Final
April 162004

# Answer all questions in the space provided 

Make sure that you have 8 pages

Student Last Name: $\qquad$
Student Given Name: $\qquad$
Student Id. No: $\qquad$

| Question | Value | Score |
| :---: | ---: | ---: |
| 1 | 60 |  |
| 2 | $40 / 50$ |  |
| TOTAL | $100 / 110$ |  |

Question 1. [ 60 points]

1. [4 points] In what cases do we use the Extended Kalman Filter?
2. [4 points] In which of the two steps of the Kalman filter does the uncertainty decrease?
3. [4 points] What is the fundamental assumption used in most optical flow algorithms?
4. [4 points] Name one advantage of the correlation method versus straight sum of squared differences for pattern matching.
5. [4 points] If a robot can see two landmarks and the angle between the lines of sight is $\phi$, what do we know about the 2-D position of the robot? (we know the positions of the landmarks)
6. [4 points] What is the Markov property?
7. [4 points] Name three data structures that are commonly used to represent multidimensional space.
8. [4 points] What problem does the Non-Maximum-Suppression solve?
9. [4 points] What is the main disadvantage with the potential fields approach where every obstacle has an electric charge similar to the robot?
10. [4 points] What is the bug algorithm?
11. [4 points] What is "static stability" for a limbed robot?
12. [4 points] How many pairs of 3-D points and their images do we need to compute the projection matrix.
13. [4 points] What is the most popular mathematical tool for the analysis of convolutions?
14. [ 4 points] How many edges does a tangent graph have?
15. [4 points] Name the three components of Bayes' rule

$$
P(W \mid R)=\frac{P(R \mid W) P(W)}{P(R)}
$$

Question 2. [80/95 points]

1. [10 points] A robotic kangaroo in a discrete two dimensional world is in position [3,3] with probability 0.5 and in position [3, 4] with probability 0.5 as well. The kangaroo jumps 3 places to the right and lands on the intended place with probability 0.2 and with probability 0.1 lands on one of the eight neighboring places. What is the probability distribution of its position after the jump?

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
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2. [10 points] The above kangaroo is in the same initial position with the same probabilities and fires his omnidirectional sonar and the response says that he is 2 tiles away from the nearest wall. The sonar is accurate with probability 0.5 and the rest it undershoots or overshoots by one place with probability 0.25 . Mark the likelihood based on the measurement on the map below as well the probabilities of the robot being there. (The wall is the thicker line on the map).

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
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## 3. [10 points]

Assume that we have two images and they both contain the projections of a set of points which are $P_{i}$ for $i=1 . . N$ in the coordinate system of the one camera and $P_{i}^{\prime}$ for $i=1 . . N$ in the coordinate system of the other camera. Assume that we know the distance (e.g. the $Z$ component) of the points in the coordinate system of the first camera but not the second. We also do not know the relative position of the two cameras.
(1) What is the minimum number of points to solve for all the unknowns.
(2) Eliminate the unkown $Z_{i}$ from each point.
(3) If we want to "linearize" the problem and treat every element of the rotation matrix as an unknown, how many equations do we need?

## 4. [10 points]

A robot detects a landmark in the direction $\hat{p}$ (unit vector) with respect to its coordinate system. If another instrument informs the robot that the landmark is in position $P$, then find the distance $\lambda$ of the landmark such that $\lambda \hat{p}-P$ is as small as posible.
5. [10 points] [GRADS] A 2-D rotation matrix is a function of a single independent parameter. A 3-D rotation matrix is a function of two such parameters. A 4-D rotation matrix is a function of how many such parameters? [Hint 1: Do not try to visualize it.] [Hint 2: Start from the fundamental property of the rotation matrices and count the constraints]

