Faculty of Science and Engineering

MATH1090. Problem Set No1

Posted: Sept. 20, 2008

Due: Oct. 3, 2008; in the course assignment box.

It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

1. (3 MARKS) Prove that the *first* symbol of a formula cannot be $\equiv$.

2. (3 MARKS) Prove that the complexity of a formula is half its number of brackets.

Hint. Analyse formula-calculations.

3. (6 MARKS) Recall that a schema is a tautology iff all its instances are tautologies.

Which of the following schemata are tautologies? Show the whole process that led to your answers.

I note that in the six sub-questions below I am not using all the formally necessary brackets.

- $((A \to B) \to A) \to A$
- $A \wedge B \to A \equiv B$
- $A \equiv B \to A \wedge B$
- $A \to B \equiv \neg B \to \neg A$
- $A \wedge (B \equiv C) \equiv A \wedge B \equiv A \wedge C$
- $A \vee (B \equiv C) \equiv A \vee B \equiv A \vee C$

4. (5 MARKS) Prove that if we have $A, B, C \models_{\text{taut}} D$, then we also have $\models_{\text{taut}} A \to B \to C \to D$ and conversely. Or as we usually put it: "$A, B, C \models_{\text{taut}} D$ iff $\models_{\text{taut}} A \to B \to C \to D$".

Here, using truth tables or truth-table tricks, you will show that if you have one side, then you must have the other. There are two directions in your proof!

5. (5 MARKS) By using truth tables, or using related shortcuts, examine whether or not the following tautological implications are correct.

Show the whole process that led to each of your answers.

- $p \vee q \models_{\text{taut}} p$
- $A \models_{\text{taut}} A \wedge B$
- $A, A \to B \models_{\text{taut}} B$
- $B, A \to B \models_{\text{taut}} A$
- $p \wedge q \models_{\text{taut}} p$

6. (6 MARKS) Compute the most simplified result of the following substitutions, *whenever the requested substitution makes sense*. Whenever a requested substitution does not make sense, explain exactly why it does not.

Show the whole process that led to each of your answers in each case.

Remember the priorities of the various connectives as well as of the meta-expression "$[\mathbf{p} := \ldots]$"! The following formulae have not been written with all the formally required brackets.

- $p \vee q \to p[p := r]$
- $(p \vee q)[p := \mathbf{f}]$
- $(p \vee q)[p := \bot]$
- $(\bot \vee q)[\bot := p]$
- $p \vee q \wedge r[p := A]$ (where A is some formula)
- $p \vee (q \wedge r)[r := A]$ (where A is some formula)