1) (8 marks) Consider the following formulae:

\[
\begin{align*}
A &: p \rightarrow (q \lor r) \\
B &: q \equiv r \\
C &: \neg q \land (p \lor r)
\end{align*}
\]

a) Using truth tables, which of the above formulae is satisfiable?
b) Which of the above formulae is a contradiction?
c) Which of the following sets are satisfiable?
   a. \{A, B\}
   b. \{B, C\}
   c. \{A, B, C\}
d) Which of the above sets i-iii are inconsistent?

Explain your answers.

2) (5 marks) Consider the following formula in domain of natural numbers:

\[
(\forall x)(\forall y)(x > y \lor x + 1 = 5)
\]

In this formula,
(a) List all object variables and constants.
(b) What are the predicates? What are the functions?
(c) List all terms.
(d) List all atomic formulas.
(e) Is the formula semantically true?
3) (10 marks) Convert the following formulae to conjunctive normal form (CNF). Clearly show ALL steps.

(i) \( p \rightarrow q \rightarrow \neg r \)

(ii) \((s \equiv r) \rightarrow (p \land q)\)

(iii) \((\forall X)(\exists Y)lt(X, Y) \rightarrow (\exists Y)(\forall Y)gt(X, Y)\)

4) (10 marks) For each of the formulae in question 3,

(a) Write the formula in logic programming notation.

(b) Identify the Horn clauses in your notation in part (a). For Horn clauses, identify facts, rules, and queries.

(c) Indicate whether your notation in part (a) results in a definite program or not. Why?

5) (5 marks) Using what we covered in class about arithmetic in Prolog,

(a) Write a simple predicate convert(SQm, SQcm) that converts area in square meters to area in square centimetres.

(b) Write a query that converts 5 square meters to square centimetres given the above predicate. What does Prolog return as an answer to your query? Explain why.

(c) Write a query that converts 5 square centimetres to square meters given the above predicate. What does Prolog return as an answer to your query? Explain why.

6) (5 marks) Consider the following Boolean variables and their corresponding English phrases:

p: weather is warm
q: John is swimming
r: John is outdoors
s: John is studying at home

Given the following statements:

- If the weather is warm and John is not studying at home then John is outdoors.
- John is swimming provided he is outdoors.
- The weather is warm.

And the following query:

- John is swimming if he is not studying at home.

Use the logic programming notation and a refutation tree to find the answer to above query.

7) (10 marks) Given the following facts and rules:

- The basic parts of a bike are front frame, back frame, and saddle.
- A subpart of a bike is a part of a basic part.
- A handle and a fork are parts of the front frame.
the following program is coded in Prolog (numbered from C0 to C5):
C0: basicpart(front).
C1: basicpart(back).
C2: basicpart(saddle).
C3: subpart(X) :- basicpart(Y), partof(X,Y).
C4: partof(handle, front).
C5: partof(fork, front).

Draw the search tree based on linear refutation for the following query (let’s call it G0):
    :- subpart(X).
Use the above labels (C0 - C5 and G0) in your search tree. Label all branches to show resolution and unifier. Under each leaf node, mention what will happen and why. If there are any outputs by Prolog, indicate them too.

Please not that this assignment is due on January 30, 2012 at 3:30pm in the course drop box next to LAS 1003 (formerly CSE1003). No late assignments!