In Chapter 4, we covered a wide range of digital modulation schemes including Pulse Shift Keying (PSK), Amplitude Shift Keying (ASK), Phase Shift Keying (PSK), and Amplitude Phase Keying (APK). We also covered coherent (or synchronous) detection of PSK, where the phase and frequency of the carrier signal is known precisely. Please cover the coherent detection for FSK (section 4.4.4), which was not covered explicitly in the class. Noncoherent detection schemes do not require exact knowledge of the phase and the frequency of the carrier to demodulate the transmitted signal. The coverage of the noncoherent detection schemes is beyond the scope of the course.

An important measure of performance used to compare the performance of the digital modulation schemes is the probability of error $P_E$. The derivations of the probability of error for various modulation schemes were not explicitly covered in the lecture but you should know the following expressions for the probability of error $P_E$ for BPSK and BFSK corrupted with additive White Gaussian noise (AWGN) and detected using coherent detection schemes.

**BPSK (Coherent Detection):**

$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

**BFSK (Coherent Detection):**

$$P_B = Q\left(\frac{E_b(1-\rho)}{N_0}\right)$$

where $E_b$ is the signal energy per binary symbol, $N_0/2$ is the power spectral density (PSD) of the AWGN, and $\rho$ is the correlation coefficient between the two transmitted waveforms used in BFSK.

Problems 5 to 10 apply these formulas for calculating the probability of bit error including the derivation of the expression for probability of bit error for BPSK.

**Problem 1 (Intersymbol Interference):** A voice signal, whose significant frequency components lie in the range of 300Hz to 3300Hz is samples at 16000 samples/s. The following two schemes are designed to transmit the sampled voice signal.

**Scheme 1:** transmits the voice samples directly as PAM.

**Scheme 2:** converts each sample to a PCM format and use binary PCM waveforms for transmission.

In both schemes, ISI is avoided with a filter roll-off characteristics of ($r = 1$)

(a) What is the minimum system bandwidth required for the detection of PAM pulses used in scheme 1?
(b) What is the system bandwidth required for the detection of PCM pulses used in scheme 2 if the samples are quantized to eight levels.
(c) Repeat (b) for 256 quantization levels.

**Problem 2 (NRZ Baseband Transmission):** Consider a baseband communication system that uses NRZ binary pulses to transmit binary data at a rate of 56kbit/s along a cable that: (i) attenuates the signal power by 3dB from the transmitter to the receiver; and (ii) distorts the pulses with AWGN having $N_0 = 10^{-6}$ Watt/Hz. What is the minimum amount of power needed at the transmitter in order to maintain a bit-error probability of $P_B = 10^{-3}$ at the receiver if coherent detection is utilized?
Problem 3 (Transversal Equalizer): Recall that the desired overall response \( h_i(t) \) of a distortionless communication system is given by \( h_i(t) = \delta(t) \), where \( \delta(t) \) is the impulse function. Assume that the channel introduces ISI so that the overall response of the communication channel is given by

\[
h_i(t) = \delta(t) + \alpha \delta(t - T).
\]

Design a 5-tap transversal filter that reduces the ISI. Demonstrate that the designed filter suppresses the ISI. How can the ISI be reduced even further?

Problem 4 (Transversal Equalizer): The impulse response \( h(t) \) of a communication system is zero except for the following non-zero values

\[
h(-3T) = 0.1, \ h(-2T) = 0.3, \ h(-T) = -0.2, \ h(0) = 1.0, \ h(T) = 0.4, \ h(2T) = -0.1, \ h(3T) = 0.1.
\]

Design a 3-tap transversal equalizer that forces the ISI to be zero at one sampling point on each side of the mainlobe. After equalization, what is the largest magnitude sample contributing to ISI, and what is the sum of all the ISI magnitudes.

Problem 5 (Binary PSK with ASK Modulation): A binary modulation scheme combines BPSK with BASK such that the transmitted signals are given by

\[
s_1(t) = \sqrt{\frac{k}{T}} \cos(\omega_0 t) \quad (0 \leq t \leq T), \text{ for bit 1} \\
s_2(t) = \sqrt{\frac{k}{T}} \cos(\omega_0 t + \pi) \quad (0 \leq t \leq T), \text{ for bit 0}
\]

Assuming a basis function of \( \psi(t) = \sqrt{\frac{k}{T}} \cos(\omega_0 t), (0 \leq t \leq T) \), construct the signal constellation diagram for the communication system. Determine the optimum threshold for the matched filter output \( z(T) = [s_1(t) \otimes s_1(T-t)] \) if bits 0 and 1 are equally probable in the transmitted message.

Problem 6 (Binary FSK with orthogonal signals): Consider the coherent matched filter detection of equally likely binary FSK symbols

\[
s_1(t) = 0.5 \cos(2000\pi t) \quad (0 \leq t \leq 0.01s), \text{ for bit 1} \\
s_2(t) = 0.5 \cos(2200\pi t) \quad (0 \leq t \leq 0.01s), \text{ for bit 0}
\]

Compute the probability of bit error for the coherent matched filter detection of the equally likely binary FSK signals if the two sided AWGN has a power spectral density given by \( N_0/2 = 0.001 \). Draw the signal constellation diagram and indicate the decision region associated with the detector.

Problem 7 (BPSK Detection): A coherent BPSK receiver receives data at a rate of 5000 bits/s. The transmitted digital waveforms corresponding to bits 0 and 1 are given by

\[
s_1(t) = A \cos \omega_0 t \quad \text{for bit 1} \\
s_2(t) = -A \cos \omega_0 t \quad \text{for bit 0}
\]

where the amplitude \( A \) of the carrier wave is given by \( A = 1\text{mV} \). Assuming that the PSD of the AWGN introduced by the channel is given by \( N_0 = 10^{-11} \text{ Watt/Hz} \), compute the expected number of bit errors made by the system in 1 day.

Problem 8 (BPSK versus BFSK Modulation Schemes): We are interested in replacing a BPSK modulation scheme with a BFSK modulation scheme that uses orthogonal signals (\( \rho = 0 \)). Given that the available \( E_b/N_0 \)
ratio for a coherent BPSK receiver is 8dB, what should be the minimum value of the $E_b/N_0$ ratio for a coherent BFSK receiver that outperforms the coherent BPSK receiver.

**Problem 9 (Derivation of BPSK Probability of Bit Error):** Derive the expression for probability of bit error

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

for coherent detection for BPSK communication

$$s_1(t) = \sqrt{\frac{2E}{T}} \cos \omega_0 t \quad (0 \leq t \leq T), \text{ for bit 1}$$
$$s_2(t) = -\sqrt{\frac{2E}{T}} \cos \omega_0 t \quad (0 \leq t \leq T), \text{ for bit 0}$$

if bit 1 and bit 0 are equiprobable.

**Problem 10 (BFSK with nonorthogonal signals):** Consider the coherent matched filter detection of equally likely binary FSK symbols

$$s_1(t) = 0.5 \cos(2000\pi t) \quad (0 \leq t \leq 0.01s), \text{ for bit 1}$$
$$s_2(t) = 0.5 \cos(2020\pi t) \quad (0 \leq t \leq 0.01s), \text{ for bit 0}$$

Compute the probability of bit error for the coherent matched filter detection of the equally likely binary FSK signals if the two sided AWGN has a power spectral density given by $N_0/2 = 0.0001$. 