# York University
# CSE 2001 Fall 2017 – Assignment 1 of 4
**Instructor: Jeff Edmonds**

Family Name: _____ Given Name: _____

Student #: _____ Email: _____

| | | |
|---|---|---|
| 1) Design TM | 50 | |
| 2) LI | 10 | |
| 3) Black Board | 10 | |
| 4) TM1 | 10 | |
| 5) TM2 | 10 | |
| 0) Art | 2 | |
| Total | 102 marks | |

This exam is designed to be completed in 1/2 an hour. Students are expected to complete the exam in that period. If unable to so so, they may continue based on the discretion of the instructor.
Keep your answers short and clear.

0) (2 marks) Art therapy question: When half done the exam, draw a picture of how you are feeling.

1. (50 marks) **Turing Machine:** Write all the transition rules for a Turing Machine that solves the palindrome problem. A palindrome is a string that is the same if you reverse it. For example, 9235329 is an odd length palindrome and 923329 is an even length one. Your TM given a string of characters from $\{0, 1, ..., 9\}$ will answer with either yes or no. The input will have a blank $b$ at the beginning and end, i.e. $b9235329b$. The head will be at the first character, which is a $c$.

   (a) Writing down TM descriptions is hard. Instead, start by writing pseudo code using variables and loops. The only allowed actions are to read and write to the tape where the head is and to move the head to the left and right. I am also a big believer in *loop invariants*. This is a clear picture of what the tape looks like at the beginning of each iteration.
   Hint: The loop invariant should be that either the TM has already halted because it has already discovered that the string is not to a palindrome or for some $i \geq 0$, the first $i$ characters and the last $i$ characters have been correctly matched and blanked out. What remains is to check that the remaining string is a palindrome. For example, with $i = 2$ the tape would contain $bbb353bbb$. The head will be at the first remaining character, which is stored in $c$.

   (b) Translate this code into Turing Machine Transitions

   $$\delta(q_{\langle line=?,x=? \rangle} = \langle q_{\langle next\ state \rangle}, char\ to\ write,\ direction\ move\ head \rangle$$

   . Recall the states are named with the line number and the value of each variable.

   (c) How many states does your TM have?

2. (10 marks) What is a *Loop Invariant* and how is it useful in the designing of a Turing Machine?

3. (10 marks) Does a TM have lines of code that it executes and a pooh bear small black board it can write on? Explain these concepts.

4. (10 marks) Explain in your own words what it means and why it is true or why it is not.
   Use the game taught in class to prove either it or its negation.
   When appropriate sketch in a few sentences what the TM does and how many states it has.
   $\forall$ integers $k$, $\exists$ TM $M$, $\forall x, y \leq k$, $M$ multiplies $x \times y$ in one time step,
   i.e. it can read in $\langle x, y \rangle$ and then simply know the answer.

5. (10 marks) Explain in your own words what it means and why it is true or why it is not.
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   i.e. it can read in $\langle x, y \rangle$ and then simply know the answer.