Faculty of Science and Engineering

MATH1090- Problem Set No 3

Due: November 15, 2012 at 2:00pm; in the course assignment box.

Reminder: Any assignment that is a full or partial copy of another person’s assignment is an offence against the standards of Academic Honesty, and will therefore lead to penalties as outlined in Senate’s regulations.

1. (3 marks) Prove $A \lor B \nleq A \land B$.

2. (3 marks) (Question 2(a), section 3.6 of book) Let $A$ be a formula in which the variables $p, q, r$ occur, but no others, and whose truth table has a result $\mathbf{t}$ only in the rows $\mathbf{f}, \mathbf{t}, \mathbf{f}$ (state for $p, q, r$ in that order) and $\mathbf{t}, \mathbf{f}, \mathbf{f}$. Show that $A$ is provably equivalent to the formula $\neg p \land q \land \neg r \lor p \land \neg q \land \neg r$.
   Hint. Prove that $\models_{\text{taut}} A \equiv \neg p \land q \land \neg r \lor p \land \neg q \land \neg r$ instead.

3. Consider the following formula:

   $$(\forall x)x = x \land \neg(\forall x)x = x \equiv \bot$$

   (a) (2 marks) Identify free and bound $x$. For each bound occurrence of $x$, find which $(\forall x)$ it belongs to.
   (b) (2 marks) Identify all subformulae of above formula with complexity $< 2$.
   (c) (2 marks) Write the abstraction of the above formula.
   (d) (3 marks) Prove the above formula. Explain your steps.
   (e) (2 marks) Based on your answer in part (d), prove

   $$(\forall y)((\forall x)x = x \land \neg(\forall x)x = x \equiv \bot)$$
4. (2 marks each) Apply the following substitutions if legal and defined. Otherwise, explain why illegal or undefined.

(a) \(((\forall x)x = g(x) \land x = g(y))[x := f(x, y)]\)
(b) \(((\forall y)x = g(x) \land x = g(y))[x := f(x, y)]\)
(c) \(((\forall x)(\forall z)p \equiv \psi(x, z))[p \setminus f(x, y)]\)
(d) \(((\forall z)(\forall x)p \equiv \psi(x, z))[p := (\forall x)x = y]\)

5. (3 marks) (Question 8, section 4.3 of book) Is \((\forall x)(\forall y)x = y \rightarrow (\forall y)y = y\) an instance of \textbf{Ax2}? Why?

6. (3 marks) (Question 9, section 4.3 of book) Prove \(\vdash (\forall x)(\forall y)x = y \rightarrow (\forall y)y = y\).

7. (3 marks) (Question 3, section 6.6 of book) Prove

\(\vdash (\forall x)(A \lor B \rightarrow C) \rightarrow (\forall x)(A \rightarrow C)\)

8. (3 marks) (Question 4, section 4.3 of book) Prove

\(\vdash (\forall x)(A \rightarrow B \land C) \rightarrow (\forall x)(A \rightarrow B)\)

9. (3 marks) Prove \(\vdash (\forall x)_A(B \equiv C) \rightarrow ((\forall x)_A B \equiv (\forall x)_A C)\).