York University
CSE 2001 – Unit 5.2 Reductions for Undecidability
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Read Jeff's notes. Read the book. Go to class. Ask lots of question. Study the slides. Work hard on solving these questions on your own. Talk to your friends about it. Talk to Jeff about it. Only after this should you read the posted solutions. Study the solutions. Understand the solutions. Memorize the solutions. The questions on the tests will be different. But the answers will be surprisingly close.

1. Reductions: Let $P_{TM\ Eq} = \{\langle M, M'\rangle \mid \forall I,\ M(I) = M'(I)\}$. The "Reductions For Uncomputability" slides prove that $Halting \leq P_{TM\ Eq}$. Review this proof.

   (a) Later in the same slides stated but did not prove that $\neg Halting \leq P_{TM\ Eq}$. Do this (yes to yes, no to no) reduction. What does this say about the language $P_{TM\ Eq}$ with respect to it being undecidable, unrecognizable, and/or co-recognizable.
   Hint: We still say that $M(I) = M'(I) = \infty$ even if neither machines halt on I.

   (b) Remember we like Karp reductions that take Yes instances to Yes instances and No to no. Prove $Halting \leq \neg P_{TM\ Eq}$.
   Hint: Or give the changes to a previous proof.

2. Let $P_{reverse} = \{$"M" $\mid M$ is a TM that accepts I^R (reverse) whenever it accepts I $\}$, i.e. computational problem P says "yes" on inputs "M" in this set and says "no" on inputs not in this set.

   (a) Use a reduction to prove that this is undecidable.

   (b) Can you directly use Rice's Theorem to prove this?

   (c) Don't do an additional reduction for this question, but given the reductions we have seen, do you think $P_{reverse}$ is unrecognizable and/or co-recognizable.

3. Let $P = \{\langle$"M"$, I\rangle \mid$ TM M on input I never tries to move its head left when it is already on the left hand most cell of the tape. $\}$.

   (a) Use a reduction to prove that this is undecidable.
   OOPS: The problem P is really equivalent to the negation of the halting problem. The halting problem asks for a "yes" if a given event happens while P asks for a "no". This means that P is in $co-recognizable$. It is hence impossible to prove $Halting \leq_{poly} P$. But it is possible to flip the "yes" and the "no" and prove $\neg Halting \leq_{poly} P$. This is sufficient to prove P is undecidable.

   (b) Can you directly use Rice's Theorem to prove this?

4. Let $P = \{\langle$"M"$, I\rangle \mid$ TM M on input I never tries to move its head left. $\}$. (Assume that TMs never leave their head stationary.)

   (a) Which other model of computation does a TM that never moves its head left remind you of?

   (b) After the TM has moved its right past the input and moved right for a while on the blank tape what must eventually happen to the state that it is in (i.e. whats written on its black board)?

   (c) Give an algorithm that decides the problem.