Chapter 2

Chapter Goals

- **Know the different types of numbers**
- **Describe positional notation**
- **Convert numbers in other bases to base 10**
- **Convert base 10 numbers into numbers of other bases**
- **Describe the relationship between bases 2, 8, and 16**
- **Explain computing and bases that are powers of 2**

Numbers

Natural Numbers

Zero and any number obtained by repeatedly adding one to it.

Examples: 100, 0, 45645, 32

Negative Numbers

A value less than 0, with a – sign

Examples: -24, -1, -45645, -32

Numbers

Integers

A natural number, a negative number, zero

Examples: 249, 0, - 45645, - 32

Rational Numbers

An integer or the quotient of two integers

Examples: -249, -1, 0, 3/7, -2/5

Natural Numbers

How many ones are there in 642?

600 + 40 + 2 ?

Or is it

384 + 32 + 2 ?

Or maybe…

1536 + 64 + 2 ?

Natural Numbers

Aha!

642 is 600 + 40 + 2 in **BASE 10**

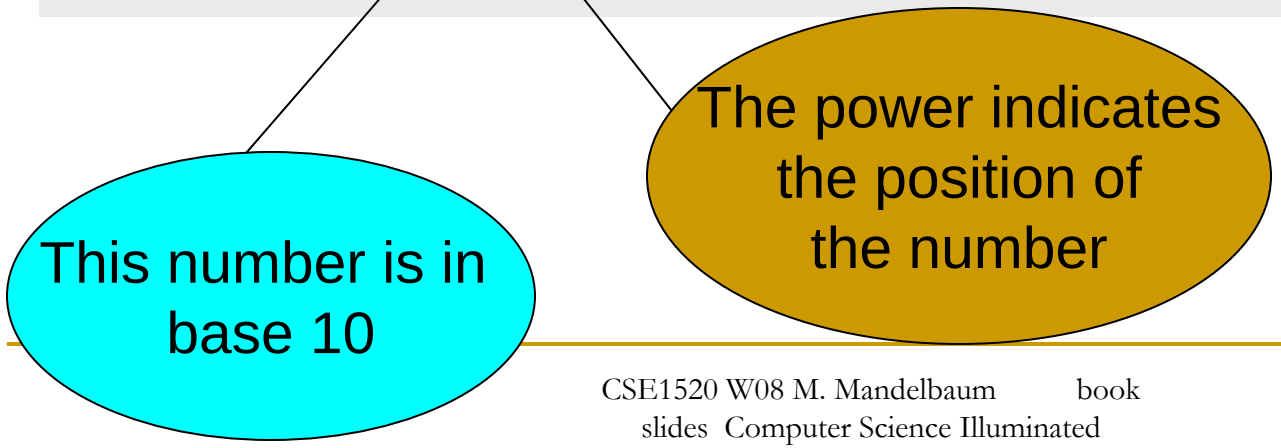
The **base** of a number determines the number of digits and the value of digit positions

Positional Notation

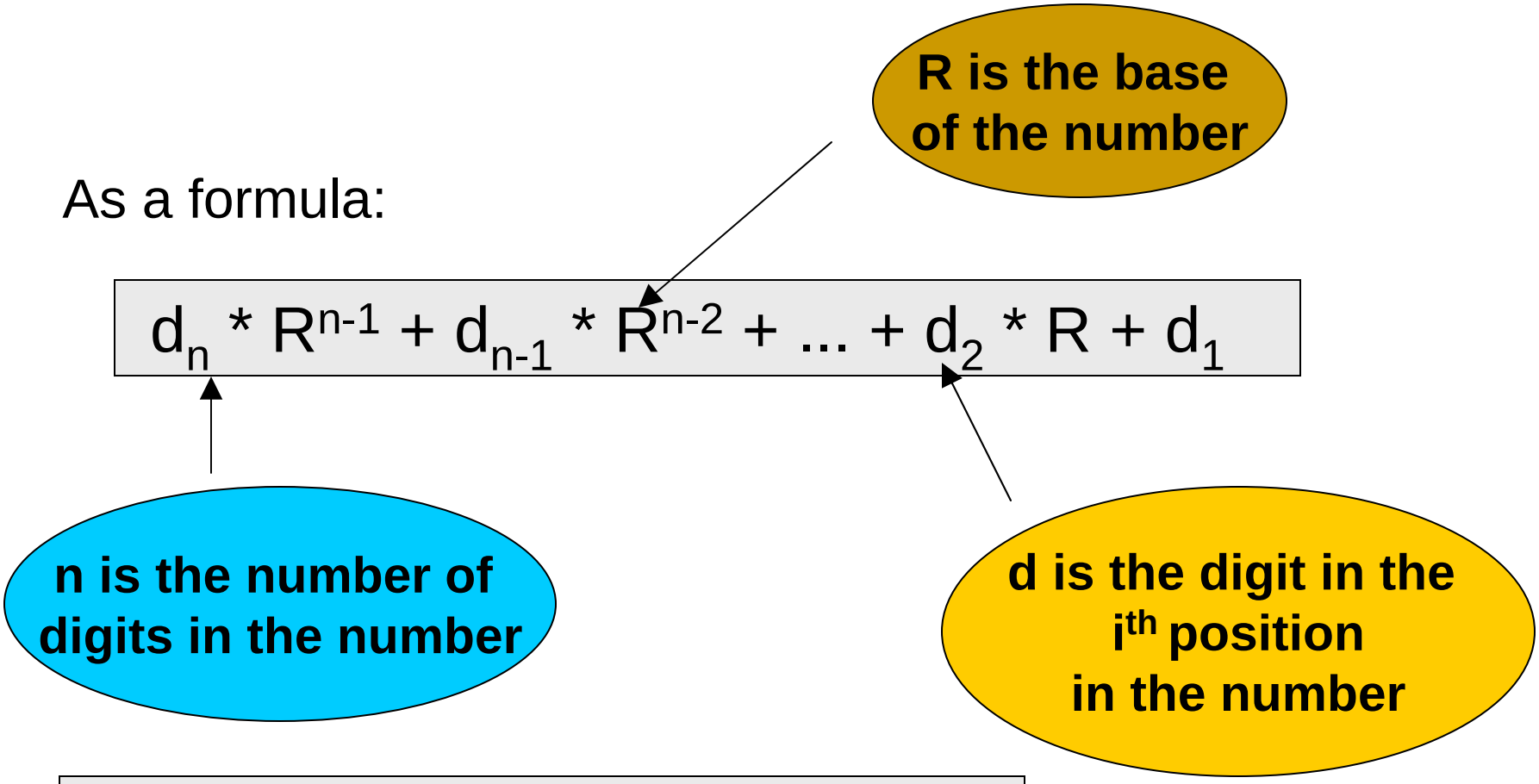
Continuing with our example…

642 in base 10 *positional notation* is:

$$6 \times 10^2 = 6 \times 100 = 600$$
$$+ \; 4 \times 10^1 = 4 \times 10 = 40$$
$$+ \; 2 \times 10^0 = 2 \times 1 = 2 = 642 \text{ in base } 10$$



Positional Notation



642 is $6_3 * 10^2 + 4_2 * 10 + 2_1$

Positional Notation

What if 642 has the base of 13?

$$+ 6 \times 13^2 = 6 \times 169 = 1014$$
$$+ 4 \times 13^1 = 4 \times 13 = 52$$
$$+ 2 \times 13^0 = 2 \times 1 = 2$$
$$= 1068 \text{ in base } 10$$

642 in base 13 is equivalent to 1068 in base 10

Binary

Decimal is base 10 and has 10 digits:

0,1,2,3,4,5,6,7,8,9

Binary is base 2 and has 2 digits:

0,1

For a number to exist in a given number system, the number system must include those digits. For example, the number 284 only exists in base 9 and higher.

Bases Higher than 10

How are digits in bases higher than 10 represented?

With distinct symbols for 10 and above.

Base 16 has 16 digits:
0,1,2,3,4,5,6,7,8,9,A,B,C,D,E, and F

Converting Octal to Decimal

What is the decimal equivalent of the octal number 642?

$$
\begin{aligned}
6 \times 8^2 &= 6 \times 64 &&= 384 \\
+\ 4 \times 8^1 &= 4 \times 8 &&= 32 \\
+\ 2 \times 8^0 &= 2 \times 1 &&= 2 \\
&&&= 418 \text{ in base } 10
\end{aligned}
$$

Converting Hexadecimal to Decimal

What is the decimal equivalent of the hexadecimal number DEF?

$$
\begin{aligned}
D \times 16^2 &= 13 \times 256 = 3328 \\
+\ E \times 16^1 &= 14 \times 16 = 224 \\
+\ F \times 16^0 &= 15 \times 1 = 15 \\
&= 3567 \text{ in base } 10
\end{aligned}
$$

Remember, the digits in base 16 are 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Converting Binary to Decimal

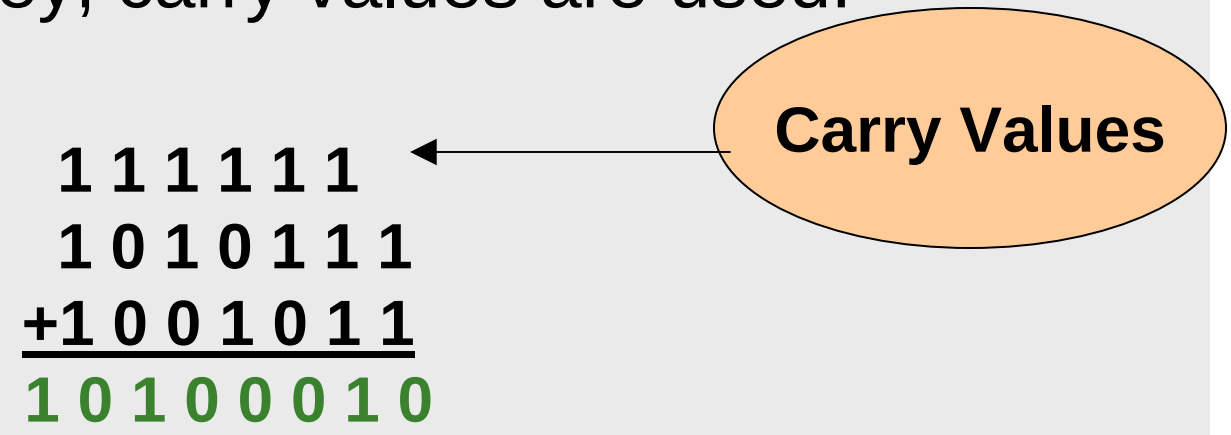
What is the decimal equivalent of the binary number 1011110?

$$
\begin{aligned}
&1 \times 2^6 = 1 \times 64 = 64 \\
+\ &1 \times 2^5 = 1 \times 32 = 32 \\
+\ &0 \times 2^4 = 0 \times 16 = 0 \\
+\ &1 \times 2^3 = 1 \times 8 = 8 \\
+\ &1 \times 2^2 = 1 \times 4 = 4 \\
+\ &1 \times 2^1 = 1 \times 2 = 2 \\
+\ &0 \times 2^0 = 0 \times 1 = 0 \\
&= 110 \text{ in base } 10
\end{aligned}
$$

Arithmetic in Binary

Remember that there are only 2 digits in binary, 0 and 1

Position is key, carry values are used:



Subtracting Binary Numbers

Remember borrowing? **Apply that concept here:**

```
   1 2
   2 0 2
 1 0 1 0 1 1 1
-  1 1 1 0 1 1
 0 0 1 1 1 0 0
```

Power of 2 Number System

Binary	Octal	Decimal
000	0	0
001	1	1
010	2	2
011	3	3
100	4	4
101	5	5
110	6	6
111	7	7
100	10	8
1001	11	9
1010	12	10

Converting Binary to Octal

- Groups of Three (from right)
- Convert each group

10101011 **10 101 011**

2 5 3

10101011 is 253 in base 8

Converting Binary to Hexadecimal

- Groups of Four (from right)
- Convert each group

10101011 **1010 1011**
A B

10101011 is AB in base 16

Converting Decimal to Other Bases

Algorithm for converting base 10 to other bases

While the quotient is ***not*** zero

Divide the decimal number by the new base

Make the remainder the next digit to the left in the answer

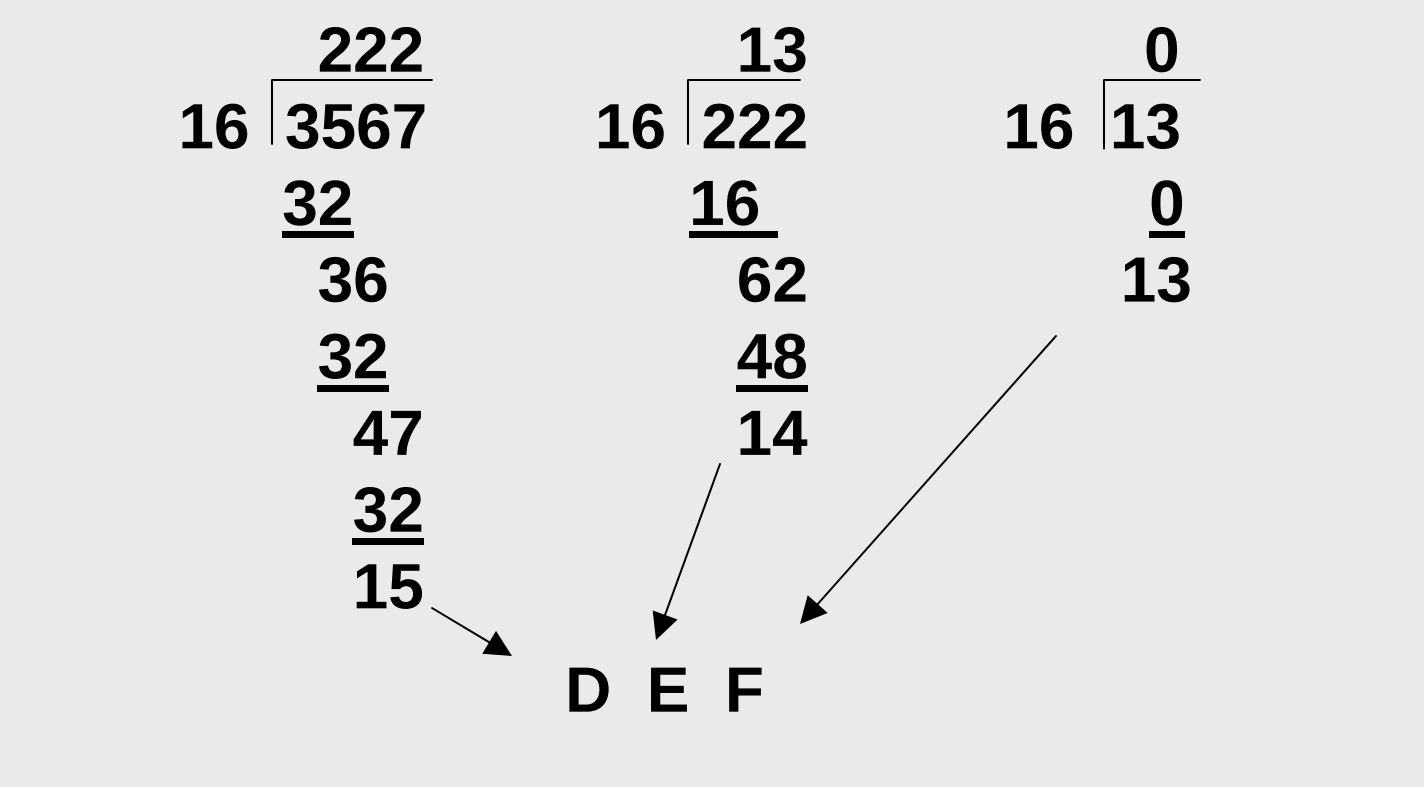
Replace the original dividend with the quotient

Converting Decimal to Hexadecimal

Try a Conversion

The base 10 number 3567 is what number in base 16?

Converting Decimal to Hexadecimal



Binary and Computers

Binary computers have storage units called binary digits or bits

Low Voltage = 0
High Voltage = 1 all bits have 0 or 1

Binary and Computers

Byte 8 bits

The number of bits in a word determines the word length of the computer, but it is usually a multiple of 8

- 32-bit machines
- 64-bit machines etc.