Review Questions

Regular Languages

1. Let L_1 be the language of all binary strings that contain an even number of 0's and an odd number of 1's. Design a DFA that accepts L_1. For each state of your machine, describe the set of strings that take the machine to that state.

2. Let L_2 be the language of all binary strings that do not contain 011 as a substring. Design an NFA that accepts L_2.

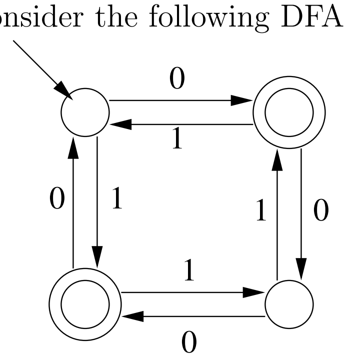
3. Give regular expressions for L_1 and for L_2, defined above.

4. Let $L_4 = \{x\#y : x, y \in \{0,1\}^*$ and y contains x^R as a substring$\}$. Is L_4 regular? Prove your answer is correct.

5. Let L_5 be the set of all binary strings that contain either 011 or 101 as a substring. Is L_5 regular? Prove your answer is correct.

6. Given a language L over alphabet Σ, define $\textsc{Prefix}(L)$ to be $\{x :$ there exists $y \in \Sigma^*$ such that $xy \in L\}$. Is the class of regular languages closed under $\textsc{Prefix}$?

7. Consider the following DFA.



Give a careful proof that no string of even length is accepted by this machine. (Also, describe in one sentence the set of strings that are accepted by this machine.)

8. Write an algorithm that takes as input a regular expression and outputs yes if the regular expression generates (at least) one string that contains 001 as a substring and outputs no otherwise.

Context-Free Languages

9. Let $L_9 = \{x2^n : n \in \mathbb{N} \text{ and } x \in \{0,1\}^* \text{ and } |x| > n\}$. Give a context-free grammar for L_9.

10. Let $L_{10} = \{0^n 1^m : m \geq n \text{ and } n \text{ is even}\}$. Design a PDA that accepts L_{10}

11. Let $L_{11} = \{a^i b^j c^k : i - j = k\}$. Consider the following CFG G with starting symbol S:

$$\begin{aligned} S &\rightarrow aSc \mid T \\ T &\rightarrow aTb \mid \varepsilon \end{aligned}$$

Give a formal proof that G generates L_{11}. (I.e., prove that G generates every string in L_{11} and no others.)

12. Let $L_{12} = \{a^i b^j c^k d^\ell : i - j = k - \ell\}$. Is L_{12} context-free? If it is, give a grammar or a PDA for it. If it is not, prove it is not.

13. Is L_4 context-free? If it is, give a grammar or a PDA for it. If it is not, prove it is not.

14. Let $L_{14} = \{x\#y : x, y \in \{0,1\}^* \text{ and } y \text{ contains } x \text{ as a substring}\}$. Is L_{14} context-free? If it is, give a grammar or a PDA for it. If it is not, prove it is not.

Decidable and Recognizable Languages

You may use Church's thesis freely when answering the questions below.

15. Give a high-level description of a Turing Machine that decides L_{14}. (By "high-level description", I mean the kind of description given in Example 3.12 in the textbook.)

16. Show that the class of decidable languages is closed under intersection.

17. Let $L_{17} = \{\langle M \rangle : M \text{ is a Turing machine that accepts } \varepsilon \text{ and at least one other string}\}$. Show that L_{17} is recognizable.

18. Show that L_{17} is not decidable.

19. Is $\overline{L_{17}}$ recognizable? Show your answer is correct.

20. Show that the union of two recognizable sets is recognizable.

21. Let $L_{21} = \{\langle M, x \rangle : \text{Turing machine } M \text{ accepts input string } x \text{ in fewer than 489 steps}\}$. Is L_{21} recognizable? Is L_{21} decidable? Show your answer is correct.

22. Let $L_{22} = \{\langle M \rangle : \text{ Turing machine } M \text{ accepts no strings of length 7}\}$. Is L_{22} recognizable? Is L_{22} decidable? Show your answer is correct.

23. Let $L_{23} = \{\langle M_1, M_2 \rangle : \text{there is some string that is accepted by both of the Turing machines } M_1 \text{ and } M_2\}$. Is L_{23} recognizable? Is it decidable? Show your answer is correct.

General

24. Show that the union of two countable sets is countable.

25. Alice and Bob have a telephone conversation. Eve wants to eavesdrop. Eve has bugged Alice and Bob's rooms. One of Eve's minions listens to what happens in Alice's room. The minion can hear what Alice says on the telephone but not what Bob says. The minion types up a transcript a of what he hears Alice say. Another minion listens to what happens in Bob's room and can hear what Bob says, but not what Alice says. That minion types up a transcript b of what he hears Bob say. Eve's minions are not very careful, so they don't write down where there are pauses in Alice's or Bob's speech.

Eve gets a and b from her minions and wants to reconstruct the entire conversation c (with Alice and Bob's words spliced together in their proper order). Assume Alice and Bob take turns talking, never talking simultaneously.

(a) Give an algorithm that takes 3 strings, a, b and c as inputs and decides whether c can be obtained by splicing together a and b, as described above.

(b) Let $\textsc{Splice}(L_1, L_2)$ be the set of all strings c that can be obtained by splicing together a string a from L_1 and a string b from L_2. Prove that if L_1 and L_2 are both regular languages, then $\textsc{Splice}(L_1, L_2)$ is also regular.

(c) Prove that if L_1 and L_2 are both decidable languages, then $\textsc{Splice}(L_1, L_2)$ is also decidable.