Homework Assignment #10
Due: Monday July 30, 2012 at 7:00 p.m.

1. Let $L_1 = \{a^i b^j c^k : i = j \text{ or } j = k\}$.

  (a) Give a context-free grammar for L_1.

  You do not have to prove that your answer is correct, but you should give a precise description of the strings that can be generated by each variable in your grammar.

  (b) Show that your grammar is ambiguous. (This statement will be true if your solution in part (a) is correct.)

2. Let $L_2 = \{a^i b^j : i \leq j < 2i\}$. Let G_2 be the grammar with starting symbol S and the following rules.
Rule 1: $S \rightarrow aSbb$
Rule 2: $S \rightarrow T$
Rule 3: $T \rightarrow aTb$
Rule 4: $T \rightarrow ab$
The goal of this question is to prove that G_2 generates the language L_2.

  (a) For each i and j satisfying $i \leq j < 2i$, describe a leftmost derivation of $a^i b^j$ using the grammar G_2. (That is say how many times to apply each rule, and in what order.)

  (b) Give a formal proof that every string that can be generated from T is of the form $a^i b^i$ for some positive integer i.

  (c) Give a formal proof that every string generated by grammar G_2 is in L_2.