assignment only if you have read and understood the policy on academic honesty on the course web page. If you have questions or concerns, please contact the instructor.
4. Use the dropbox near the EECS main office to submit your assignments, OR submit your assignment online using the `submit` command from a EECS computer or the web interface to submit from any computer (follow instructions on the class webpage). No late submissions will be accepted. Please do not send files by email.
5. Your answers should be precise and concise. Points may be deducted for long, rambling arguments.
6. Assume $\mathbb{R}$ to denote the real numbers, $\mathbb{Z}$ to denote the set of integers $(\ldots, -2, -1, 0, 1, 2, \ldots)$ and $\mathbb{N}$ to denote the natural numbers $(1, 2, 3, \ldots)$.

Question 1

[4 points] Prove using Mathematical Induction that for all natural numbers n,

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} \geq \sqrt{n}$$

Question 2

[4 points] Prove that $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9.
Hint: Use Mathematical Induction.

Question 3

[4 points] Recall that a number y is rational if it satisfies $y = p/q$ where p, q are integers and $q \neq 0$. Prove that any number of the form $a + b\sqrt{5}$ is irrational, where a, b are integers and $b \neq 0$.

Question 4

[4 points] Prove the following statement: If a, b, c are odd integers, then $ax^2 + bx + c = 0$ does not have a rational number solution.

Question 5

[4 points] Prove that among any given $n + 1$ positive integers, there are always two whose difference is divisible by n.
Hint: Use the Pigeonhole Principle.