EECS 1090 – Oracle Game Problems

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*** I banged this out with global search and replace from

`https://www.eecs.yorku.ca/~jeff/courses/1090/ppt/1090-9-Quesions.pptx`

Let me know if there are typos.
I dont have solutions.

1. Use the oricle−prover−adversary game (or formal proof) to prove each of the following:
   (a) $[\neg\forall y\ (F(y)\ \&\ G(y))] \to [\exists y\ (\neg F(y) \vee \neg G(y))]$
   (b) $[\forall w\ (L(w) \to M(w))\ \&\ \forall y\ (M(y) \to N(y))] \to [\forall w\ (L(w) \to N(w))]$
   (c) $[\exists x\ (G(x)\ \&\ A(x))\ \&\ \forall y\ (C(y) \to \neg G(y))] \to [\exists z(A(z)\ \&\ \neg C(z))]$
   (d) $[\neg\exists x\ (\neg R(x)\ \&\ S(x,x))\ \&\ S(j,j)] \to R(j)$
   (e) $[\forall x\ ((\neg C(x,b) \vee H(x)) \to I(x,x))\ \&\ \exists y\ \neg I(y,y)] \to \exists x\ C(x,b)$
   (f) $[\forall x\ F(x)\ \&\ \forall z\ H(z)] \to [\neg\exists y\ (\neg F(y) \vee \neg H(y))]$

2. Use the oricle−prover−adversary game (or formal proof) to prove each of the following:
   (a) $[(\forall x\ \neg J(x))\ \&\ (\exists y\ (H(y) \vee R(y,y))) \to \exists x\ J(x)] \to [\forall y\ \neg(H(y) \vee R(y,y))]$
   (b) $[\neg\exists x\ \forall y\ (P(x,y)\ \&\ \neg Q(x,y))] \to [\forall x\ \exists y\ (P(x,y) \to Q(x,y))]$
   (c) $[(\forall x\ \neg(\forall y\ (H(y)(x) \vee T(x))))\ \&\ \neg\exists y\ (T(y) \vee \exists x\ \neg H(x,y))] \to [\forall x\ \forall y\ H(x,y)\ \&\ \forall x\ \neg T(x)]$
   (d) $[(\forall z\ (L(z) \text{ iff } H(z)))\ \&\ (\forall x\ \neg(H(x) \vee \neg B(x)))] \to \neg L(b)$
   (e) $[(\forall z\ [K(z) \to (M(z)\ \&\ N(z))])\ \&\ (\exists z\ \neg N(z))] \to [\exists x\ \neg K(x,x)]$
   (f) $[(\exists x\ (\neg B(x,m)\ \&\ \forall y\ (C(y) \to \neg G(x,y))))$
   $\&\ (\forall z\ (\neg\forall y\ (W(y) \to G(z,y)) \to B(z,m)))]$
   $\to [\forall x\ (C(x) \to \neg W(x))]$
   (g) $[(\exists z\ \ Q(z)\ \ \to\ \ \forall w\ \ (L(w,w)\ \ \to\ \ \neg H(w)))\ \ \&\ \ (\exists x\ \ B(x)\ \ \to\ \ \forall y\ \ (A(y)\ \ \to\ \ H(y)))]\ \ \to$ $[\exists w\ (Q(w)\ \&\ B(w)) \to \forall y\ (L(y)(y) \to \neg A(y))]$
   (h) $[\forall y\ (K(b,y) \to \neg H(y))] \to [\forall x\ [\exists y\ (K(b,y)\ \&\ Q(x,y)) \to \exists z\ (\neg Hz\ \&\ Q(x)z)]]$
   (i) $[(\neg\forall x\ (\neg G(x) \vee \neg H(x))\ \to\ \forall(C(x)\ \&\ \forall y\ (I(y)\ \to\ A(x,y))))\ \&\ (\exists x\ [H(x)\ \&\ \ \&\ \forall y\ (L(y)\ \to$ $A(x,y))] \to \forall x\ (F(x)\ \&\ \forall y\ B(x,y)))] \to [\neg\forall x\ \forall y\ B(x,y) \to \forall x\ (\neg G(x) \vee \neg H(x))]$

3. Use the oricle−prover−adversary game (or formal proof) to prove each of the following:
   (a) $\forall x\ (A(x) \to B(x)) \to \forall x\ (B(x) \vee \neg A(x))$
   (b) $\forall x\ (A(x) \to (A(x) \to B(x))) \to \forall x\ (A(x) \to B(x))$
   (c) $\neg\exists x\ (A(x) \vee B(x)) \to \forall x\ \neg A(x)$
   (d) $\forall x\ (A(x) \to B(x)) \vee \exists x\ A(x)$
   (e) $(\exists x\ A(x) \to \exists x\ B(x)) \to \exists x\ (A(x) \to B(x))$
   (f) $\forall x\ \exists y\ (A(x) \vee B(y)) \text{ iff } \exists y\ \forall x\ (A(x) \vee B(y))$

4. Show that the members of each of the following pairs of sentences are equivalent.
   (a) $\neg\forall x\ (A(x) \to B(x)) \text{ iff } \exists x\ (A(x)\neg B(x))$
   (b) $(\exists x\ \exists y\ A(x,y)) \to A(a,b)$
   iff $(\exists x\ \exists y\ A(x,y)) \text{ iff } A(a,b)$
   (c) $\neg\forall x\ \neg[(A(x)\ \&\ B(x)) \to C(x)]$
   iff $\exists x\ [\neg A(x) \vee (\neg C(x) \to \neg B(x))]$
   (d) $\neg\forall x\ \exists y\ [(A(x)\ \&\ B(x)) \vee C(y)]$
   iff $\exists x\ \forall y\ [\neg(C(y) \vee A(x)) \vee \neg(C(y) \vee B(x))]$

(e) $\forall x\ (A(x) \text{ iff } B(x))$
iff $\neg\exists x\ [(\neg A(x) \vee \neg B(x))\ \&\ (A(x) \vee B(x))]$

(f) $\forall x\ (A(x)\ \&\ \exists y\ \neg$
$B(x,y))$ iff $\neg\exists x\ [\neg A(x) \vee \forall y\ (B(x,y)\ \&\ B(x,y))]$

5. Show that each of the following sets of sentences is inconsistent.

(a) $[[\forall x\ (M(x) \text{ iff } [(x))\ \&\ \neg Mc]\ \&\ \forall x\ ]$

(b) $[\neg Fa, \neg\exists x\ (\neg F(x) \vee \neg F(x))]$

(c) $[\forall x\ \forall y\ l(x,y) \rightarrow \neg\exists z\ T(z)$
$(V(x))(V(y))L(x,y)(((w))C(w)(w)v(3(z))T(z))$
$(\neg\forall x\ \forall y\ L(x,y) \vee \forall z\ B(z)(z)k)$
$(\neg(V(z))B(z)(z)kv\neg(3)C(w)(w))$
$\forall x\ \forall y\ [(x,y))$

(d) $[\exists x\ \forall y\ (H(x,y) \rightarrow \forall w\ \rfloor(w)(w))$
$(3(x))\neg J(x,x)$
$\neg(7(x))\neg H(x)m)$

(e) $[\forall x\ \forall y\ (G(x,y) \rightarrow Hc)$
$(3(x))Gi(x)\ \&\ \sqrt{x})((y))(V(z))L(x,y,z), Lcib$
$v\neg(Hc \vee Hc)]$

(f) $[\forall x\ [(S(x)\ \&\ B(x,x)) \rightarrow Ka(x)]$
$(\sqrt{x})(H(x)B(x,x))$
$(3(x))(S(x)\ \&\ H(x)$
$\forall x\ \neg(Ka(x)\ \&\ H(x))|$