Faculty of Science and Engineering

MATH1090- Problem Set No 1

Due: September 25, 2012 at 2:00pm; in the course assignment box.

1. (3 marks) Consider the string ($\top \land (\neg(p \rightarrow q)))$.
   (a) Show that the above string is a well-formed formula (by the procedural definition).
   (b) Show the top-down parsing for this formula.
   (c) Show this formula in least parenthesized notation.

2. (3 marks) Prove by induction on formulae that the complexity of a wff equals the number of its left brackets.

3. (3 marks) Prove that the last symbol of a Boolean formula is never the symbol $\neg$. Hint. Use analysis of formula calculation, or prove by induction on formulae.

4. (4 marks) Which of the following are tautologies? Show your work. Note that to prove that a schema is not a tautology, you must show an instance of it that is not a tautology.
   - $p \land q \rightarrow p \lor q$
   - $p \land q$
   - $A \lor B \rightarrow A \land B$
   - $A \lor (B \equiv C) \equiv A \lor B \equiv A \lor C$
5. (4 marks) Use truth tables (with explanations) or truth table shortcuts to show if the following statements are valid. For any invalid schema, show an instance that is not a tautological implication.

- $A, B \models_{\text{taut}} A \land B$
- $p, q \models_{\text{taut}} q$
- $B \rightarrow A, A \models_{\text{taut}} B$
- $A, A \equiv B \equiv C \models_{\text{taut}} C$

6. (2 marks) Use the truth table shortcut method to show that

$$C, A \rightarrow (B \equiv C) \models_{\text{taut}} A \rightarrow B$$

7. (2 marks) Which of the following sets is satisfiable?

- $\{p \lor q, q \lor r, r \lor \neg p\}$
- $\{p \lor q, p \land \neg p, \top\}$

8. (4 marks) Calculate the wff obtained after the following substitutions, if they are valid. Otherwise explain why the substitution is illegal. Show all steps for the first valid substitution only, and only the final answer for the rest.

- $p \rightarrow (q \equiv p)[p := \top]$
- $(p \rightarrow q) \equiv p[p := \mathsf{t}]$
- $((p \rightarrow q) \equiv p)[(p \rightarrow q) := A]$ (where $A$ is a formula)
- $q[p := A \lor B]$ (where $A$ and $B$ are formulae)