

## York University

Faculties of Science and Engineering, Arts, Atkinson

**MATH 1090. Problem Set #1**

**Posted September 22, 2005**

**Due: October 6, 2005; 4:00pm,  
in the course box (location in CSEB: TBA)**

### Section A



Worth reproducing (from the course outline):

“The homework must be each individual’s own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, at the end of all this consultation each student will have to produce an individual report rather than a copy (full or partial) of somebody else’s report.

See <http://www.yorku.ca/secretariat/legislation/senate/acadhone.htm> to familiarise yourselves with Senate’s expectations regarding Academic Honesty.

The concept of late assignments does not exist.”



1. (5 MARKS) Prove that a Boolean formula has as first symbol one of

- a variable
- a constant (*true* or *false*)
- a left bracket.

The proof is required to be either by *induction on the complexity of formulae*, or by analysing *formula-calculations*.

2. (6 MARKS) Which of the following are tautologies? Show all work.

I am using “least parenthesised notation”.

- $((A \Rightarrow B) \Rightarrow A) \Rightarrow A$
- $A \wedge B \Rightarrow A \vee B$
- $A \vee B \Rightarrow A \wedge B$
- $A \Rightarrow B \equiv \neg B \Rightarrow \neg A$

- $A \wedge (B \equiv C) \equiv A \wedge B \equiv A \wedge C$
  - $A \vee (B \equiv C) \equiv A \vee B \equiv A \vee C$
3. (5 MARKS) Prove that if we have  $A, B, C \models_{\text{taut}} D$ , then we also have  $\models_{\text{taut}} A \Rightarrow B \Rightarrow C \Rightarrow D$  and conversely. Or as we usually put it: “ $A, B, C \models_{\text{taut}} D$  iff  $\models_{\text{taut}} A \Rightarrow B \Rightarrow C \Rightarrow D$ ”.

Here, using truth tables or truth-table tricks, you will show that if you have one side, then you must have the other. There are two directions in your proof!

4. (5 MARKS) Using truth tables or truth-table tricks determine the validity of the following. Show all your work.
- $p \models_{\text{taut}} p \wedge q$
  - $A, B \models_{\text{taut}} A \wedge B$
  - $A, A \Rightarrow B \models_{\text{taut}} B$
  - $B, A \Rightarrow B \models_{\text{taut}} A$
  - $p \wedge q \models_{\text{taut}} p$

5. (5 MARKS) Calculate the following (show/explain all work!).

**NB.** The first bullet below must be done using Definition 0.3.13, step by step. For the rest you are free to rely on the intuitive definition of substitution/replacement. Some of the replacements I ask you to do may be “illegal”. If so, explain precisely why they are illegal and don’t do them!



Review priorities!



- $p \vee (q \Rightarrow p)[p := r]$
- $(p \vee q)[p := \mathbf{t}]$
- $(p \vee q)[p := \text{true}]$
- $p \vee q \wedge r[q := A]$  (where  $A$  is some formula, we don’t care which)
- $p \vee (q \wedge r)[q := A]$  (where  $A$  is some formula, we don’t care which)