Lassonde School of Engineering

Dept. of EECS
Professor G. Tourlakis
MATH1090 A. Problem Set No 4
Posted: Nov. 22, 2022

Due: Dec. 7, 2022; by 3:00pm, in eClass, "Assignment #4"

Q: **How do I submit?**

A:

(1) **Submission must be ONLY ONE file**

(2) **Accepted File Types: PDF, RTF, MS WORD, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

In what follows, "give a proof of $\vdash A$" or "show $\vdash A$" means to give an Equational or Hilbert-style proof of A, unless some other proof style is required (e.g., Resolution).

Annotation is always required!

Do the following problems (**5 MARKS/Each**).

1. Prove using **soundness** (**Required**):

$$\nvdash (\forall \mathbf{x})(A \vee B) \to (\forall \mathbf{x})A \vee (\forall \mathbf{x})B$$

2. Prove $\vdash (\forall \mathbf{x})A \vee (\forall \mathbf{x})B \to (\forall \mathbf{x})(A \vee B)$.

3. Use the $\exists$ elimination technique —**Required**— to show $\vdash (\exists \mathbf{x})(A \wedge B) \to (\exists \mathbf{x})A \wedge (\exists \mathbf{x})B$.

4. Use the $\exists$ elimination technique —**Required**— to show
$$\vdash (\exists \mathbf{x})(A \equiv \neg A) \to A \wedge \neg A$$

5.

(4 MARKS) For any binary predicate ϕ, prove $\vdash (\forall x)(\forall y)\phi(x, y) \to (\forall y)\phi(y, y)$.

(1 MARK) Also explain precisely why the above is NOT an instance of **Ax2**.