Lassonde School of Engineering

Dept. of EECS
Professor G. Tourlakis

EECS 1028 Z. Problem Set No1

Posted: Jan. 13, 2024

Due: Feb. 2, 2024; by 6:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is NOT accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of “late assignments” does not exist in this course, as you recall.

1. True or False **and Why**. (**NOTE**: NO Why – NO Points)

(a) (2 MARKS) $\{\{a\},\{b\}\} = \{a, b\}$

(b) (2 MARKS) $\emptyset \in \emptyset$.

(c) (2 MARKS) $\bigcup\{\{c\},\{d\}\} = \{c, d\}$

(d) (2 MARKS) $\emptyset \subseteq \emptyset$

(e) (2 MARKS) $\emptyset \in \{1\}$

2. (3 MARKS) Is the class $\{\{x\} : \text{all } \underline{\text{atoms}}\ x\}$ a set? **Why** yes or no exactly?

3. (5 MARKS) Is the class $\{\{x, y, z\} : \text{for all } \underline{\text{sets}} \text{ and } \underline{\text{atoms}}\ x, y, and\ z\}$ a set? **Why** yes or no exactly?

4. (3 MARKS) Let A, B, C be sets or atoms. Prove that $\{A, B, C\}$ *is a set, without* using any of Principles 0, 1, 2. *Rather use results (theorems)* that we already established in class/Notes.

5. (5 MARKS) Prove that Principle 2 implies that we have infinitely many stages available.

Hint. Arguing by contradiction, assume instead that we only have **finitely many** stages. So repeatedly applying Principle 2 we can form a non ending sequence of stages

$$\cdots < \Sigma' < \Sigma'' < \Sigma''' < \Sigma'''' < \cdots \tag{1}$$

If the sequence (1) contains only a *finite* number of distinct $\Sigma^{''\cdots'}$, then at least two of the $\Sigma^{''\cdots'}$ in (1) are the same stage. Use this conclusion and properties of “$<$” to get a contradiction

6. (4 MARKS) Prove that, for any *set* A we have that $\mathbb{U} - B$ is *a proper class*.

7. (4 MARKS) Prove for any classes $\mathbb{A}, \mathbb{B}$, that $\mathbb{A} - \mathbb{B} = \mathbb{A} - \mathbb{A} \cap \mathbb{B}$.

Hint. This is a simple case of proving $lhs \subseteq rhs$ by doing "Let $x \in lhs$. BLA BLA BLA and concluding $x \in rhs$", and then *ALSO* doing $rhs \subseteq lhs$ by doing "Let $x \in rhs$. BLA BLA BLA and concluding $x \in lhs$".

8. Use notation by explicitly listing **all the members** of each rhs $\{???\}$ to complete the following incomplete equalities:

(a) (2 MARKS) $2^{\emptyset} = \{???\}$

(b) (2 MARKS) $2^{\{1,2,3\}} = \{???\}$