York University
CSE 2001 Fall 2017 – Midterm A
Instructor: Jeff Edmonds

Family Name: _____ Given Name: _____

Student #: _____ Email: _____

1) Language	6	
2) Short	6	
3) Logic	8	
4) TM	20	
5) DFA	20	
6) Bumping	20	
7) CFG	20	
0) Art	2	
Total	102 marks	

This exam is designed to be completed in 1.5 hours, but we will give you two hours.
Keep your answers short and clear.

0) (2 marks) Art therapy question: When half done the exam, draw a picture of how you are feeling.

1. Consider the alphabet $\Sigma = \{a, b\}$. Prove or disprove the following:

   (a) $bbabbaab \in \Sigma^* a \Sigma^* b \Sigma^* a \Sigma^*$.

   (b) $baab \in \Sigma^* a \Sigma^* b \Sigma^* a \Sigma^*$.

2. Short Answers: What is the 'N' in NFA for? How is it's Transition Function δ different than that of an DFA and what effect does this have on its computation? Given an NFA M and a string α, how do you know whether or not the string is accepted by the machine.

3. First Order Logic: We state as follows that single-tape Turing machines can compute everything that multi-tape can.

   $\forall$ multi-tape TM M_{multi}, $\exists$ a single tape TM M_{single}, $\forall$ inputs I, $M_{multi}(I) = M_{single}(I)$.

   How is the *First Order Logic Game* game played for this statement?

   Explain how this game states whether M_{single} is likely to have more states or fewer than M_{multi}.

4. **Turing Machine:** (25 marks) Write all the transition rules for a Turing Machine that solves the computational problem that takes as input a string of characters from $\{0, 1, ..., 9\}$ and replaces every 9 with the character that appeared two cells before it in the string. (If the first or second is a 9, replace it with a zero.)

   Input: 94398279293193
   Output: 04348272293133

   Pseudo Code:

   0: Put head on first character.
   c = getCharAtHead()
   $pp = 0$ and $prev = 0$
   1: loop ⟨***loop−invariant***⟩**:** The variables pp and $prev \in \{0, ..., 9\}$ remember the previous two characters read and we are looking at a new character c.
   $pp = prev$ In preparation for shifting head update previous two characters.
   $prev = c$
   if($c = 9$) then this character must be replaced by the previous previous character.
   write pp
   elseif(c =blank)
   Halt
   else
   Nothing to do
   Move right to next character
   c = getCharAtHead()
   end loop

   (a) Translate this code into Turing Machine Transitions. Recall the states are named with the line number and the value of each variable.

   (b) How many states does your TM have?

5. DFA: Draw a DFA for the following language.
   $L = \{\alpha \in \{0,1\}^* \mid \alpha \text{ does NOT contain the substring } 001\}$.
   Example: The strings 111<u>001</u>00 contains 001 and hence in not in the language. The strings 111010100 does not so is.
   Be sure to label the states of your DFA with names indicating what the DFA "knows" when in that state.

6. Bumping Lemma is stated as follows:
   [$\exists$ infinite set S, $\forall \alpha, \beta \in S$ with $\alpha \neq \beta$, $\exists \zeta$, $L(\alpha\zeta) \neq L(\beta\zeta)$] $\Rightarrow$ [$\forall$ DFA M, $\exists$ an input I, $M(I) \neq L(I)$].

   (a) No need for a full proof, but give reasonable intuition to why it is true.

   (b) For each of these languages, either prove that it is not regular or prove that it is. Give the intuition.

      i. $L = \{a, b\}^n \#\{u, v\}^n$ or $^{(n+1)}$
      ii. $L = \{a, b\}^m \#\{u, v\}^n$ or $^{(n+1)}$

7. For each of these languages, either give a context free grammar for it or argue that it can't be done.

   (a) $L = 0^{2n+7}\ a^m\{x, y\}^s 1^{2m}\{u, v\}^{3n+8}$

   (b) $L = 0^{2m+7}\ a^n\{x, y\}^s 1^{2m}\{u, v\}^{3n+8}$