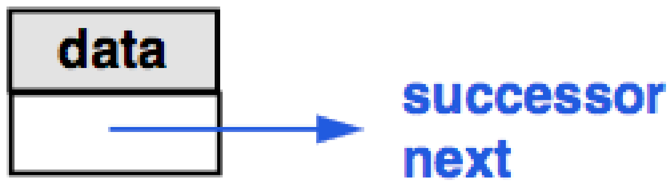
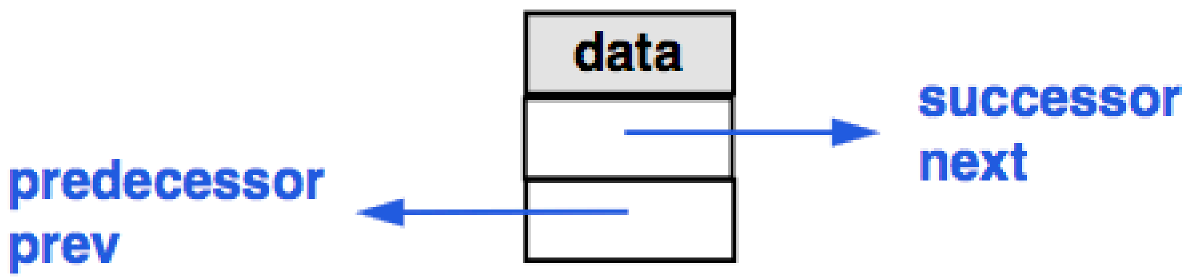
Tree Definitions
&
Types of Trees

On Pointers

- Pointers represent relationships between objects
  - » In a singly linked list they show the successor (next) relationship

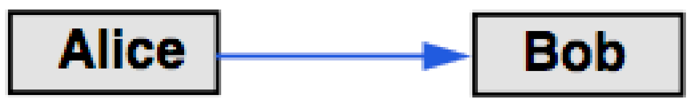


  - » In a doubly linked list one pointer shows the successor relationship and the other pointer shows the predecessor (prev) relationship



On Pointers – 2

- Pointers in singly and doubly linked lists show an ordering relationship
- A pointer can be used to show any binary relationship

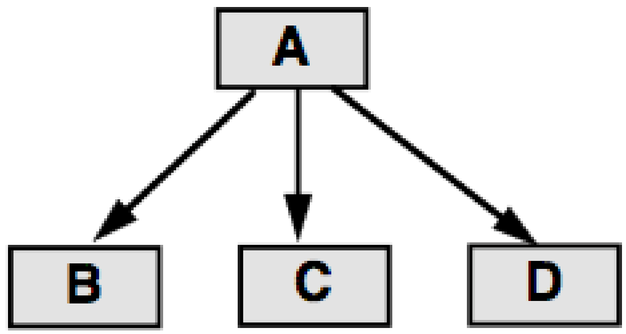


The above can represent

» **Alice is the mother of Bob**
Alice is the mentor of Bob
Alice telephones Bob
Alice emails Bob
etc.

On Pointers – 3

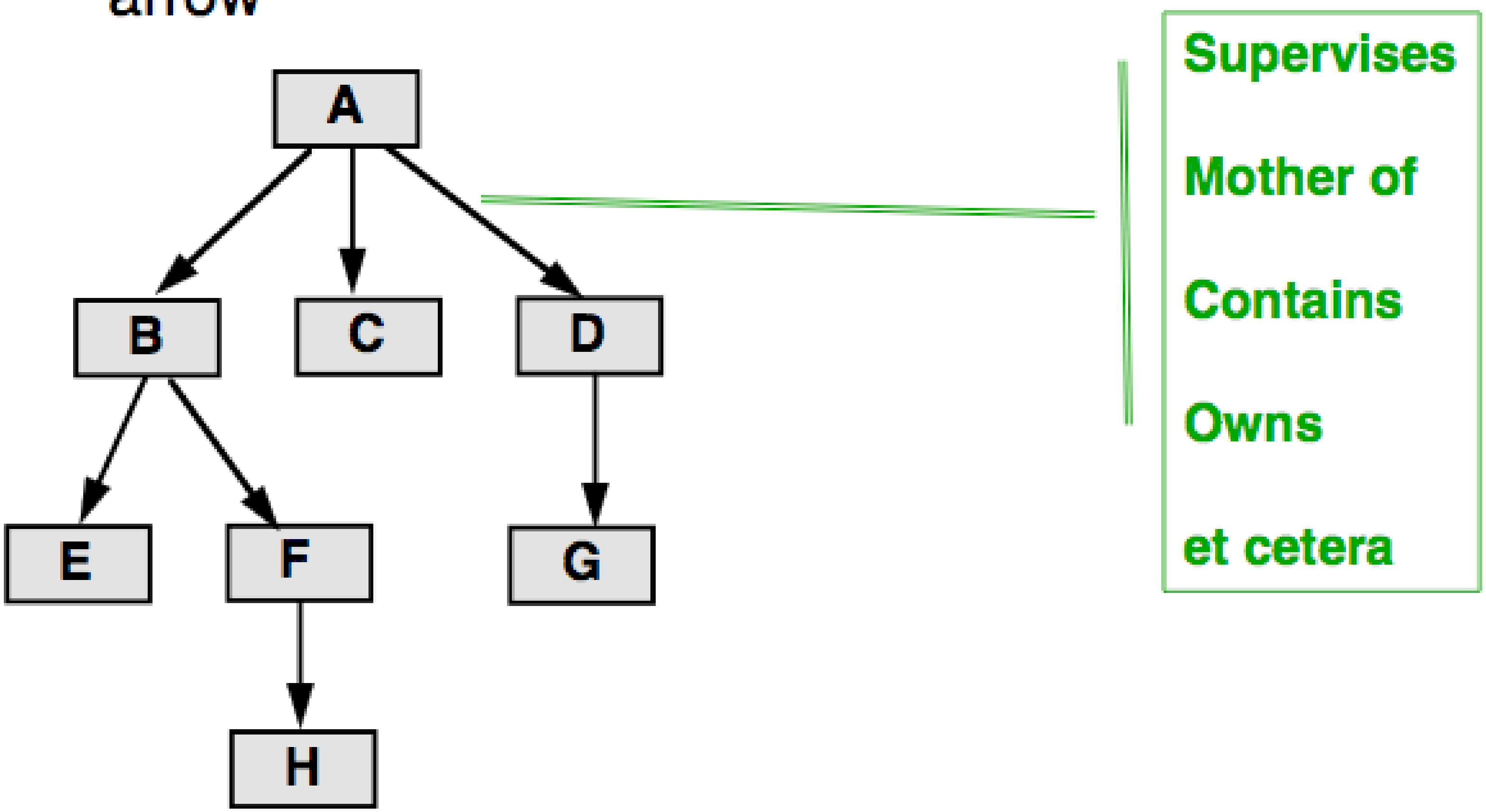
- Objects can have the same relationship with more than one other object



» A is the mother of B, C and D
A is the mentor of B, C and D
A telephones B, C and D
A emails B, C and D
etc.

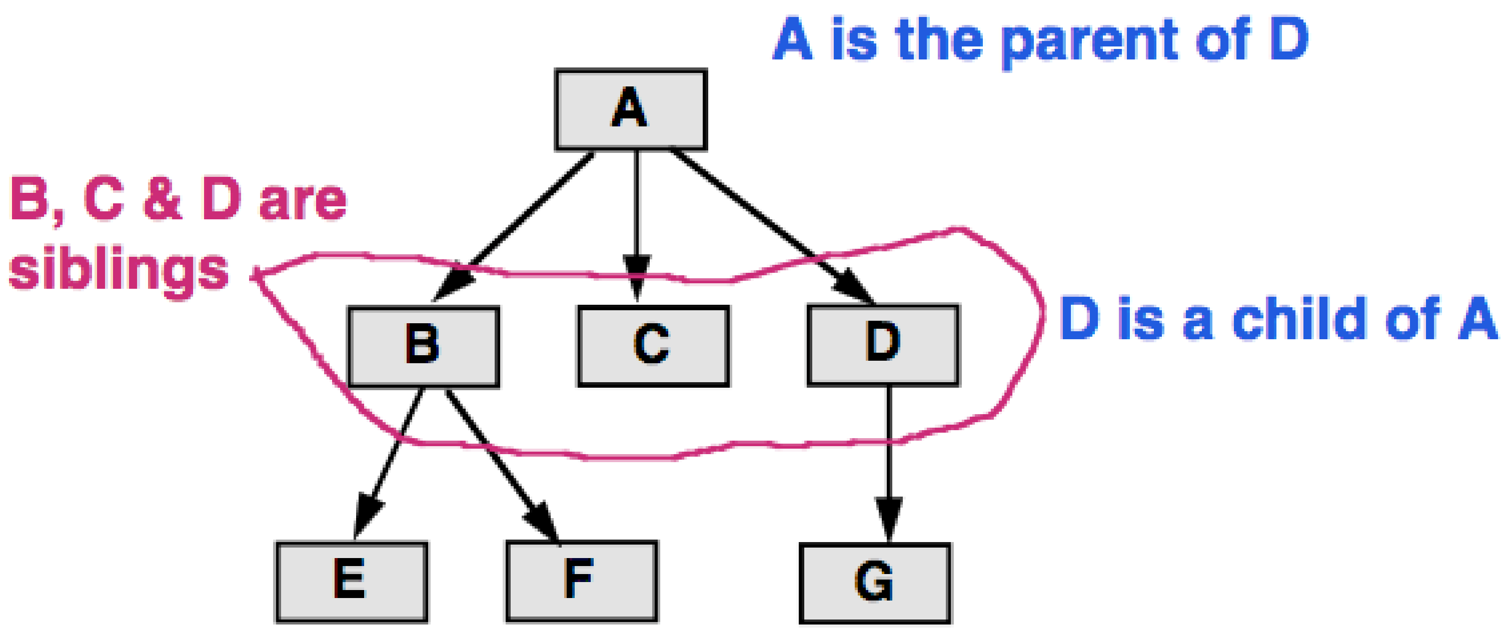
Trees

- Trees show a relationship among a collection of objects (**nodes**), where relationships are one way and only one object (**node**) is at the head of every arrow



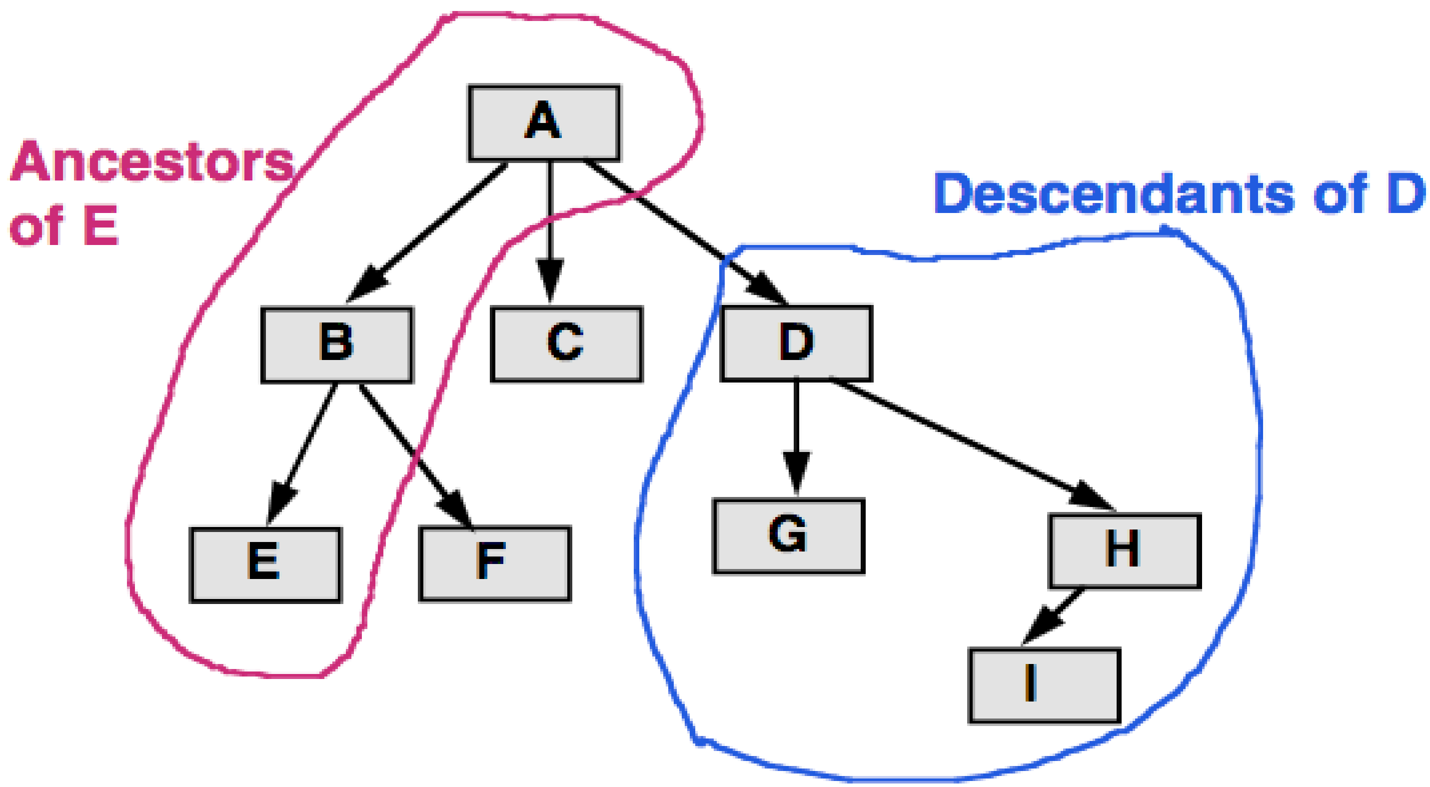
Terminology

- **Parent** – node at the tail of an arrow
- **Child** – node at the head of an arrow
- **Siblings** – nodes with the same parent



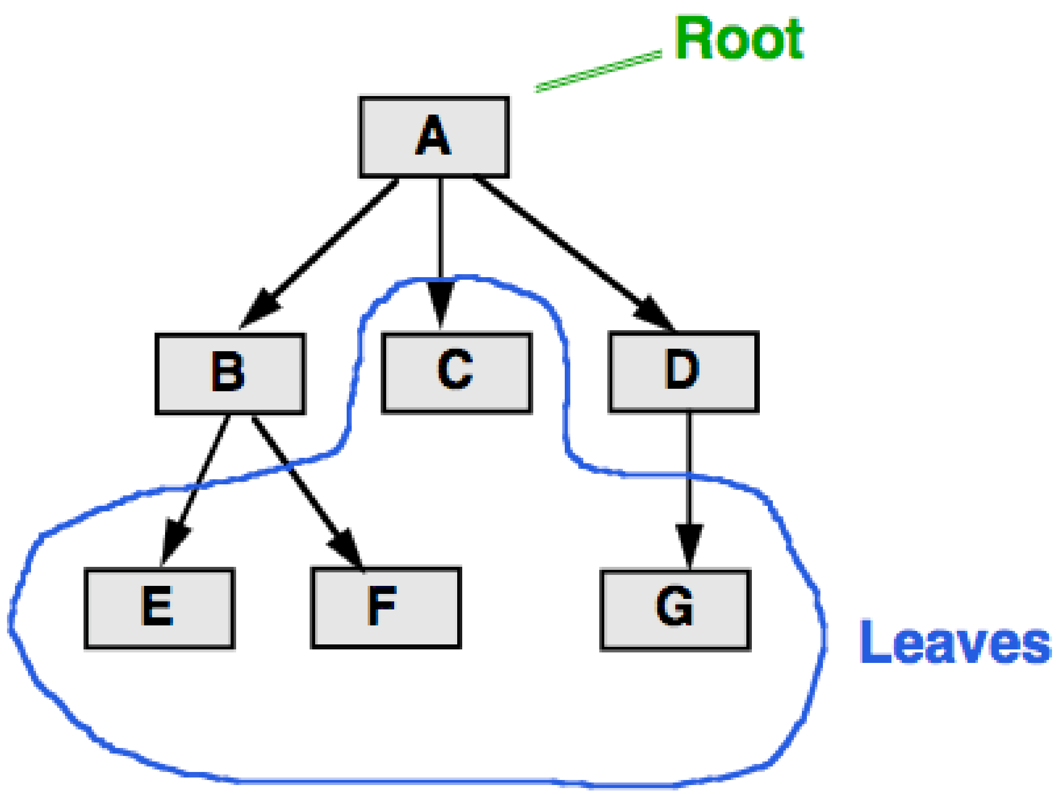
Terminology – 2

- **Ancestor** – the node itself, parent, parent of parent, etc.
- **Descendent** – the node itself, child, child of child, etc.



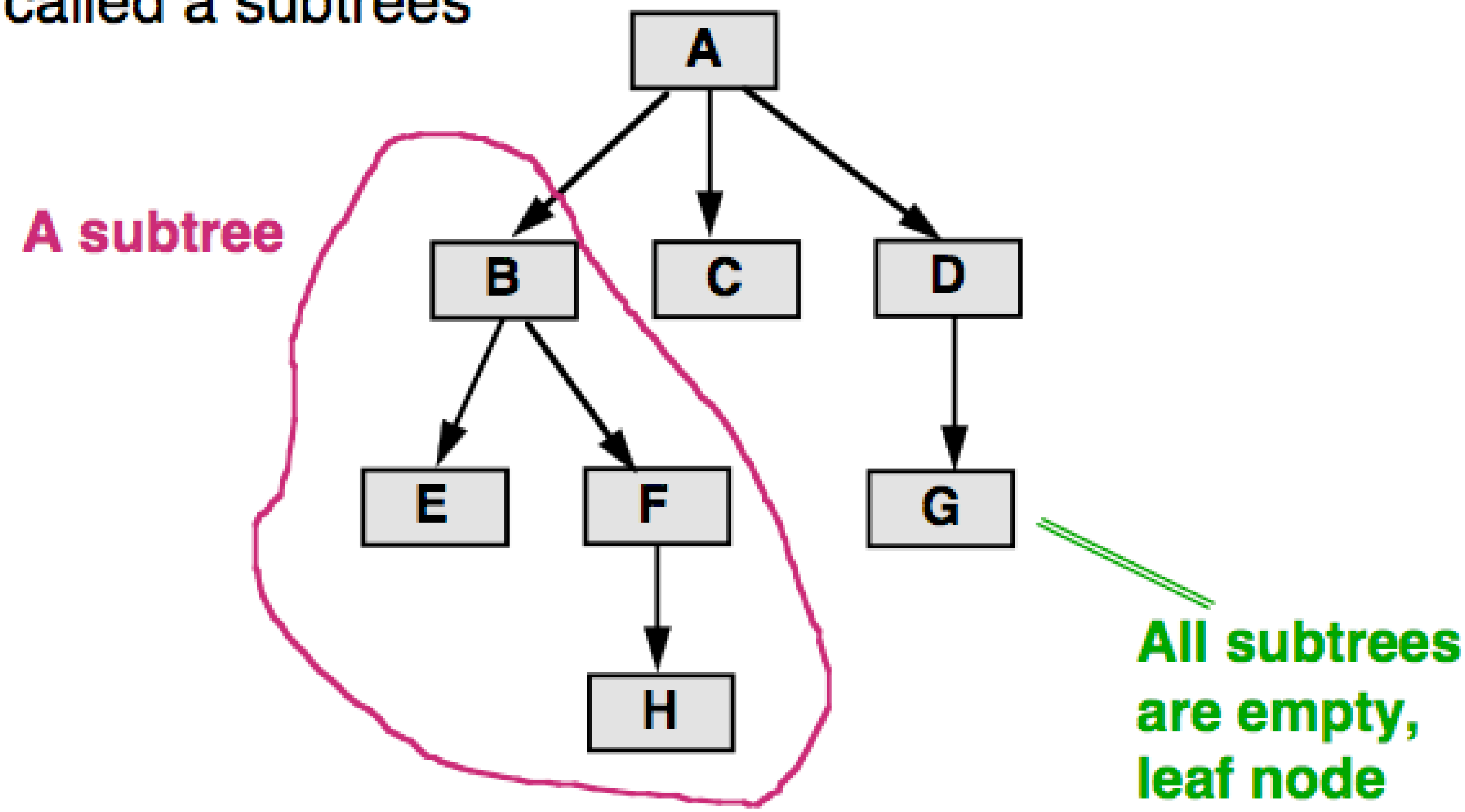
Terminology – 3

- **Root** – node with no parent
- **Leaf** – node with no children – also called **external** (all other nodes are **internal**)



Tree Definition

- An empty tree is a tree
- A node of type T with a finite number of children where each child is a disjoint tree of base type T, called a subtrees

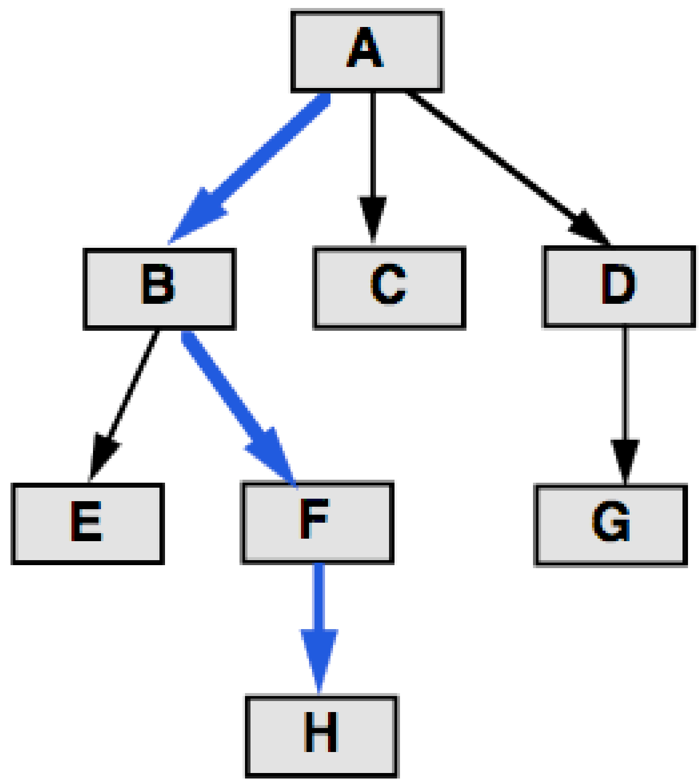


Terminology – 4

- **Path** – the set of edges from the root to a node
- **Path length** – the number of edges in a path

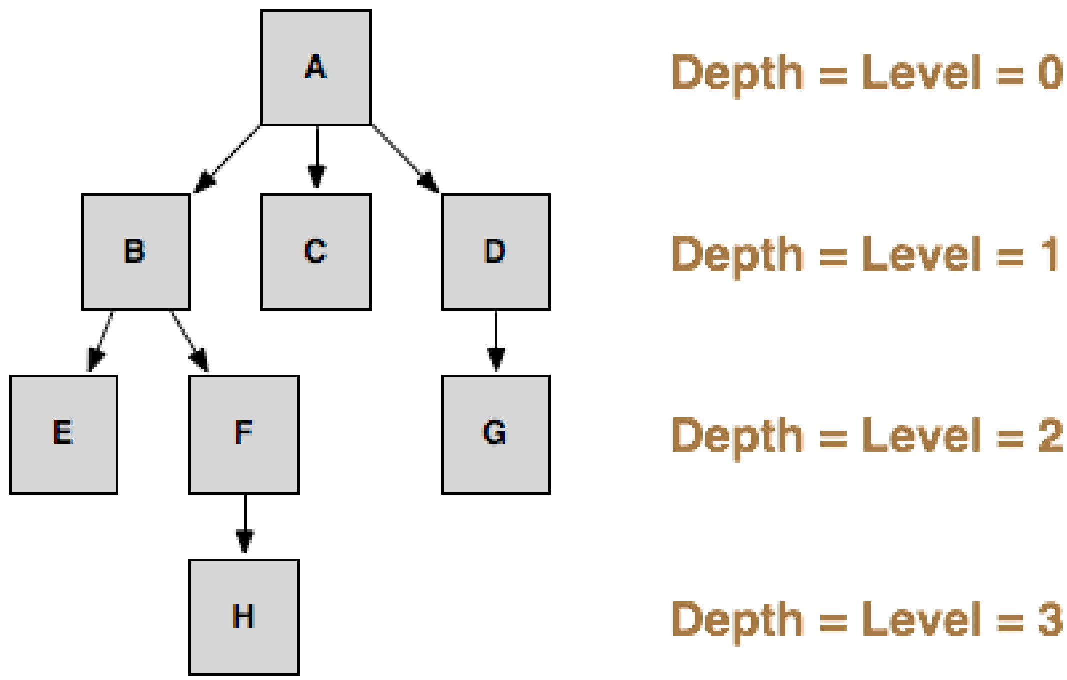
Path from A to H is
<A,B> <B,F> <F,H>

Length is 3



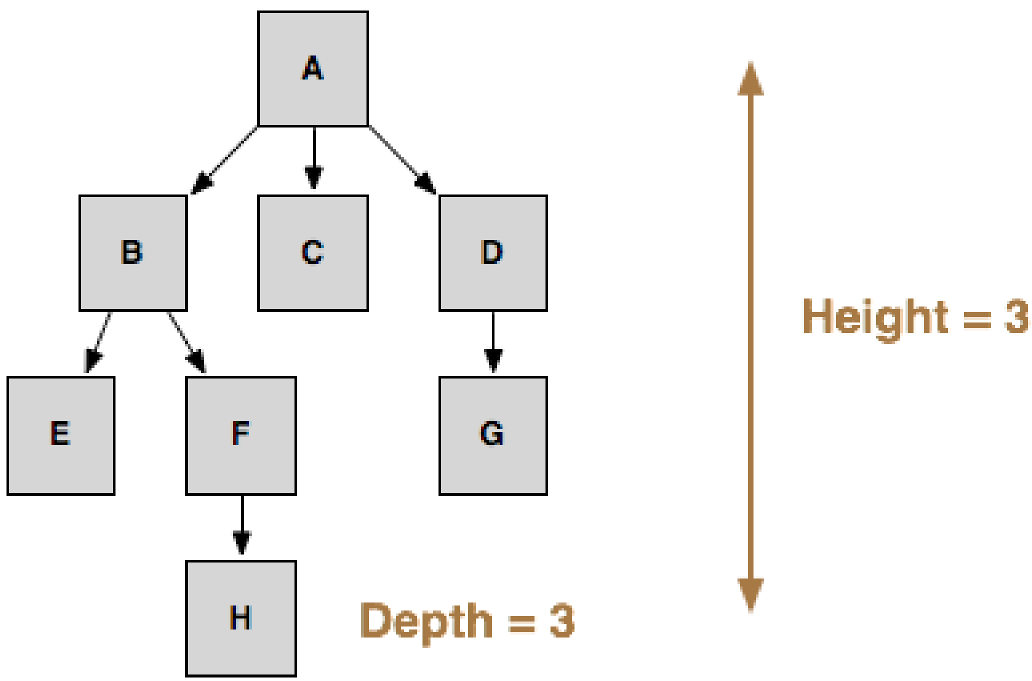
Terminology – 5

- **Node level** & **node depth** – the path length from the root
  - > **The root is level 0 and depth 0**
  - > **Other nodes depth is 1 + depth of parent**



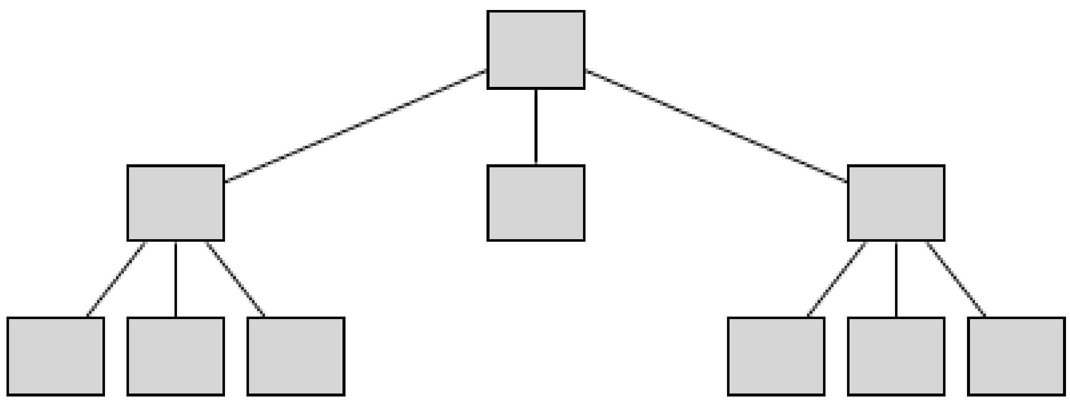
Terminology – 6

- **Height of a tree** – The longest path length from the root to a leaf.
  - > **Non empty tree: Height = max depth**



Terminology – 7

- **Degree** – the maximum number of possible children for every node in the tree
- **Proper tree (full tree)** – Nodes have all children non-void, or all children are void
  - > **A full tree of height 2 and degree 3**

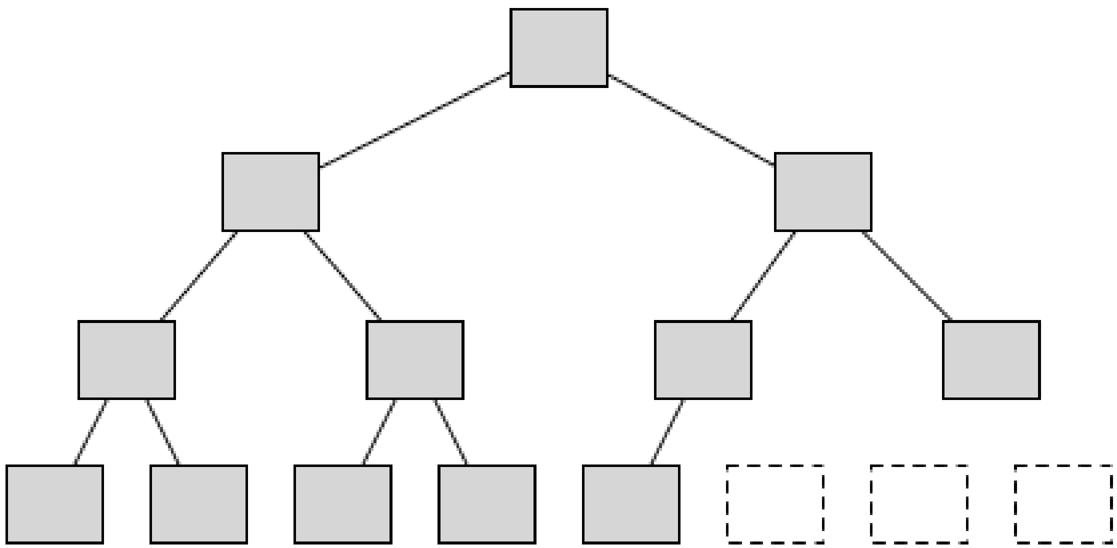


  - > **If some of the nodes have fewer actual children the tree is still of degree 3**

Terminology – 8

- **Complete** – All levels are full except for the deepest level, which is partially filled from the left

  > **A complete binary tree of degree 2**



Terminology – 9

- **Balanced** – Different definitions depending upon type of tree
  - » **Having 1/N of the nodes in each of N children**
  - » **Height of all subtrees within constant K**
    - > **In a binary tree**
      - – **Height(left_subtree) – Height(right_subtree) ≤ K**
  - » **max_level(leafNode) – min_level(leafNode) ≤ K**
    - > **For a complete tree K= 1**
- **Balance** – Redistribute the nodes to restore balance constraint while maintaining the ordering property

Types of Trees

- General tree
  - » **Every node can have any number of sub-trees, there is no maximum**
  - » **Different number is possible of each node**
- N'ary tree
  - » **Every node has at most N sub-trees**
    - > **Special case N= 2 is a binary tree**
    - > **Sub-trees may be empty – pointer is void**

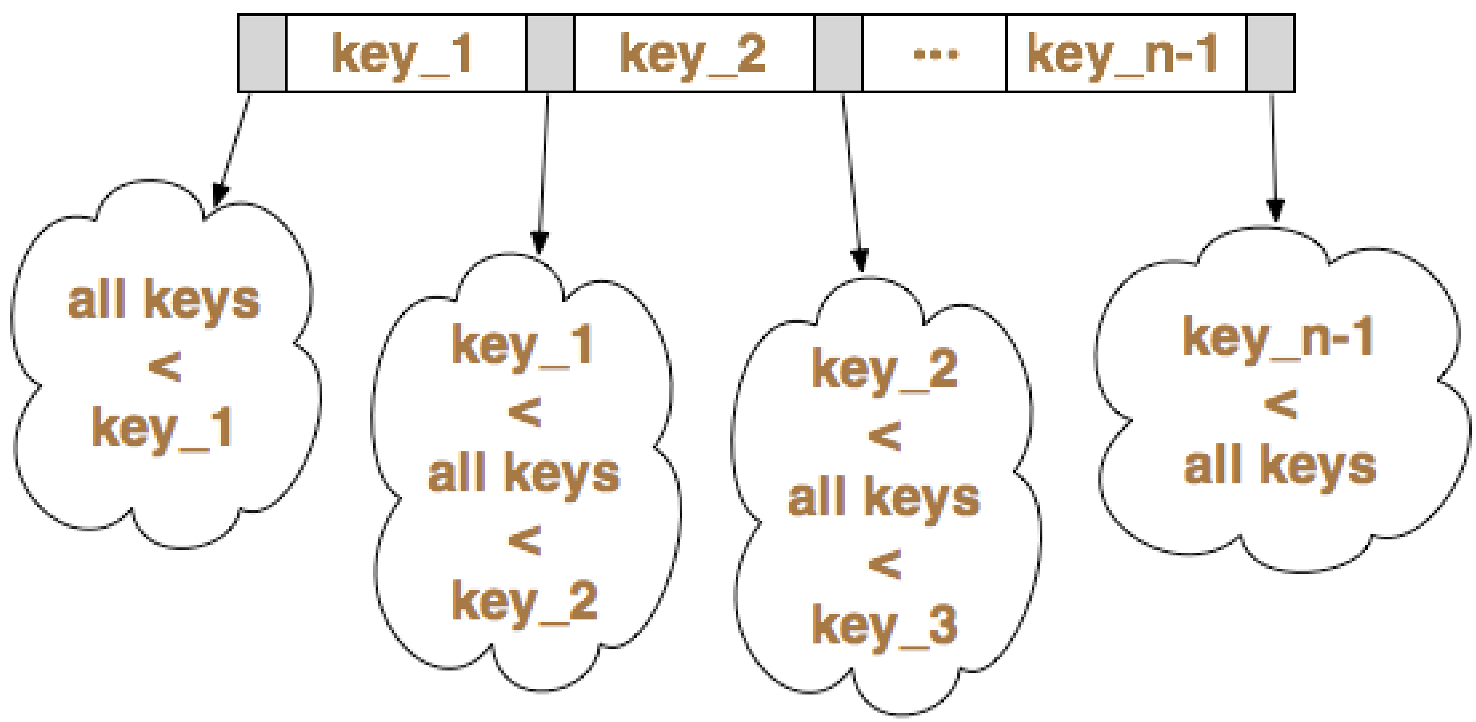
Ordered Trees

- Can be general or N'ary but have one additional constraint
  - » **An ordering is imposed between the parent and its children**
  - » **Each node has one or more keys that are in an order relationship (< ≤ ≥ >) with the keys in its children**
    - > **The same relationship pattern is used throughout for all nodes in the tree**

Ordered Trees – N'ary case

- Each note contains 1 to N–1 keys

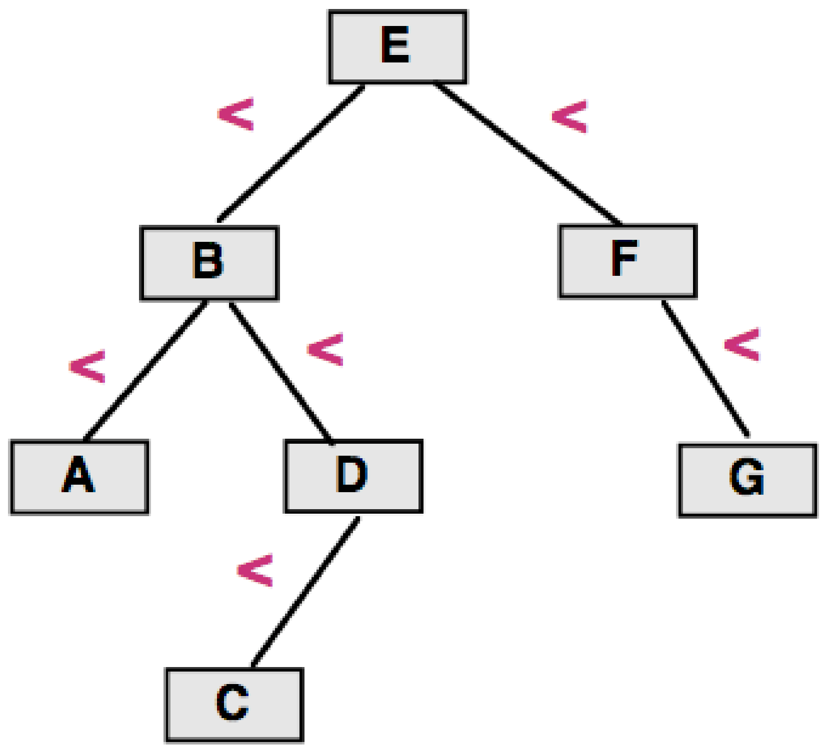
key_1 < key_2 < … < key_n-1



Ordered Trees – Binary Search Tree

- Special case N'ary with N=2
  - » **Complementary relationship of the parent with two children**

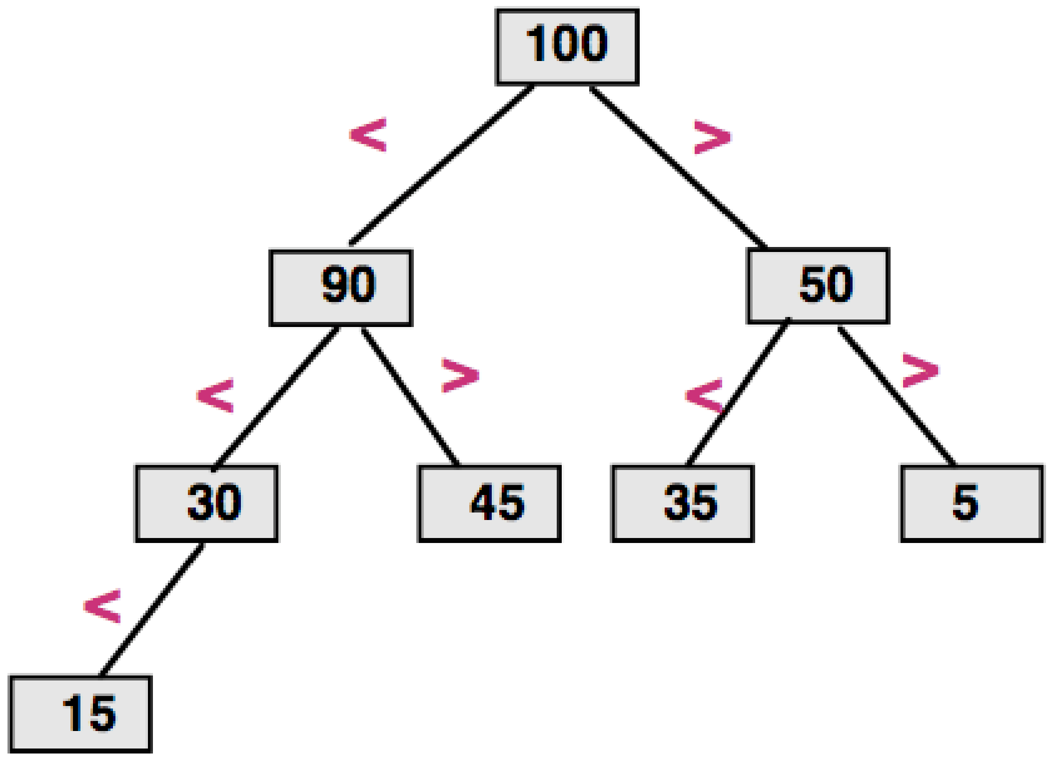
    key_left_child < key_parent < key_right_child



Ordered Trees – Heap

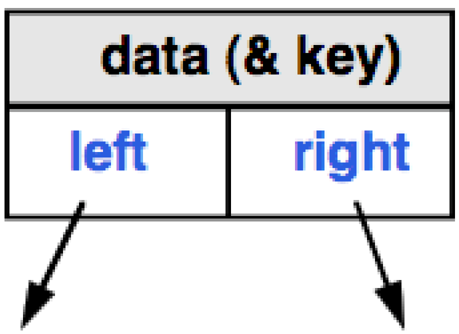
- Special case N'ary with N=2 and a complete binary tree
  - » **Same relationship between the parent and each child**

key_left_child < key_parent > key_right_child



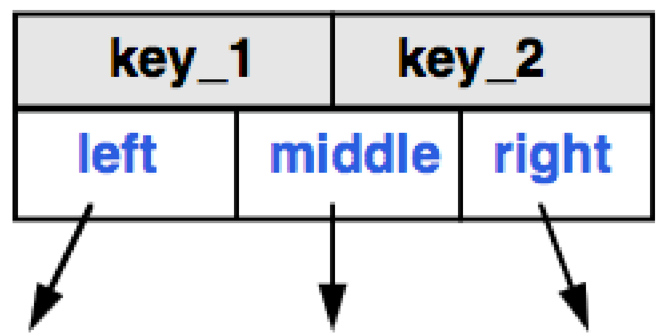
N’ary Tree Nodes

- Unordered & ordered trees – for small N
  - » **Data + specific names for pointer fields**



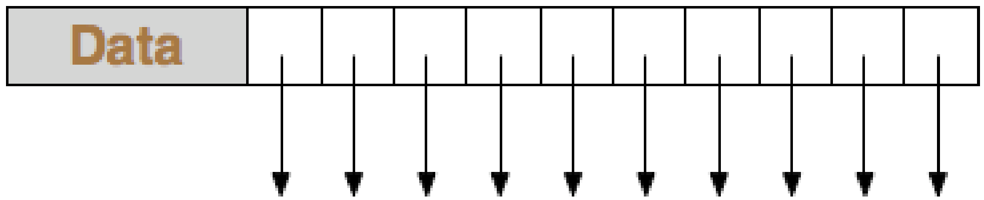
binary tree

ternary tree

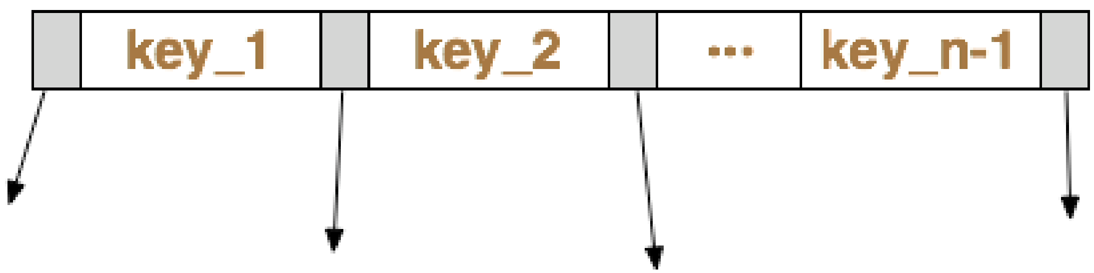


N'ary Tree Nodes – 2

- Unordered trees – large N
  - » **Data + array of pointers**

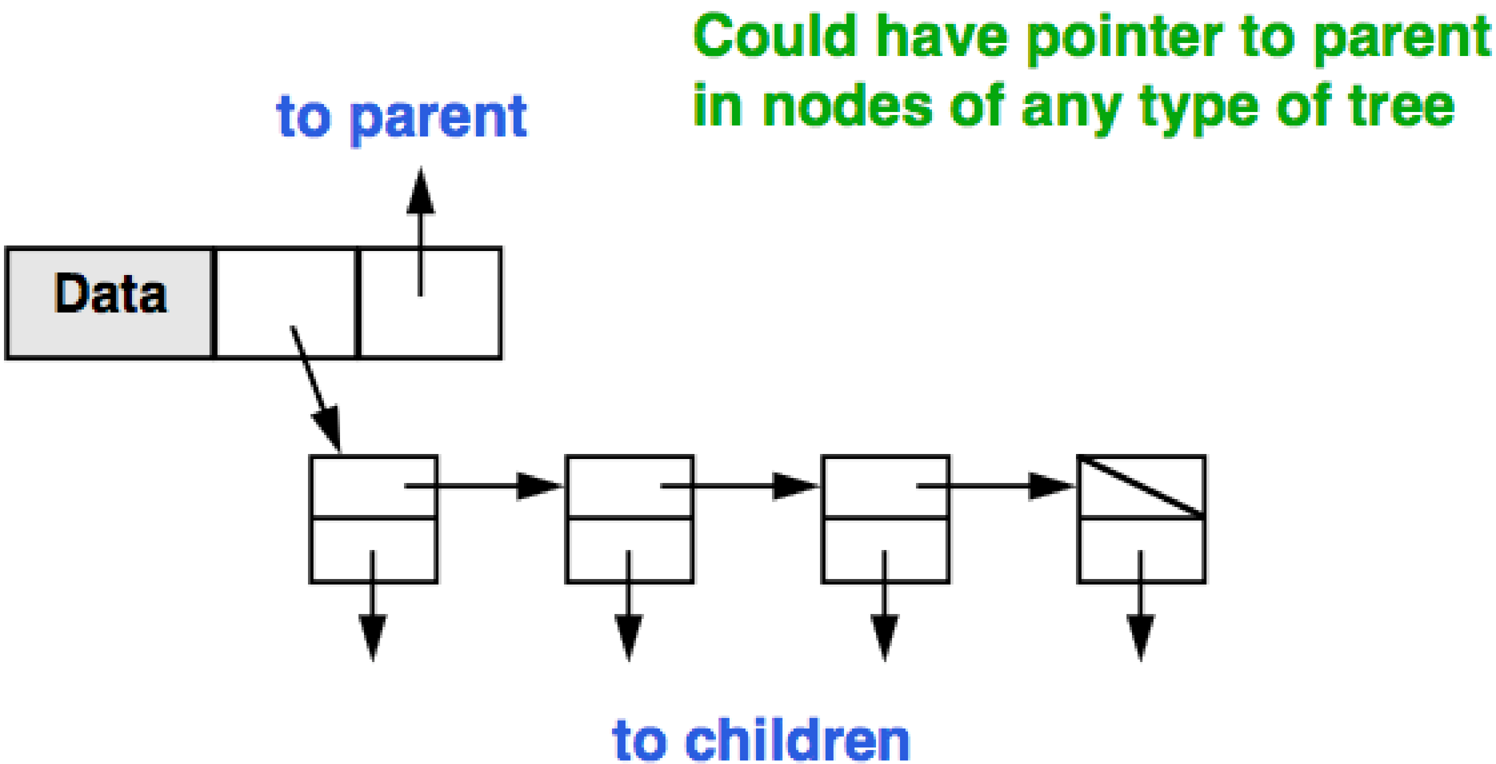


- Ordered trees – large N
  - » **Array of keys and an array of pointers that are logically interspered**



General Tree Nodes

- Use list of pointers – for any number of children



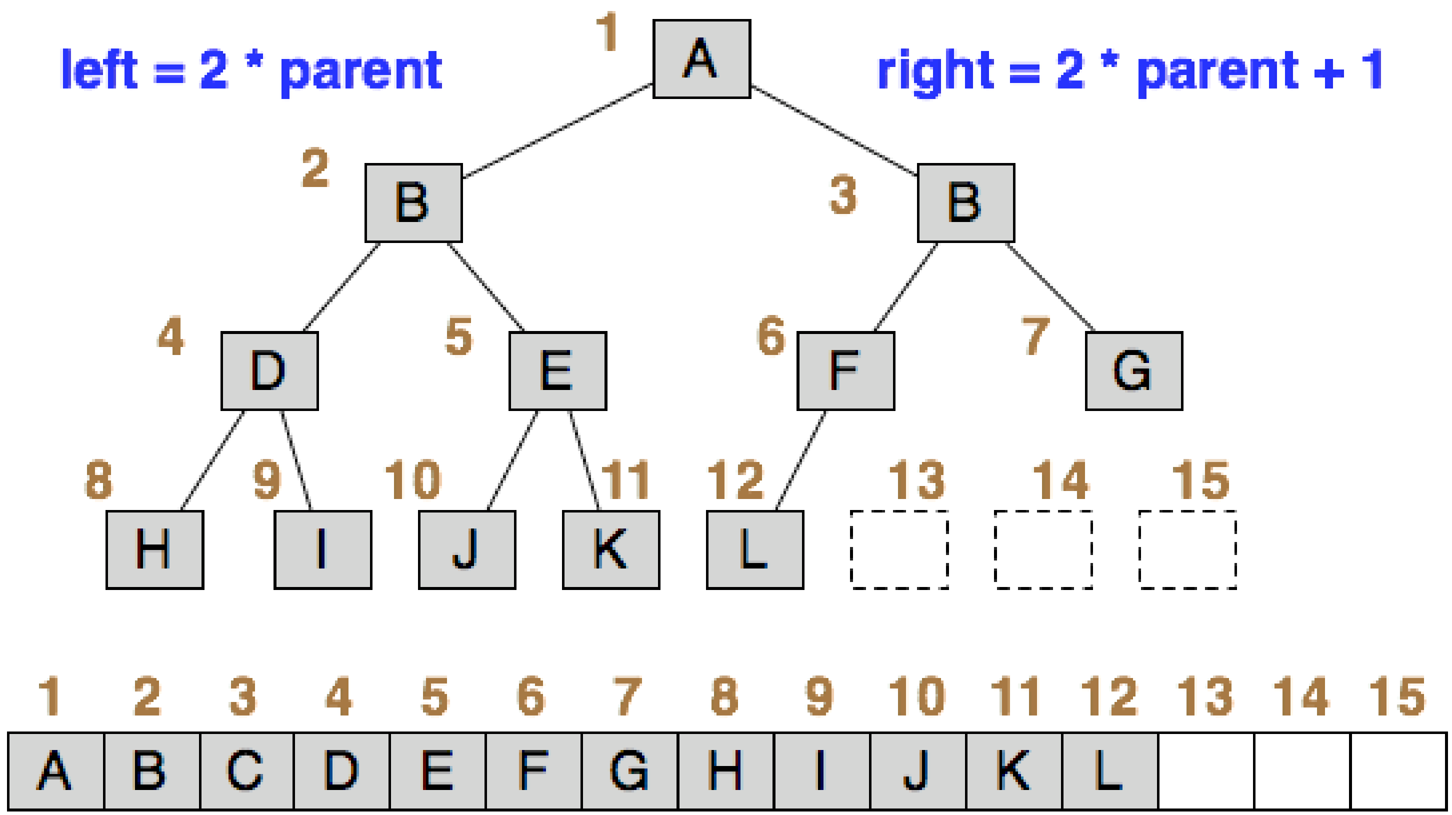
Array Representation of N'ary Trees

- If N'ary trees are complete, then can use arrays to store the data.
  - » **Pointers are indices to the array (addresses relative to the start of the array scaled by the size of a pointer)**
  - » **Use arithmetic to compute where the children are**

- Binary trees are a special case
  - » **Heaps are usully implemented using arrays to represent a complete binary tree**

Array Representation – 2

- Mapping between a complete binary tree and an array



Array Representation – 3

- In general for an N'ary tree the following set of relationships holds – **the root must have index 0**

  child_1 = N * parent + 1
  child_2 = N * parent + 2
  child_3 = N * parent + 3
  …
  child_N = N * parent + N

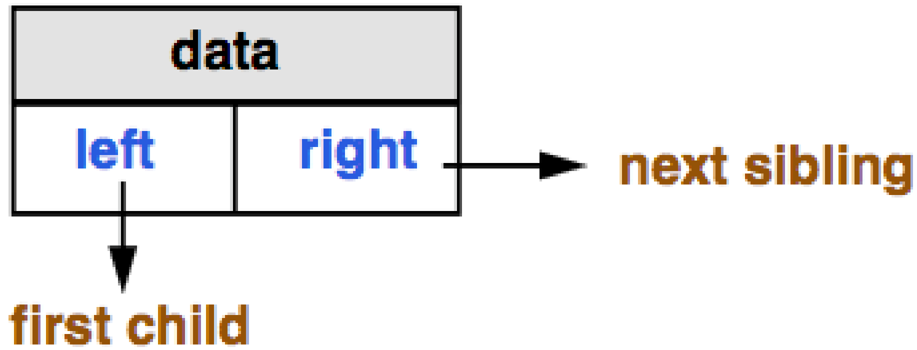
  Nodes are numbered in breadth traversal order

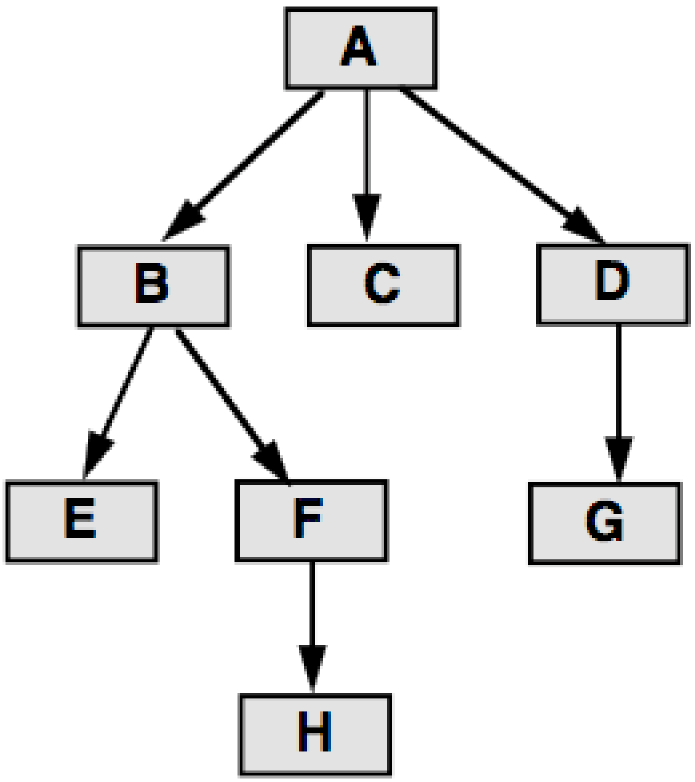
- The binary case is an exception where the root can be index 1 because **2*1 = 2**, the index adjacent to the root
  - » **This gives the pair 2 * parent & 2 * parent + 1, which is less arithmetic than the above, and the inverse to find the parent is easier to compute.**

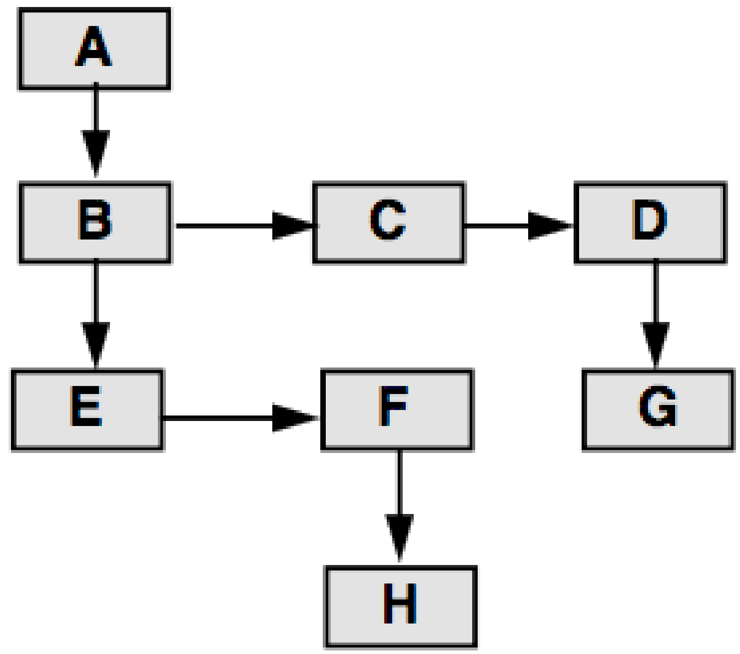
Representing General Trees as Binary Trees

- Binary trees are all that are logically necessary
  - » **Lisp programming language for artificial intelligence applications has binary trees as its fundamental (and in theory only) data structure.**

An Example


data
left
right
next sibling
first child


A
B
C
D
E
F
G
H


A
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