Lassonde Faculty of Engineering
EECS
MATH1090. Problem Set No. 4
Posted: November 16, 2018

Due: Dec. 4, 2018, by 2:30pm; in the course assignment box.

Administrative Stuff. It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

A brief but full justification of each proof step is required!

Do all the following problems; (5 Points Each).

Important Notes; Read First!

"Show that —or prove that— $\Gamma \vdash A$" means "write a Γ-proof that establishes A". The proof can be Equational or Hilbert-style. Equational is rather easier in Boolean Logic. But it is your choice, unless a problem explicitly asks for a particular proof style.

"**Required Method**" means that any other method will get a 0-grade.

1. Prove $\vdash (\forall \mathbf{x})(\forall \mathbf{x})A \equiv (\forall \mathbf{x})A$.

2. Prove $\vdash (\exists \mathbf{x})(\exists \mathbf{x})A \equiv (\exists \mathbf{x})A$.

3. Prove **Equationally** (required method)

$\vdash (\forall \mathbf{x})(A \vee B \to C) \equiv (\forall \mathbf{x})(A \to C) \wedge (\forall \mathbf{x})(B \to C)$.

4. Show that $(\exists \mathbf{x})A \wedge (\exists \mathbf{x})B \to (\exists \mathbf{x})(A \wedge B)$ is NOT a theorem schema.

To this end find specific $A, \mathbf{x}$ and B so that

$$\not\models (\exists \mathbf{x})A \wedge (\exists \mathbf{x})B \rightarrow (\exists \mathbf{x})(A \wedge B)$$

and invoke 1st-order soundness.

Hint. Always prefer "uncomplicated" Interpretations!

5. Prove $\vdash \mathbf{x} = \mathbf{y} \wedge \mathbf{y} = \mathbf{z} \rightarrow \mathbf{x} = \mathbf{z}$.

From the text, Section 6.6 (p. 190) do:

6. #18

7. #35

From the text, Section 8.3 (p. 209) do:

8. #8