

Notes:

1. The assignment can be handwritten or typed. It MUST be legible.
2. You must do this assignment individually.
3. Submit this assignment only if you have read and understood the policy on academic honesty on the course web page. If you have questions or concerns, please contact the instructor.
4. Use the dropbox near the EECS main office to submit your assignments, OR submit your assignment online using the `submit` command from a EECS computer (follow instructions on the class webpage). No late submissions will be accepted. Please do not send files by email.
5. Your answers should be precise and concise. Points may be deducted for long, rambling arguments.
6. Assume $\mathbb{R}$ to denote the real numbers, $\mathbb{Z}$ to denote the set of integers $(\dots, -2, -1, 0, 1, 2, \dots)$ and $\mathbb{N}$ to denote the natural numbers $(1, 2, 3, \dots)$.

Question 1

[2 points] Form the contrapositive of these statements:

1. If you drop the course, you will not get a grade for the course at the end of the term.
2. If a triangle is equilateral, it has 3 equal angles.

Question 2

[6 points] Decide whether the inferences are valid in each case. Give the reason behind each step. Do not use truth tables in this question.

1. $(p \vee q) \rightarrow r$
 p
 $\therefore r$
2. $p \rightarrow r$
 $p \vee q$
 $\neg q$
 $\therefore r$

3. $\forall x \in \mathbb{R}, p(x) \vee q(x)$
 $a \in \mathbb{R}$
 $q(a) \rightarrow r(a)$
 $\therefore p(a) \vee r(a)$

Question 3

[4 points] Write down the following sentences in predicate logic.

1. The equation $x^2 + 2x + 1 = 0$ has no solutions over the natural numbers.
2. Every positive real number has a **unique** positive real square root.

Question 4

[4 points] Write down the negations of the following expressions, so that the $\neg$ symbol does not arise to the left of any quantifier. Indicate whether the negated statement is true.

1. $\forall x \in \mathbb{N}, x^2 \in \mathbb{N}$ and $\frac{1}{2x} \notin \mathbb{N}$.
2. $\forall x \in \mathbb{R}, \exists n \in \mathbb{Z}, x^n > 0$.

Question 5

[4 points] Negate the following sentences

1. Truth is not always popular, but it is always right.
2. Some young people are not athletic.