CSE 2001—Winter 2008

Problem Set No. 3
Posted: March 13, 2008
Due: April 7, 2008

All reports must be typed (except for diagrams). All assignments are due by 2:00pm on the due date **in the course box**.

1. Prove constructively that if a context free grammar G has only rules of the types $A \to Ba$ and $A \to a$, then $L(G)$ is regular.

Hint. Construct an appropriate FA or NFA M and prove $L(G) = L(M)$.

2. Prove that regular languages are closed under reversal, that is, if L is regular then so is

$$L^R \stackrel{\text{by def}}{=} \{x^R : x \in L\}$$

Hint. Either use the previous problem, or induction on the length of regular expressions.

3. Prove that CFL are also closed under reversal, that is, if L is a CFL then so is L^R.

Hint. Recommended to use grammars in CNF.

4. In class we constructed a ES-PDA M such that $L(M) = \{0^n 1^n : n \geq 0\}$. Find (probably by modifying M) an ES-PDA N such that $L(N) = \{0^n 1^n : n \geq 1\}$.

Recall that ϕ_x denotes the function ***of one argument*** computed by the Turing Machine, M_x, found in position x of the "standard" (algorithmic) enumeration of all Turing Machines.

5. Prove that $\{x : \text{dom}(\phi_x) = \emptyset\}$ is not decidable.

Note. By "dom" we mean "domain", that is, for any function f of one argument

$$\text{dom}(f) = \{x : (\exists z) f(x) = z\}$$

6. **YES/NO**, but ***with proof*** please!

(a) The problem $\varepsilon \in L(G)$ is decidable for any CFG G.

(b) The problem $0 \in \text{dom}(\phi_x)$ is decidable.

(c) The problem $0 \in \text{dom}(\phi_x)$ is semi-decidable.

Rice's Theorem must not be used.