Computer Science 5324.03/4421.03

Midterm Test

Feb. 23 2004

Answer all questions in the space provided

Make sure that you have 9 pages

Student Last Name: ______________________________

Student Given Name: _____________________________

Student Id. No: ____________________________________

Question	Value	Score
1	48	
2	50/60	
TOTAL	98/108	

Question 1. [48 points]

1. [4 points] What is the difference between a rotation matrix and an orthonormal matrix?

2. [4 points] What is a necessary and sufficient condition for rigid motion?

3. [4 points] What is the name of the condition where the motors of a robot for no apparent reason seem to work at their limit while the end-effector (gripper) is just moving at a steady speed?

4. [4 points] What do we do if we want to estimate parameter $\hat{x}$ using N measurements x_i of equal variance.

5. [4 points] Name the two quite different situations where we used Jacobians in this course.

6. [4 points] What is statically stable legged locomotion?

7. [4 points] Name three popular wheel configurations for robots.

8. [4 points] What is the main disadvantage of dead reckoning?

9. [4 points] What is the variance of a sum of two measurements that are statistically independent?

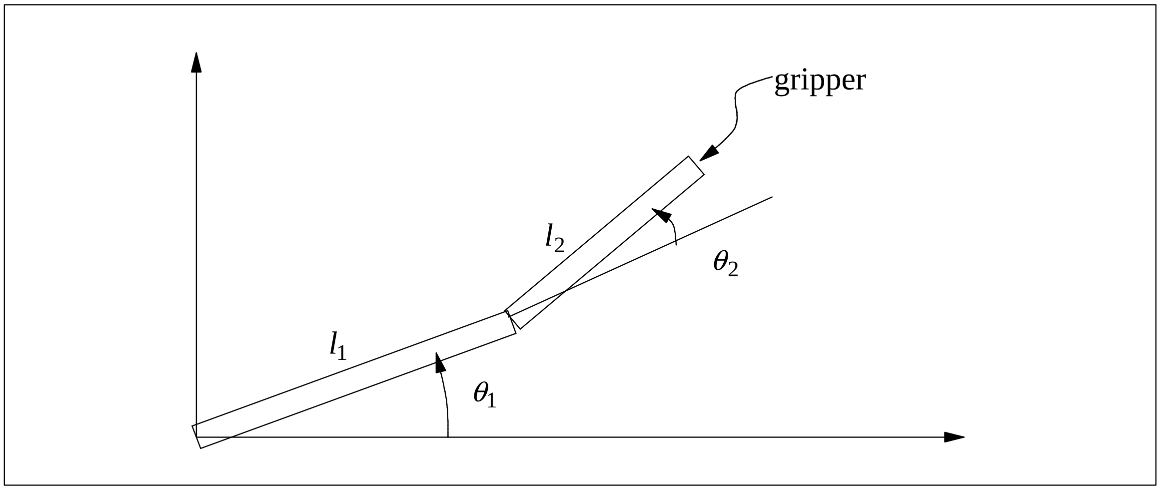
10. [4 points] How many satelites are needed to establish the position of a GPS device (in one shot)?

11. [4 points] What is the main reason that we use the homogeneous coordinates?

12. [4 points] What is the highest ω such that the signal $\cos\omega x + \sin\frac{\omega}{2}x + \sin\frac{\omega}{4}x$ can be represented accurately by discrete samples? Assume that x is sampled at $x = 0, 1, 2, 3 \cdots$ etc.

Question 2. [50/60 points]

1. [10 points] Someone bought a used robot without shaft encoders. The robot has only two degrees of freedom and link lengths l_1 and l_2. So he decided to use a Kalman filter as follows. He built a computer vision system that gave him the approximate coordinates of the end effector (gripper) as x_k and y_k with a covariance matrix C_w. The robot was controlled by inputs $\delta\theta_1$ and $\delta\theta_2$, in a stop and go fashion. Due to the lack of shaft encoders, when the robot was instructed to move by $\delta\theta_1$, $\delta\theta_2$ was only moving approximately with error of zero mean and covariance C_v. Write the state transition equation (plant equation) and the measurement equation. You do not need to write the Jacobians if any.



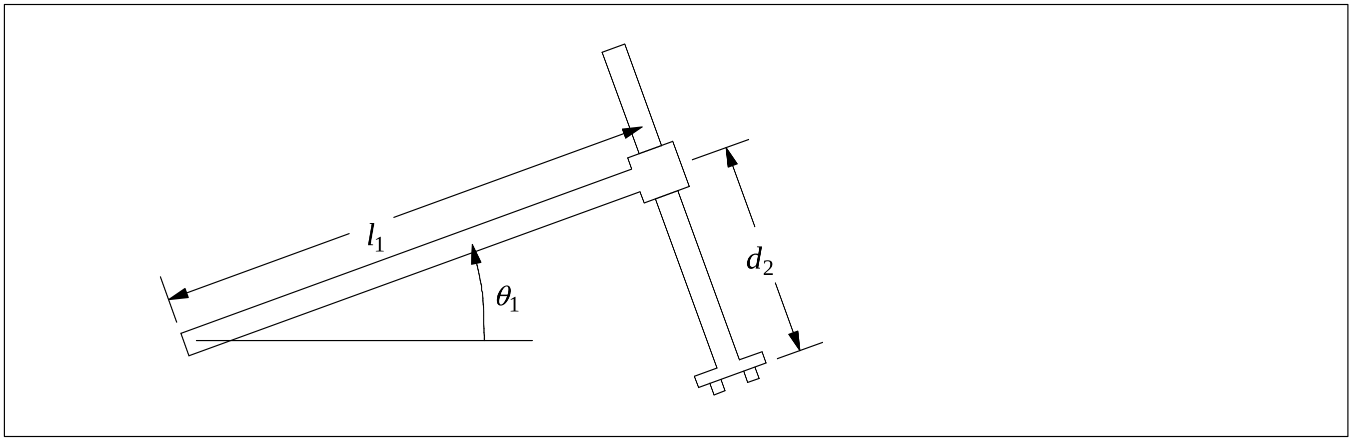
2. [15 points] A robot has the following forward kinematics:

$$x = d_1 + l_2 c_2$$

$$y = l_2 s_2$$

where the joint variables are d_1 and θ_2. Write the Jacobian and find the singularities.

2. [15 points] A robot has one rotary joint and a telescopic joint as depicted in the figure. Write the forward and the inverse kinematics.



4. [10 points] **GRADS** Two random variables x_1 and x_2 have means μ_1 and μ_2, variances σ_1^2 and σ_2^2 and covariance σ_{12}. What is the expected value of their product?

5. [10 points] We have a camera with focal length 1 cm, that has a CCD sensor with 100×100 pixels and physical size 1 cm by 1 cm. In front of it we place a white paper that has black vertical lines. There are 10 lines per cm, each line being 0.05cm wide separated by 0.05cm of white space. How far away from the lens of the camera can we put the paper before we hit the Nyquist limit?

$$\mathbf{x}_{k+1} = \Phi_k \mathbf{x}_k + \Gamma_k \mathbf{u}_k + \mathbf{v}_k$$

$$\mathbf{z}_{k+1} = \Lambda_{k+1} \mathbf{x}_{k+1} + b w_{k+1}.$$

$$\hat{\mathbf{x}}_{k+1}^{k} = \Phi_k \hat{\mathbf{x}}_k^k + \Gamma_k \mathbf{u}_k$$

$$P_{k+1}^{k} = \Phi_k P_k^k \Phi_k^T + C_{v,k}$$

$$K_{k+1} = P_{k+1}^{k} \Lambda_{k+1}^T \left(\Lambda_{k+1} P_{k+1}^{k} \Lambda_{k+1}^T + C_{w,k+1} \right)^{-1}$$

$$P_{k+1}^{k+1} = P_{k+1}^{k} - K_{k+1} \Lambda_{k+1} P_{k+1}^{k}$$

$$\hat{\mathbf{x}}_{k+1}^{k+1} = \hat{\mathbf{x}}_{k+1}^{k} + K_{k+1} \left(\mathbf{z}_{k+1} - \Lambda_{k+1} \hat{\mathbf{x}}_{k+1}^{k} \right)$$

$$\cos(\theta_1 + \theta_2) = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$$

$$\sin(\theta_1 + \theta_2) = \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2$$