York University
Department of Electrical Engineering and Computer Science
Lassonde School of Engineering

EECS1028Z FINAL TAKE-HOME EXAM, April 22, 2024; 2:00–4:00PM

Professor George Tourlakis

Student NAME (**Clearly**):_____

Student NUMBER (**Clearly**):_____

DATE (**Cearly**):_____

By putting my name and student ID on this FINAL EXAM page, I attest to the fact that my answers included here and submitted by eClass are my own work, and that I have acted with integrity, abiding by the *Senate Policy on Academic Honesty* that the instructor discussed at the beginning of the course and *linked the full Policy to the Course Outline*.

README FIRST! **INSTRUCTIONS:**

1. *Operationally* write your answers *the same way you do with Homework Assignments*. **BUT: PAGE ONE OF THIS BOOK MUST BE COMPLETED AND SUBMITTED.**
2. Please read ALL these instructions carefully before you start writing.
3. **A repeat of the allegations and claims (aping the MIDTERM) in this EXAM regarding *WiFi* and *Computer Issues* —as happened after the MIDTERM— will NOT be tolerated. Simply, use your smart phone to provide a "hotspot" for your laptop WiFi, if you believe that your WiFi is down.**
4. **Please answer ALL the questions.**
5. This is a **TIME-LIMITED TAKE-HOME *FINAL EXAM***. You have **120 MIN to answer the EXAM questions**. **NO EXTENSIONS will be granted FOR ANY REASON**. ABSOLUTELY last opportunity to upload is **BY 4:00 (pm)**
6. **Just like Assignments, here too Only a SINGLE file *of SIZE $\leq$ 10MB* can be uploaded per student.**
7. **eClass will reject files bigger than 10MB. So will I.**
8. If you submit photographed copy **it still must be ONE file that you submit**. Either ZIP the PNG or JPEG images **OR** import them in MS Word and submit *ONE* Word *file* with the photos attached.
9. **Be sure to select the lowest resolution setting** of your photographs before you ZIP the images!!!

Question	MAX POINTS	MARK
1	5	
2	4	
3	3	
4	5	
5	4	
6	6	
7	4	
8	7	
TOTAL	38	

Question 1. (a) (1 MARK) Define precisely the term "**Set A is Finite**".

(b) (4 MARKS) Let $n \in \mathbb{N}$ and $n > 0$. Let $X \subseteq \{x \in \mathbb{N} : x \leq n\}$. Prove that X is finite.

Question 2. (4 MARKS) Prove that an enumerable set is infinite.

Question 3. (3 MARKS) Prove that the set $\{1\}$ is countable.

Question 4. (a) (1 MARK) Prove that the class $\{7^m : m \geq 0\}$ is a **set**.

(b) (4 MARKS) Prove that the set $\{7^m : m \geq 0\}$ is **enumerable**.

Question 5. (4 MARKS) Prove using techniques of predicate logic and a Hilbert Proof,

$$\vdash (\exists x)(A \to B) \to (\forall x)A \to (\exists x)B$$

Question 6. (a) (2 MARKS) Let A be a formula of Predicate Logic. What does the notation "$A(x)$" mean **exactly**? ONE sentence please!

(b) (4 MARKS) Consider $(\exists x)A(x) \to A(x)$.
Show that it cannot possibly be valid, and **do so** by finding a simple formula A over $\mathbb{N}$ that provides a counterexample to validity.

Question 7. (4 MARKS) Use induction to prove that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

Question 8. Consider the inductive definition of the set B as $\mathrm{Cl}(\mathcal{I}, \mathcal{O})$ —that is, we set $B = \mathrm{Cl}(\mathcal{I}, \mathcal{O})$— where

(a) $\mathcal{I} = \{\lambda\}$

(b) $\mathcal{O}$ contains ONLY two operations,

i. $(X, Y) \longrightarrow \boxed{concat} \longrightarrow XY$ **Comment**: Concatenation of X and Y in that order. and

ii. $X \longrightarrow \boxed{paren} \longrightarrow (X)$ **Comment**: Concatenation of "(", "X" and ")" in that order.

Prove:

- (3 MARKS) The strings
$$(), (()), \text{ and } ()(()) \text{ are in } B$$
- (4 MARKS) If $X \in B$, then X has as many left brackets as it has right brackets.