EECS 1090 – Test 1

Instructor: Jeff Edmonds

1. (20 marks) Fill out the table with all of the rules.

	Proof Techniques/Lemmas			
	Using		Proving	
	From:	Conclude	From:	Conclude
And $\land$:	Separating And		Eval/Build/Simplify $\land$	
Or $\lor$:	Selecting Or		Eval/Build/Simplify $\lor$	
	Cases		Excluded Middle	
Implies $\rightarrow$:	Modus Ponens		Deduction	
	Cases		Eval/Build/Simplify $\rightarrow$	
	Equivalence		Contrapositive	
	Transitivity		De Morgan's Law	

2. (13 marks) Find all possible assignments of the variables that makes the following expression true/satisfied.
   Explain all of the steps in your search for the assignment and in proving that this assignment works.
   Use Purple table reasoning, not a table.
   Hint: Start with proof/search by cases with the $p \vee q$, then see how you can force the values of other variables.
   $$[p \vee q] \wedge [p \rightarrow s] \wedge [\neg p \vee \neg s] \wedge [t \rightarrow \neg q] \wedge [u \vee t] \wedge [u \oplus v] \wedge [w \rightarrow \neg w] \wedge [y \rightarrow (x \wedge \neg x)].$$
   How many different satisfying assignments are there?

3. Nicer Father

   (a) (8 marks) It is not true that
   $$[(\alpha \rightarrow \gamma) \ \mathit{and} \ (\beta \rightarrow \gamma)] \text{ iff } [(\alpha \ \mathit{and} \ \beta) \rightarrow \gamma].$$
   Thinking of $\rightarrow$ as causality, argue in English as you would to someone who does not know logic why this statement is false.

(b) Use the purple table to prove the following two statements.
Use deduction. Do NOT convert the $\rightarrow$ into *and* or *or*.

i. (13 marks) $[(\alpha \rightarrow \gamma)\ and\ (\beta \rightarrow \gamma)] \rightarrow [(\alpha\ or\ \beta) \rightarrow \gamma]$.

ii. (13 marks) $[(\alpha \rightarrow \gamma)\ and\ (\beta \rightarrow \gamma)] \leftarrow [(\alpha\ or\ \beta) \rightarrow \gamma]$.

(c) (7 marks) Jeff told a story about a grumpy father, doing well at school, following rules, and being loved. Change the contract of the father to be $[(\alpha \rightarrow \gamma)\ and\ (\beta \rightarrow \gamma)]$. How is this father different than in Jeff's story.

4. (13 marks) The game *Ping* has two rounds. Player-A goes first. Let m_1^A denote his first move. Player-B goes next. Let m_1^B denote his move. Then player-A goes m_2^A and player-B goes m_2^B. The relation $AWins(m_1^A, m_1^B, m_2^A, m_2^B)$ is true iff player-A wins with these moves.

   Use universal and existential quantifiers to express the fact that player-A has a strategy in which she wins no matter what player-B does. Use $m_1^A, m_1^B, m_2^A, m_2^B$ as variables. Explain.

5. (13 marks) We say that the sequence $f = f(1), f(2), f(3), \ldots$ converges if
   $\exists c\ \forall \epsilon > 0\ \exists n_0\ \forall n \geq n_0\ |f(n) - c| \leq \epsilon$.
   Play the Jeff's game in order to prove that the sequence $f = \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \ldots$, i.e., $f(i) = \frac{1}{i}$ converges.