Lassonde Faculty of Engineering
EECS
EECS2001Z. Problem Set No1 —Hints
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General HINT. Please do *not* provide derivations or the $prim(h, g)$ notation in your solutions *unless I asked for them*. Also do not always go back to Adam and Eve. We have shown quite a few primitive recursive functions and relation to be primitive recursive. E.g., (without λ) $x + y$, $x \times y$, if-then-else, x^y, $x = y, x < y, x \leq y$, $x|y$, $Pr(x)$, $\pi(x)$, p_n, etc. Use them —**along with the closure properties we know**, e.g., *prim*, *comp*, Grz-Ops, $(\mu y)_{<z}$, def. by cases for functions; Boolean ops and $(\exists y)_{<z}$ and $(\forall y)_{<z}$ for relations— to build new functions and relations that are primitive recursive.

1. (5 MARKS) Write a correct URM which simulates the assignment statement $\mathbf{x} \leftarrow \mathbf{z}$ without changing the original value of $\mathbf{z}$.

You must provide a brief coherent argument of correctness.

Hint. The part in red means that after $\mathbf{x}$ received the value of $\mathbf{z}$, i.e., when we are done with $\mathbf{x}$, we take steps to restore the original value of $\mathbf{z}$ (that now resides in $\mathbf{x}$).

2. (5 MARKS) Prove that the function

$$x \text{ 2s } \left\{ 2^{2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}} \right.$$

is in $\mathcal{PR}$.

Hint. One primitive recursion will do it. Note that in

$$x \text{ 2s } \left\{ 2^{2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}} \right.$$

brackets are inserted right-to-left (top-down) as is common with unary functions. That is, for example, 2^{2^2} means $2^{(2^2)}$, NOT $(2^2)^2$. While these two give the same result, this is not the case when the ladder of 2s becomes longer.

For example, $2^{2^{2^2}} = 2^{(2^{(2^2)})} = 65536$ while (the incorrect) $((2^2)^2)^2 = 256$.

3. p.234 of the text, Section 2.12: Do

(a) Problem 3 (5 MARKS)
Hint. "A total function $\lambda\vec{x}_n.f(\vec{x}_n)$ is 1-1", as you know from EECS 1019, *means* that

$$f(\vec{x}_n) = f(\vec{y}_n) \to x_i = y_i, \text{ for } i = 1, \ldots, n$$

To show $\lambda xy.2^{x+y+2} + 2^{y+1}$ is 1-1, think what precisely is the binary representation of the number $2^{x+y+2} + 2^{y+1}$. You do not need to solve for K and L as the book does.

(b) Problem 4 (5 MARKS)
Hint. To show $\lambda xy.(x + y)^2 + x$ is 1-1, start with "let $(x + y)^2 + x = (x' + y')^2 + x'$" and show

$$x = x' \wedge y = y'$$

But how? Well, try to show that the "let" implies $x + y = x' + y'$. You must *not* solve for K and L at any time during your proof.

(c) Problem 6 (5 MARKS)
Hint. A relation Q is a table (finite or infinite) of vectors $\vec{x}_n$. If $n = 1$ we have a relation consisting of numbers. We write the relation most often as $Q(\vec{x}_n)$, but also as $\vec{x}_n \in Q$, or just Q.
Thus $\mathbb{N}$ is a relation (of numbers, not vectors) just like the $Pr(x)$ we discussed this week. We may write $\mathbb{N}(x)$ or $x \in \mathbb{N}$ for this relation.
Now, in our very first class we reviewed set theory and one of the things we recalled was that

$$x \in \{a_1, a_2, a_3, \ldots, a_n\} \equiv x = a_1 \vee x = a_2 \vee x = a_3 \vee \ldots x = a_n$$

(d) Problem 7 (5 MARKS)
Hint. Use the definition of $\mathbb{N}(x) \in \mathcal{PR}_*$ or $\mathbb{N} \in \mathcal{PR}_*$. It requires that $c_{\mathbb{N}} \in \mathcal{PR}$.
So, what *IS* $\lambda x.c_{\mathbb{N}}(x)$, and can you prove it in $\mathcal{PR}$?