Rules of inference

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Argument

Argument:

1. "If you have the current password, then you can log onto the network."
2. "You have a current password."

Therefore,

3. "You can log onto the network."

An **Argument** is a sequence of propositions.

Premises and conclusion

Argument:

1. "If you have the current password, then you can log onto the network."
2. "You have a current password."

premises

Therefore,

3. "You can log onto the network."

conclusion

All but the final proposition are called **premises.** The final proposition is called the **conclusion.**

Valid argument

Argument:

1. "If you have the current password, then you can log onto the network."
2. "You have a current password."

Therefore,

3. "You can log onto the network."

premises true

conclusion true

An argument is **valid** if the truth of all premises implies that conclusion is true.

Valid argument

Argument:

p

1. "If $\boxed{\text{you have the current password}}$ then

q $\boxed{\text{you can log onto the network}}$"

2. "You have a current password."

Therefore,

3. "You can log onto the network."

1. "If p, then q."
2. "p."

Therefore, 3. "q."

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

$$\boxed{((p \rightarrow q) \wedge p) \rightarrow q}$$

Tautology

Rules of inference

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

$$\boxed{((p \rightarrow q) \wedge p) \rightarrow q}$$

Modus ponens
Law of detachment

Rules of inference (example)

Assume "if you go out tonight, you will come back late" and "you go out tonight" are true.

Show the truth value of "you will come back late".

Solution:

- Determine individual propositions
  - p: you go out tonight
  - q: you will come back late
- Form the argument and truth value of the conclusion

$p \rightarrow q$	true
p	true
$\therefore q$	true

Rules of inference (example)

Determine the following argument is valid or not.

If 27196 is multiple of 17, then 27196+17 is multiple of 17.
27196 is multiple of 17.
Therefore, 27196+17 is multiple of 17.

Solution:

- Check if premises are true then the conclusion is true
  - p: 27196 is multiple of 17
  - q: 27196+17 is multiple of 17

If p, then q.	if true
p.	if true
Therefore, q.	true

- By modus ponens the argument is valid.

Rules of inference (example)

Determine the conclusion of the following argument must be true or not.

If 27196 is multiple of 17, then 27196+17 is multiple of 17.

27196 is multiple of 17.

Therefore, 27196+17 is multiple of 17.

Solution:

- Check if all premises are true
  - "27196 is multiple of 17" is false.
- The conclusion "27196+17 is multiple of 17" is not true.

Rules of inference

Argument:

It is sunny today.

Therefore,

it is either sunny or cloudy today.

premise true

conclusion true

p.
Therefore, $p \vee q$

$$\frac{p}{\therefore\ p \vee q}$$

$p \rightarrow (p \vee q)$

Tautology

Rules of inference (example)

State which rule of inference is applied in the following argument.

Tofu is healthy to eat.

Therefore, either tofu or cheeseburger is healthy to eat.

Solution:

- Determine individual propositions
  - p: tofu is healthy to eat.
  - q: cheeseburger is healthy to eat.
- Determine the argument using p and q

  p.

  Therefore, $p \lor q$.

$$\boxed{\begin{array}{l} p \\ \hline \therefore\ p \lor q \end{array}}$$

Rules of inference

Argument:

It is cloudy and raining. **premise true**

Therefore,

it is cloudy. **conclusion true**

$p \wedge q$.
Therefore, p.

$$\frac{p \wedge q}{\therefore\ p}$$

$$(p \wedge q) \rightarrow p$$

Tautology

Rules of inference (example)

State which rule of inference is applied in the following argument.

You are clever and lucky.

Therefore, you are clever.

Solution:

- Determine individual propositions
  - p: you are clever.
  - q: you are lucky.
- Determine the argument using p and q

  $p \land q.$

  Therefore, p.

$$\boxed{\begin{array}{l} p \land q \\ \hline \therefore\ p \end{array}}$$

Rules of inference

Argument:

If I go swimming, then I will stay in the sun too long.

If I stay in the sun too long, then I will sunburn.

Therefore,

if I go swimming, then I will sunburn.

premises true

conclusion true

$p \rightarrow q$.
$q \rightarrow r$.
Therefore, $p \rightarrow r$.

$$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r} \\ \therefore\ p \rightarrow r \end{array}$$

$(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$

Tautology

Rules of inference (example)

State which rule of inference is applied in the following argument.

If today is sunny, then she goes shopping.

If she goes shopping, then she spends money.

Therefore, if today is sunny, then she spends money.

Solution:

- Determine individual propositions
  - p: today is sunny.
  - q: she goes shopping.
  - r: she spends money.
- Determine the argument using p and q

  $p \to q.$

  $q \to r.$

  Therefore, $p \to r.$

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

Rules of inference

Argument:

If it is sunny today, then I go swimming today.

I do not go swimming today.

premises true

Therefore,

it is not sunny today.

conclusion true

$p \rightarrow q$.

$\neg q$.

Therefore, $\neg p$.

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

$((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$

Tautology

Rules of inference (example)

State which rule of inference is applied in the following argument.

If today is sunny, then she goes shopping.

She does not go shopping.

Therefore, today is not sunny.

Solution:

- Determine individual propositions
  - p: today is sunny.
  - q: she goes shopping.
- Determine the argument using p and q

$p \rightarrow q$.

$\neg q$.

Therefore, $\neg p$.

$$\boxed{\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}}$$

Rules of inference for quantified statement

Argument:

All women are wise.

Therefore,

Lisa is wise.

premises true

$\Downarrow$

conclusion true

∀x P(x).

Therefore, P(c).

(c is a particular member of the domain.)

$$\frac{\forall x\, P(x)}{\therefore\ P(c)}$$

Universal instantiation

Rules of inference for quantified statement (example)

State which rule of inference is applied in the following argument.

All dogs are cute.

Therefore, his dog is cute.

Solution:

- Determine individual propositional function
  - P(x): x is cute. Domain: all dogs
- Determine the argument using P(x)

  $\forall x\, P(x)$. Domain: all dogs

  Therefore, P(his dog).

$$\boxed{\begin{array}{l} \forall x\, P(x) \\ \hline \therefore\ P(c) \end{array}}$$

Rules of inference for quantified statement

Let s be any student.

Argument:

student s has taken calculus. **premises true**

Therefore,

all student has taken calculus. **conclusion true**

P(s).
Therefore, ∀x P(x).
(s is an arbitrary member of the domain.)

$$\frac{P(s)}{\therefore\ \forall x\, P(x)}$$

Universal generalization

Rules of inference for quantified statement (example)

State which rule of inference is applied in the following argument.

Let s be any student.

Student s has a personal computer.

Therefore, all student has a personal computer.

Solution:

- Determine individual propositional function
  - P(x): x has a personal computer. Domain: all students
- Determine the argument using P(x)

  P(s).

  Therefore, $\forall x\, P(x)$.

  Domain: all students

  (s is an arbitrary element of the domain.)

$$\boxed{\begin{array}{l} P(s) \\ \hline \therefore\ \forall x\, P(x) \end{array}}$$

Rules of inference for quantified statement (example)

Show if $\forall x\,(P(x) \wedge Q(x))$ is true then $\forall x\,P(x) \wedge \forall x\,Q(x)$ is true. (using direct technique)

Solution:

- Assume $\forall x\,(P(x) \wedge Q(x))$ is true.
- If a is in the domain then $P(a) \wedge Q(a)$ is true.
- So, $P(a)$ is true and $Q(a)$ is true.
- Element a can be any element in the domain.
- So, $\forall x\,P(x)$ is true and $\forall x\,Q(x)$ is true.
- Thus, $\forall x\,P(x) \wedge \forall x\,Q(x)$ is true.

$$\frac{\forall x\,P(x)}{\therefore\ P(a)}$$

$$\frac{P(a)}{\therefore\ \forall x\,P(x)}$$

Rules of inference for quantified statement

Argument:

There is a fish in the pool.

Therefore,

some fish c in the pool.

premises true

conclusion true

$\exists x\, P(x)$.

Therefore, $P(c)$.

(c is some member of the domain.)

$$\frac{\exists x\, P(x)}{\therefore\ P(c)}$$

Existential instantiation

Rules of inference for quantified statement (example)

State which rule of inference is applied in the following argument.

There is a person in the store.

Therefore, some person c is in the store.

Solution:

- Determine individual propositional function
  - P(x): x is in the store.

    Domain: all people
- Determine the argument using P(x)

  $\exists x\, P(x)$.

  Therefore, P(c).

  Domain: all people

  (c is some element of the domain.)

$$\boxed{\begin{array}{l} \exists x\, P(x) \\ \hline \therefore\ P(c) \end{array}}$$

Rules of inference for quantified statement

Argument:

George is in the pool.

Therefore,

there is a person in the pool.

premises true

⇓

conclusion true

P(c).
Therefore, ∃x P(x).
(c is a particular member of the domain.)

$$\frac{P(c)}{\therefore\ \exists x\, P(x)}$$

Existential generalization

Rules of inference for quantified statement (example)

State which rule of inference is applied in the following argument.

His dog is playing in the park.

Therefore, There is a dog playing in the park.

Solution:

- Determine individual propositional function
  - P(x): x is playing in the park.

    Domain: all dogs
- Determine the argument using P(x)

  P(his dog).

  Therefore, $\exists x\, P(x)$.

  Domain: all dogs

$$\boxed{\begin{array}{l} P(c) \\ \hline \therefore\ \exists x\, P(x) \end{array}}$$

Rules of inference for quantified statement (example)

Show if $\exists x\,(P(x) \wedge Q(x))$ is true, then $\exists x\,P(x) \wedge \exists x\,Q(x)$ is true. (using direct technique)

Solution:

- Assume $\exists x\,(P(x) \wedge Q(x))$ is true.
- Let a be some element of the domain, that $P(a) \wedge Q(a)$ is true.
- So, $P(a)$ is true and $Q(a)$ is true.
- So, $\exists x\,P(x)$ is true and $\exists x\,Q(x)$ is true.
- Thus, $\exists x\,P(x) \wedge \exists x\,Q(x)$ is true

$$\frac{\exists x\,P(x)}{\therefore\ P(c)}$$

$$\frac{P(a)}{\therefore\ \exists x\,P(x)}$$

Rules of inference (example)

Show the following argument is valid.

Every one in this class has taken a course in computer science.

Ali is a student in this class.

Therefore, Ali has taken a course in computer science.

Solution:

- Determine individual propositional function
  - C(x): x is in this class.
  - S(x): x has taken a course in computer science.

Rules of inference (example)

Show the following argument is valid.

Every one in this class has taken a course in computer science.

Ali is a student in this class.

Therefore, Ali has taken a course in computer science.

Solution:

- Determine premises and the conclusion using C(x) and S(x)

  C(x): x is in this class.

  S(x): x has taken a course in computer science.

  Premises: $\forall x\,(C(x) \rightarrow S(x))$ domain: all students

  $C(Ali)$

  Conclusion: $S(Ali)$

Rules of inference (example)

Show the following argument is valid.

Every one in this class has taken a course in computer science.

Ali is a student in this class.

Therefore, Ali has taken a course in computer science.

Solution:

- Assume premises are true, show the conclusion is true using rules of inference
  1. $\forall x\,(C(x) \rightarrow S(x))$ Premise
  2. $C(Ali) \rightarrow S(Ali)$ Universal instantiation
  3. $C(Ali)$ Premise
  4. $S(Ali)$ Modus ponens

Premises:

$\forall x\, C(x) \rightarrow S(x)$

$C(Ali)$

So, the conclusion is true and the argument is valid.

Rules of inference (example)

Show the following argument is valid.

There is a student such that if he knows programming, then he knows Java.

All students know programming.

Therefore, there is a student that knows either Java or C++.

Solution:

- Determine individual propositional function
  - P(x): x knows programming.
  - J(x): x knows Java.
  - C(x): x knows C++.

Rules of inference (example)

Show the following argument is valid.

There is a student such that if he knows programming, then he knows Java.

All students know programming.

Therefore, there is a student that knows either Java or C++.

Solution:

- Determine premises and the conclusion using P(x), J(x) and C(x)
  - P(x): x knows programming.
  - J(x): x knows Java.
  - C(x): x knows C++.

Premises: $\exists x\, (P(x) \rightarrow J(x))$ domain: all students

$\forall x\, P(x)$

Conclusion: $\exists x\, (J(x) \vee C(x))$

Rules of inference (example)

Show the following argument is valid.

There is a student such that if he knows programming, then he knows Java.

All students know programming.

Therefore, there is a student that knows either Java or C++.

Solution:

- Assume premises are true, show the conclusion is true

1.	$\exists x\,(P(x) \rightarrow J(x))$	Premise
2.	$P(a) \rightarrow J(a)$	Existential instantiation
3.	$\forall x\, P(x)$	Premise
4.	$P(a)$	Universal instantiation
5.	$J(a)$	Modus ponens
6.	$J(a) \vee C(a)$	
7.	$\exists x\,(J(x) \vee C(x))$	Existential generalization

Premises:

$\exists x\,(P(x) \rightarrow J(x))$

$\forall x\, P(x)$

Recommended exercises

2,3,8,9,19,23