Lassonde School of Engineering
EECS
MATH1090. Problem Set No. 3
Posted: Oct. 26, 2017

Due: Nov. 16, 2017, by 2:30pm; in the course assignment box.

It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

In what follows, "give a proof of $\vdash A$" means to give an equational or Hilbert-style proof of A, unless some other proof style is required (e.g., Resolution).

Annotation is always required!

*Do the following problems (**5 MARKS/Each**).*

1. In class we proved $\vdash A \equiv A$ using the "trick" of applying Leibniz with "mouth" a variable $\mathbf{p}$ that does *not* occur in A.

Re-prove this theorem, but this time NOT using this trick. *Be sure your proof is NOT "circular" —i.e., must not use any theorem from class that relies already on* $\vdash A \equiv A$.

2. Use Resolution —but *not* Post's theorem— to prove $\vdash (A \to B) \to (A \lor C) \to (B \lor C)$.

3. Use an *Equational* proof for *both* subquestions below, and as needed the Deduction Theorem, to prove

(a) $\vdash A \wedge B \rightarrow A \wedge (B \vee C)$.

(b) $\vdash A \rightarrow (B \rightarrow A)$.

You are NOT allowed to use Post's theorem in the above question!

4. Use Resolution (but *not* Post's theorem!) to prove

$$\vdash (A \rightarrow B) \rightarrow (A \rightarrow C) \rightarrow (A \rightarrow B \wedge C)$$

A proof by Resolution is the only *acceptable method.*

5. Use Resolution (but *not* Post's theorem!) to prove $\vdash A \vee (B \wedge C) \rightarrow A \vee B$.

A proof by Resolution is the only *acceptable method.*

6. Let ϕ be a ternary (three-place) predicate symbol.

Prove

$$\vdash (\forall x)\phi(x, y, z) \equiv (\forall x)\phi(x, y, z) \tag{1}$$

and

$$\vdash (\forall x)\Big(\phi(x, y, z) \rightarrow \phi(x, y, z)\Big) \tag{2}$$

7. Let ϕ', ψ be any *unary* predicates, x, z distinct variables, and a a constant.

Prove that

$$(\forall x)(\phi'(x) \rightarrow \psi(x)),\ (\forall z)\phi'(z) \vdash \psi(a)$$

8. Prove $\vdash (\forall x)(\forall y)(A \vee B \vee C) \equiv (\forall x)(A \vee (\forall y)(B \vee C))$, *on the condition* that y is not free in A.

9. Prove $\vdash (\exists x)(\exists y)(A \wedge B \wedge C) \equiv (\exists x)(A \wedge (\exists y)(B \wedge C))$, *on the condition* that y is not free in A.

10. Prove $\vdash (\forall \mathbf{x})((A \vee B) \rightarrow C) \rightarrow (\forall \mathbf{x})(A \rightarrow C)$.